

Dark Energy? **Backreaction?** **Inhomogeneous** **viewpoint!**

Masumi KASAI (Hirotsaki University)

2008.12.9@KEK Cosmophysics Workshop

Is our Universe really undergoing an accelerated expansion ?

Dec. 8 - 12, 2008

1

Observed SNe Ia
m-z relations
seem to require
Dark Energy (Λ).

2

Evidences of **Cosmic Acceleration**

- **Type Ia SNe**
- **CMB by WMAP**
- **etc.**

3

Evidences of **Cosmic Acceleration**

- **Type Ia SNe**
- CMB by WMAP
- etc.

4

MEASUREMENTS OF Ω AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE

S. PERLMUTTER,¹ G. ALDERING, G. GOLDBABER,¹ R. A. KNOP, P. NUGENT, P. G. CASTRO,² S. DEUSTUA, S. FABBRO,³
A. GOOBAR,⁴ D. E. GROOM, I. M. HOOK,⁵ A. G. KIM,^{1,6} M. Y. KIM, J. C. LEE,⁷ N. J. NUNES,² R. PAIN,³
C. R. PENNYPACKER,⁸ AND R. QUIMBY

Institute for Nuclear and Particle Astrophysics, E. O. Lawrence Berkeley National Laboratory, Berkeley, CA 94720

C. LIDMAN
European Southern Observatory, La Silla, Chile

R. S. ELLIS, M. IRWIN, AND R. G. MCMAHON
Institute of Astronomy, Cambridge, England, UK

P. RUIZ-LAPUENTE
Department of Astronomy, University of Barcelona, Barcelona, Spain

N. WALTON
Isaac Newton Group, La Palma, Spain

B. SCHAEFER
Department of Astronomy, Yale University, New Haven, CT

B. J. BOYLE
Anglo-Australian Observatory, Sydney, Australia

A. V. FILIPPENKO AND T. MATHESON
Department of Astronomy, University of California, Berkeley, CA

A. S. FRUCHTER AND N. PANAGIA⁹
Space Telescope Science Institute, Baltimore, MD

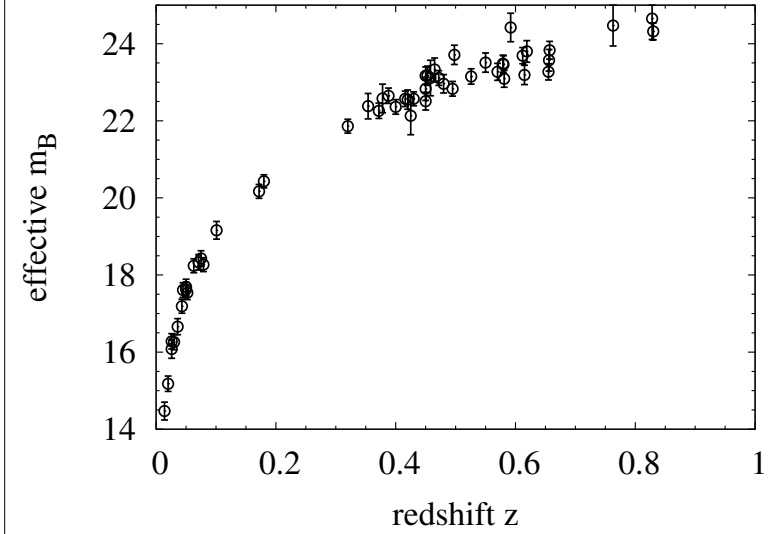
H. J. M. NEWBERG
Fermi National Laboratory, Batavia, IL

AND
W. J. COUCH

The Supernova Cosmology Project

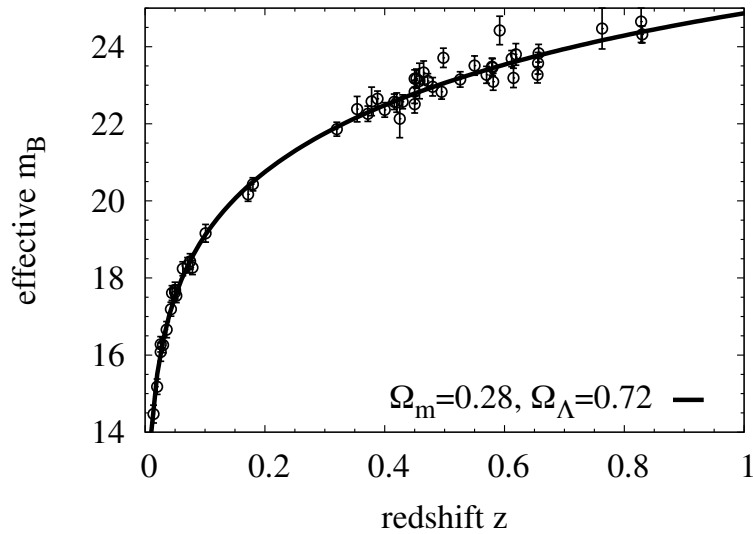
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Perlmutter et al. (1999) Primary fit C data set



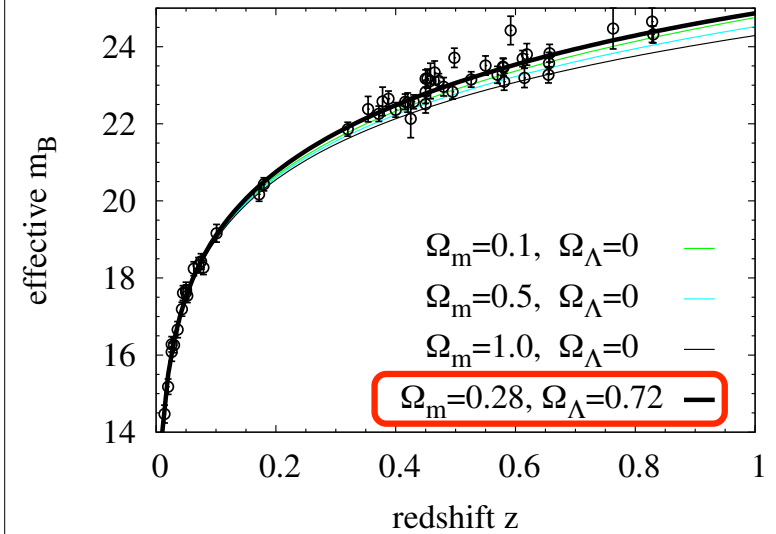
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Perlmutter et al. (1999) Primary fit C data set



7

Perlmutter et al. (1999) Primary fit C data set



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TYPE Ia SUPERNOVA DISCOVERIES AT $z > 1$ FROM THE HUBBLE SPACE TELESCOPE: EVIDENCE FOR PAST DECELERATION AND CONSTRAINTS ON DARK ENERGY EVOLUTION¹

ADAM G. RIESS,² LOUIS-GREGORY STROELGER,² JOHN TONRY,³ STEFANO CASERTANO,² HENRY C. FERGUSON,² BAHAM MOBASHER,²
PETER CHALLIS,⁴ ALEXEI V. FILIPPENKO,⁵ SAURABH JHA,⁵ WEIDONG LI,⁵ RYAN CHORNOCK,⁵ ROBERT P. KIRSHNER,⁴
BRUNO LEIBUNDGUT,⁶ MARK DICKINSON,⁷ MARIO LIVIO,² MAURO GIAVALISCO,²
CHARLES C. STEIDEL,⁷ TXITXO BENÍTEZ,⁸ AND ZLATAN TSVETANOV⁸

The Supernova Gold data set

9

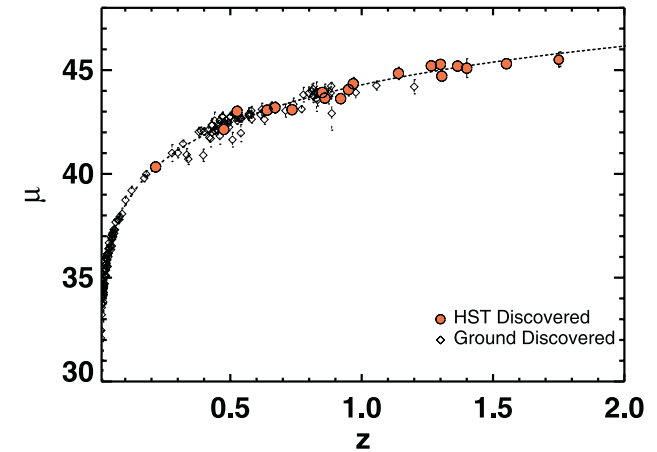


FIG. 4.—MLCS2k2 SN Ia Hubble diagram. SNe Ia from ground-based discoveries in the gold sample are shown as diamonds; *HST*-discovered SNe Ia are shown as filled symbols. Overplotted is the best fit for a flat cosmology: $\Omega_M = 0.29$, $\Omega_\Lambda = 0.71$.

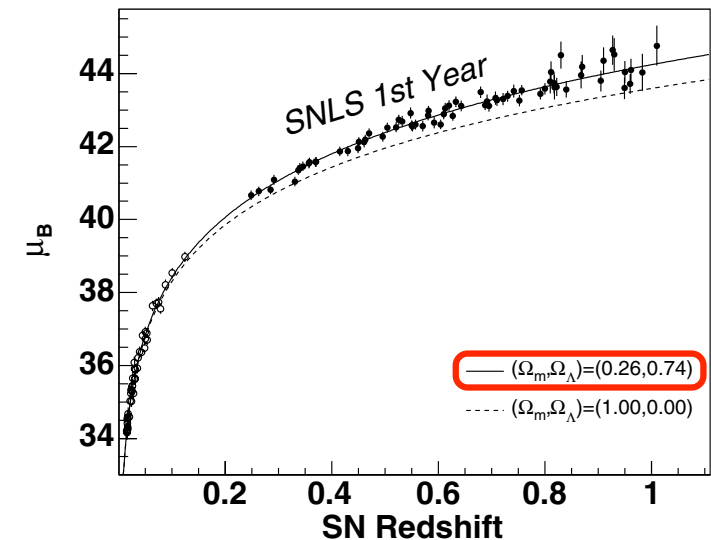
10

The Supernova Legacy Survey: measurement of Ω_M , Ω_Λ and w from the first year data set^{★,★★}

P. Astier¹, J. Guy¹, N. Regnault¹, R. Pain¹, E. Aubourg^{2,3}, D. Balam⁴, S. Basa⁵, R. G. Carlberg⁶, S. Fabbro⁷,
D. Fouchez⁸, I. M. Hook⁹, D. A. Howell⁶, H. Lafoux³, J. D. Neill⁴, N. Palanque-Delabrouille², K. Perrett⁶,
C. J. Pritcher⁴, J. Rich³, M. Sullivan⁶, R. Taulet^{1,10}, G. Aldering¹¹, P. Antilogus¹, V. Arsenijevic⁷, C. Balland^{1,2},
S. Baumont^{1,12}, J. Brondar⁹, H. Courtois¹³, R. S. Ellis¹⁴, M. Filol⁵, A. C. Gonçalves¹⁵, A. Goobar¹⁶, D. Guide¹,
D. Hardin¹, V. Lusser³, C. Lidman¹², R. McMahon¹⁷, M. Mouchet^{15,2}, A. Mourao⁷, S. Perlmutter^{11,18},
P. Ripoche⁹, C. Tao⁸, and N. Walton¹⁷

The Supernova Legacy Survey

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Cosmic Acceleration and Dark Energy

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Dark Energy

a hypothetical energy that accelerates
the cosmic expansion

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

$$w \equiv \frac{P}{\rho} < -\frac{1}{3} \Rightarrow \frac{\ddot{a}}{a} > 0$$

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Difficulties in Λ , DE

- No natural explanation for Λ of such size
- No natural candidate for **Dark Energy**

15

Then...

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Explanation of Apparent Acceleration without Dark Energy

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Inhomogeneous approach

- Tomita (2000a, 2000b, 2001, ...) local void model, **m-z relation**
- Iguchi, Nakamura, Nakao (2002) Lemaitre-Tolman-Bondi (LTB), **m-z**
- Alnes et al. (2006) LTB, **m-z** & **CMB**

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Observational indication of local void

19

THE ASTROPHYSICAL JOURNAL, 503:483–491, 1998 August 20

A LOCAL HUBBLE BUBBLE FROM TYPE Ia SUPERNOVAE?

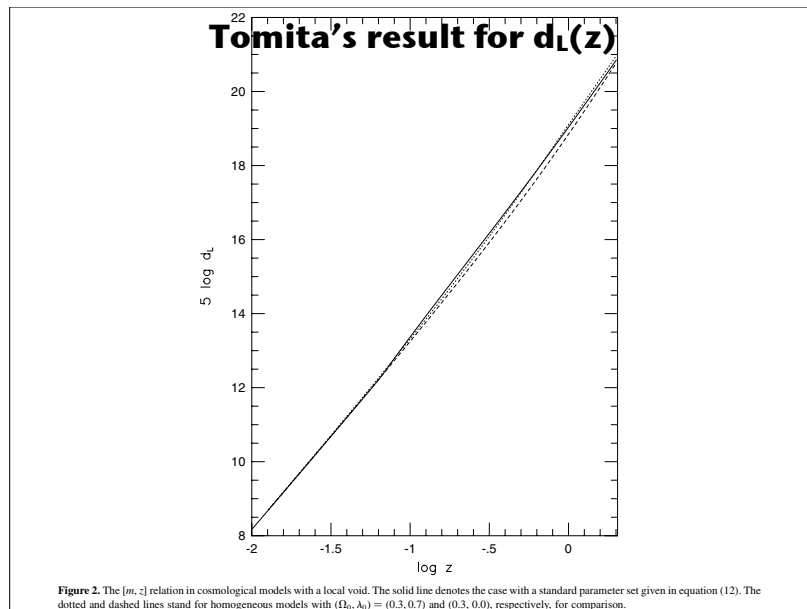
IDIT ZEHAVI,¹ ADAM G. RIESS,² ROBERT P. KIRSHNER,³ AND AVISHAI DEKEL^{1,4}

Received 1997 September 16; accepted 1998 April 1

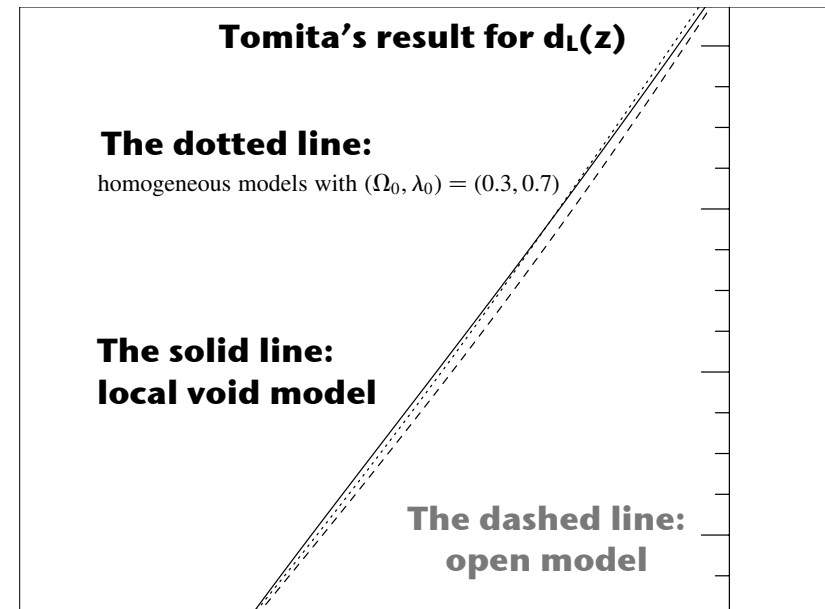
1. INTRODUCTION

Large-scale redshift surveys of galaxies show underdense regions of typical extent $\sim 50 h^{-1}$ Mpc. These “voids” appear to be bordered by dense “walls” (Kirshner et al. 1981; Huchra et al. 1983 [CfA]; Broadhurst et al. 1990; Shectman et al. 1996 [Las Campanas Redshift Survey (LCRS)]). In particular, maps of our cosmological neighborhood display the Great Wall of Coma and the Southern Wall, which appear to connect into a shell-like structure of radius $70\text{--}80 h^{-1}$ Mpc about the Local Group (Geller & Huchra 1989 [CfA2]; da Costa et al. 1994 [Southern Sky Redshift Survey 2 (SSRS2)]). The volume encompassed by this structure appears to be of lower density.

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**Essential points
have already been done
by
K. Tomita**

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**Different values
of H_0, Ω_m
here and there**

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**Tomita's local void model
with singular mass shell
might be a bit
unrealistic...**

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Progress of Theoretical Physics, Vol. 108, No. 5, November 2002

Is Dark Energy the Only Solution to the Apparent Acceleration of the Present Universe?

Hideo IGUCHI,¹ Takashi NAKAMURA^{2,*} and Ken-ichi NAKAO³

¹*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8550, Japan*

²*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

³*Department of Physics, Osaka City University, Osaka 558-8585, Japan*

(Received August 2, 2002)

Even for the observed luminosity distance $D_L(z)$, which suggests the existence of dark energy, we show that an inhomogeneous dust universe solution without dark energy is possible in general. Future observation of $D_L(z)$ for $1 \lesssim z < 1.7$ may confirm or refute this possibility.

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Progress of Theoretical Physics, Vol. 108, No. 5, November 2002

Is Dark Energy the Only Solution to the Apparent Acceleration of the Present Universe?

**The luminosity distance $D_L(z)$
in the LTB model
(as a more realistic model)
can be consistent with
observed SNe Ia data**

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PHYSICAL REVIEW D **73**, 083519 (2006)

Inhomogeneous alternative to dark energy?

Håvard Alnes,^{1,*} Morad Amarzguoui,^{2,†} and Øyvind Grøn^{1,3,‡}

¹*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*

²*Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, 0315 Oslo, Norway*

³*Oslo College, Faculty of Engineering, Cort Adelersgate 30, 0254 Oslo, Norway*

(Received 5 December 2005; published 17 April 2006)

Recently, there have been suggestions that the apparent accelerated expansion of the universe is not caused by repulsive gravitation due to dark energy, but is rather a result of inhomogeneities in the distribution of matter. In this work, we investigate the behavior of a dust-dominated inhomogeneous Lemaître-Tolman-Bondi universe model, and confront it with various astrophysical observations. We find that such a model can easily explain the observed luminosity distance-redshift relation of supernovae without the need for dark energy, when the inhomogeneity is in the form of an underdense bubble centered near the observer. With the additional assumption that the universe outside the bubble is approximately described by a homogeneous Einstein-de Sitter model, we find that the position of the first peak in the cosmic microwave background (CMB) power spectrum can be made to match the WMAP observations. Whether or not it is possible to reproduce the entire CMB angular power spectrum in an inhomogeneous model without dark energy is still an open question.

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Inhomogeneous alternative to dark energy?

Håvard Alnes,^{1,*} Morad Amarzguoui,^{2,†} and Øyvind Grøn^{1,3,‡}

**Without the need for
Dark Energy,
LTB model can explain
not only $D_L(z)$ of SNe Ia
but also CMB**

33

But...

34

**Previous works
depend on
simplified
toy models.**

35

Psychological barrier

**Such toy models can
really describe
our real world?**

36

**How to describe
the effects of inhomogeneity
without depending
specific toy models?**

37

**Note:
nonlinear
Backreaction of
inhomogeneities?**

38

**Note:
nonlinear
Backreaction of
inhomogeneities?**

NO!

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Prog. Theor. Phys. Vol. 115, No. 4, April 2006, Letters

**Toward a No-Go Theorem for an Accelerating Universe
through a Nonlinear Backreaction**

Masumi KASAI,^{1,*} Hideki ASADA^{1,**} and Toshifumi FUTAMASE^{2,***}

¹*Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan*

²*Astronomical Institute, Tohoku University, Sendai 980-8578, Japan*

Although our work does not give a complete proof, it strongly suggests the following no-go theorem: No cosmic acceleration occurs as a result of the nonlinear backreaction via averaging.

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Menu

I. Averaging

II. Backreaction

**III. Inhomogeneous
viewpoint**

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Menu

I. Averaging

II. Backreaction

III. Inhomogeneous
viewpoint

42

**Once upon a time,
there were active
discussions about...**

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**cosmic acceleration
by nonlinear
backreaction?**

44

Papers

with **positive** conclusions for the backreaction (2005 ~)

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05-08

PRESS RELEASE

March 16, 2005

Italian, US cosmologists present alternate explanation for accelerating expansion of the universe: Was Einstein right when he said he was wrong?

Why is the universe expanding at an accelerating rate, spreading its contents over ever greater dimensions of space? An original solution to this puzzle, certainly the most fascinating question in modern cosmology, was put forward by four theoretical physicists, Edward W. Kolb of the U.S. Department of Energy's Fermi National

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Primordial inflation explains why the universe is accelerating today

Edward W. Kolb*

Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500, USA
and Department of Astronomy and Astrophysics, Enrico Fermi Institute,
University of Chicago, Chicago, Illinois 60637-1433 USA

Sabino Matarrese†

Dipartimento di Fisica "G. Galilei," Università di Padova,
and INFN, Sezione di Padova, via Marzolo 8, Padova I-35131, Italy

Alessio Notari‡

Physics Department, McGill University, 3600 University Road, Montréal, QC, H3A 2T8, Canada

Antonio Riotto§

INFN, Sezione di Padova, via Marzolo 8, I-35131, Italy
(Dated: March 3, 2006)

We propose an explanation for the present accelerated expansion of the universe that does not invoke dark energy or a modification of gravity and is firmly rooted in inflationary cosmology.

arXiv:hep-th/0503117 v1 14 Mar 2005

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On cosmic acceleration without dark energy

Edward W. Kolb*

Particle Astrophysics Center, Fermi National Accelerator Laboratory,
Batavia, Illinois 60510-0500, USA
and Department of Astronomy and Astrophysics, Enrico Fermi Institute,
University of Chicago, Chicago, Illinois 60637-1433 USA

Sabino Matarrese†

Dipartimento di Fisica "G. Galilei" Università di Padova,
INFN Sezione di Padova, via Marzolo 8, Padova I-35131, Italy

Antonio Riotto‡

INFN Sezione di Padova, via Marzolo 8, I-35131, Italy

Abstract

We elaborate on the proposal that the observed acceleration of the Universe is the result of the backreaction of cosmological perturbations, rather than the effect of a negative-pressure dark-energy fluid or a modification of general relativity. Through the effective Friedmann equations describing an inhomogeneous Universe after smoothing, we demonstrate that acceleration in our

arXiv:astro-ph/0506534 v1

48

Effect of inhomogeneities on the luminosity distance-redshift relation: Is dark energy necessary in a perturbed universe?

Enrico Barausse*

*Dipartimento di Fisica "G. Galilei," Università di Padova, via Marzolo 8, Padova I-35131, Italy,
and SISSA/ISAS, via Beirut 4, I-34014 Trieste, Italy*

Sabino Matarrese†

*Dipartimento di Fisica "G. Galilei," Università di Padova, via Marzolo 8, Padova I-35131, Italy,
and INFN, Sezione di Padova, via Marzolo 8, Padova I-35131, Italy*

Antonio Riotto‡

INFN, Sezione di Padova, via Marzolo 8, I-35131, Italy
(Received 12 January 2005; published 31 March 2005)

The luminosity distance-redshift relation is one of the fundamental tools of modern cosmology. We compute the luminosity distance-redshift relation in a perturbed flat matter-dominated Universe, taking into account the presence of cosmological inhomogeneities up to second order in perturbation theory. Cosmological observations implementing the luminosity distance-redshift relation tell us that the Universe is presently undergoing a phase of accelerated expansion. This seems to call for a mysterious Dark Energy component with negative pressure. Our findings suggest that the need of a Dark Energy fluid may be challenged once a realistic inhomogeneous Universe is considered and that an accelerated expansion may be consistent with a matter-dominated Universe.

Accelerating Universe via Spatial Averaging

Yasusada Nambu* and Masayuki Tanimoto†

Department of Physics, Graduate School of Science, Nagoya University, Chikusa, Nagoya 464-8602, Japan
(Dated: June 14, 2005)

We present a model of an inhomogeneous universe that leads to accelerated expansion after taking spatial averaging. The model universe is the Tolman-Bondi solution of the Einstein equation and contains both a region with positive spatial curvature and a region with negative spatial curvature. We find that after the region with positive spatial curvature begins to re-collapse, the deceleration parameter of the spatially averaged universe becomes negative and the averaged universe starts accelerated expansion. We also discuss the generality of the condition for accelerated expansion of the spatially averaged universe.

arXiv:gr-qc/0507057

Papers with **negative** conclusions (2005 ~)

Can superhorizon cosmological perturbations explain the acceleration of the universe?

Christopher M. Hirata^{1,*} and Uroš Seljak^{1,2}

¹*Department of Physics, Jadwin Hall, Princeton University, Princeton, New Jersey 08544, USA*

²*International Center for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy*

(Dated: March 27, 2005)

We investigate the recent suggestions by Barausse et al. (astro-ph/0501152) and Kolb et al. (hep-th/0503117) that the acceleration of the universe could be explained by large superhorizon fluctuations generated by inflation. We show that no acceleration can be produced by this mechanism. We begin by showing how the application of Raychaudhuri equation to inhomogeneous cosmologies results in several "no go" theorems for accelerated expansion. Next we derive an exact solution for a specific case of initial perturbations, for which application of the Kolb et al. expressions leads to an acceleration, while the exact solution reveals that no acceleration is present. We show that the discrepancy can be traced to higher order terms that were dropped in the Kolb et al. analysis. We proceed with the analysis of initial value formulation of general relativity to argue that causality severely limits what observable effects can be derived from superhorizon perturbations. By constructing a Riemann normal coordinate system on initial slice we show that no infrared divergence terms arise in this coordinate system. Thus any divergences found previously can be eliminated by a local rescaling of coordinates and are unobservable. We perform an explicit analysis of the variance of the deceleration parameter for the case of single field inflation using usual coordinates and show that the infrared divergent terms found by Barausse et al. and Kolb et al. cancel against several additional terms not considered in their analysis. Finally, we argue that introducing isocurvature perturbations does not alter our conclusion that the accelerating expansion of the universe cannot be explained by superhorizon modes.

arXiv:astro-ph/0503582 v1 27 Mar 2005

er M. Hirata^{†, *} and Uros Seljak^{†, *}
 tall, Princeton University, Princeton, New Jersey
 eoretical Physics, Strada Costiera 11, 34014 Trieste
 (Dated: March 27, 2005)

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We show that no acceleration can be produced

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 while the exact solution reveals that no accelerati
 be traced to higher order terms that were droppe
 the analysis of initial value formulation of genera
 what observable effects can be derived from sum

53

arXiv:gr-qc/0509108 v3

Can the Acceleration of Our Universe Be
Explained by the Effects of Inhomogeneities?

Akihiro Ishibashi[†] and Robert M. Wald^{††}

*Enrico Fermi Institute[†] and Department of Physics[†]
 The University of Chicago, Chicago, IL 60637, USA*

November 14, 2005

Abstract

No. It is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or “dark energy.” We point out that our universe appears to be

54

November

Abst

No. It is simply not plausible

within the context of general relativ
 homogeneities in our universe, witho
 stant or “dark energy.” We point

55

Which is true?
Backreaction
accelerates? or not?

**We shall clear up
the confusion!**

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Basic Idea of the standard cosmology

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The Cosmological Principle

- **The universe is spatially homogeneous and isotropic**
 - **Matter distribution is smooth and homogeneous**
- i.e., the Friedmann model**

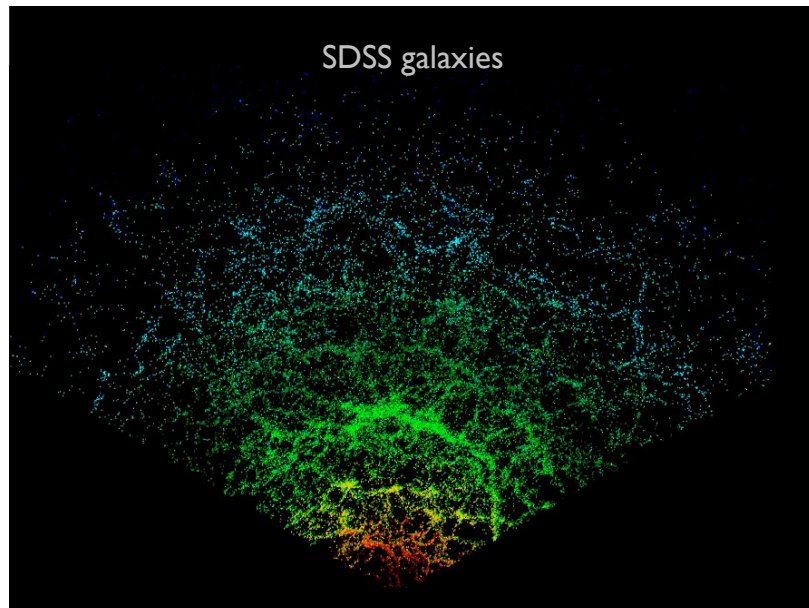
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However,

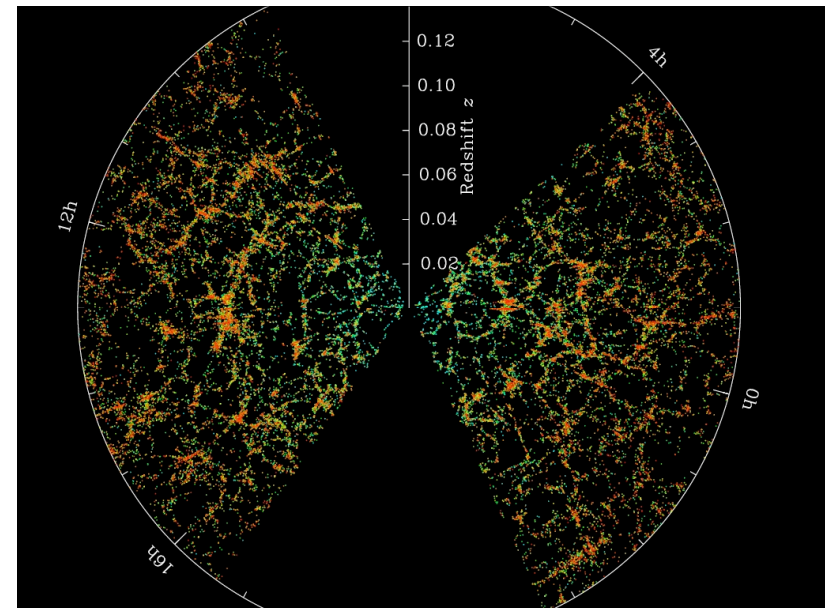
59

**The actual universe is
highly
inhomogeneous.**

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**Why
the universe is
believed to be
homogeneous and
isotropic?**

63

**an implicit
agreement
is...**

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OK, the universe is **locally**
inhomogeneous,
but
the **averaged** behavior is
described by
the Friedmann model.

65

OK, the universe is **locally**
inhomogeneous,
but
the **averaged** behavior is
described by
the Friedmann model.



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**What is
Friedmann
on average?**

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average density

$$\rho_b \equiv \langle \rho \rangle$$

scale factor

$$\frac{\dot{a}}{a} \equiv \frac{1}{3} \frac{\dot{V}}{V}$$

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averaging the Einstein eq.

$$\langle G_{\mu\nu} \rangle = 8\pi G \langle T_{\mu\nu} \rangle$$

\Downarrow

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_b + \Delta_x$$

69

If $\Delta_x = 0$, then

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_b$$

**a is driven merely by the mean density,
collectively by the clumps of matter.**

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**In general, however,
due to the
nonlinearity of
the Einstein eq. ...**

71

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_b + \Delta_x$$

**another source driving
the cosmic expansion**

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$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_b \boxed{+\Delta_x}$$

Δ_x is
the nonlinear backreaction
of inhomogeneities

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Menu

I. Averaging

II. Backreaction

III. Inhomogeneous
viewpoint

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The backreaction
accelerates
the universe?

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Pioneering works
on
the backreaction
(in 1990s and before)

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Futamase's scheme

T. Futamase, Phys. Rev. Lett. D 61, 2175 (1988)

T. Futamase, Phys. Rev. D 53, 681 (1996)

The metric:

$$ds^2 = -(1+2\phi(x)) dt^2 + a^2(t)(1-2\phi(x)) \delta_{ij} dx^i dx^j$$

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The averaging procedure:

$$\langle\langle \rho \rangle\rangle := \frac{1}{V} \int_D \rho d^3x$$

$$V := \int_D d^3x$$

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The averaged Einstein eq.:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\langle\langle \rho \rangle\rangle + \langle\langle \rho a^2 v^2 \rangle\rangle \right) + \frac{5}{3a^2} \langle\langle \phi^i \phi_{,i} \rangle\rangle$$

$$> \frac{8\pi G}{3} \langle\langle \rho \rangle\rangle$$

speed up!

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In the comoving synchronous gauge...

M. Kasai, Phys. Rev. D 52, 5605 (1995)

The metric:

$$ds^2 = -dt^2 + a^2(t) \left[\left(1 + \frac{20}{9} \Psi(x) \right) \delta_{ij} + 2a(t) \Psi_{,ij} \right] dx^i dx^j$$

80

The averaging procedure:

$$\langle \rho \rangle := \frac{1}{V_D} \int_D \rho \sqrt{{}^{(3)}g} d^3x$$

$$V_D := \int_D \sqrt{{}^{(3)}g} d^3x$$

$$\frac{\dot{a}_D}{a_D} := \frac{1}{3} \frac{\dot{V}_D}{V_D}$$

81

The averaged Einstein eq.:

$$\left(\frac{\dot{a}_D}{a_D} \right)^2 = \frac{8\pi G}{3} \langle \rho \rangle - \frac{1}{3a_D^2} \left\langle \frac{100}{81} \Psi^{,i} \Psi_{,i} \right\rangle$$
$$< \frac{8\pi G}{3} \langle \rho \rangle$$

speed down

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**In the previous works,
the effects were already
controversial (?)**

- **positive? negative?**
- **gauge dependence?**
- **averaging procedure ambiguity?**

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**Note on
the achievements
in 1990s**

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Both agree with the followings:

- the backreaction does not act as Λ .
- the backreaction behaves as a curvature term, $\propto a^{-2}$.

85

Both agree with the followings:

- the backreaction does not act as Λ .
- the backreaction behaves as a curvature term, $\propto a^{-2}$.

One disagreement in 1990s is:

- **positive/negative** contribution to \dot{a}^2

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**The backreaction
comes again
in the 21st century.
But, people often
writes...**

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an example: arXiv:astro-ph/0601699 v1 30 Jan 2006

terious dark energy to mimic Q . Of course, the averaging process is neither trivial nor unambiguous, but it is the art of physics to master it. Fortunately, there is an averaging formalism, developed mainly by Thomas Buchert[16, 17, 18, 19], which can easily be adapted to our LTB patch, having the same mass as the the FRW sphere cut out of it. In this formalism the space-average of any function $f(t, r)$ is defined by

$$\langle f \rangle \equiv \frac{1}{V_D} \int_D dV f, \quad (7)$$

where dV is the proper volume element of the 3-dimensional domain D of the patch we are considering and V_D is its volume. It has been shown[16, 20] that in

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Prof. T. Futamase

**“an averaging formalism,
developed mainly by
Thomas Buchert? Hm?”**

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another example: arXiv:0809.1183v3 [gr-qc] 24 Oct 2008

B. Buchert averaging

Buchert's averaging scheme [17] is based on the starting point that, in the case of an energy-momentum tensor for irrotational dust particles in the presence of inhomogeneities, one can choose Gaussian normal coordinates

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Prof. T. Futamase

**“Buchert averaging? Hm?”
“Who's the pioneer?
We should write a definitive paper.”**

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Prog. Theor. Phys. Vol. 115, No. 4, April 2006, Letters

Toward a No-Go Theorem for an Accelerating Universe through a Nonlinear Backreaction

Masumi KASAI,^{1,*} Hideki ASADA^{1,**} and Toshifumi FUTAMASE^{2,***}

¹*Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan*

²*Astronomical Institute, Tohoku University, Sendai 980-8578, Japan*

(Received January 6, 2006)

The backreaction of nonlinear inhomogeneities to the cosmic expansion is re-analyzed in the framework of general relativity. Apparent discrepancies regarding the effect of the nonlinear backreaction, which exist among the results of previous works in different gauges, are resolved. By defining the spatially averaged matter energy density as a conserved quantity in the large comoving volume, it is shown that the nonlinear backreaction neither accelerates nor decelerates the cosmic expansion in a matter-dominated universe. The present result in the Newtonian gauge is consistent with the previous results obtained in the comoving synchronous gauge. Although our work does not give a complete proof, it strongly suggests the following no-go theorem: No cosmic acceleration occurs as a result of the nonlinear backreaction via averaging.

So we wrote a paper.

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Averaging is a delicate procedure.

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Approximation Scheme for Constructing a Clumpy Universe in General Relativity

Toshifumi Futamase

Department of Physics, Faculty of Sciences, Hiroshima University, Hiroshima, 036, Japan,^(a) and

(background) 3-space average

The spatial average over a volume V is defined as usual,

$$\langle Q \rangle = V^{-1} \int_V Q dV, \quad (7)$$

where dV is the invariant volume element in the back-ground space.

Since the spatial average of the line element takes the following form,

$$\langle ds^2 \rangle = a^2 \{ -d\eta^2 + (\delta_{ij} + \langle \bar{h}_{ij} \rangle) dx^i dx^j \}, \quad (12)$$

average of tensors

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Averaging of a locally inhomogeneous realistic universe

Toshifumi Futamase

Astronomical Institute, Tohoku University, Sendai, 980-77, Japan

(Received 7 June 1995)

Isaacson averaging version

IV. DERIVATION OF FRW GEOMETRY BY AVERAGING

In the previous section we obtained our basic equations as perturbed equations around the background FRW model. The perturbed quantities are classified as scalar, vector, and tensor with respect to the background spatial geometry. Thus it would be natural to introduce the following averaging over the background spatial hypersurface according to Isaacson [10]:

$$\langle Q_{ij}(x) \rangle = \int g_i^{k'}(x, x') g_j^{l'}(x, x') Q_{k'l'}(x') f(x, x') d^3 x', \quad (36)$$

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Construction of Inhomogeneous Universes Which Are Friedmann-Lemaître-Robertson-Walker on Average

Masumi Kasai

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, 8046 Garching bei München, Germany and Department of Physics, Faculty of Science, Hiroshima University, 3 Bunkyo-cho, Hiroshima, 036 Japan^(a)

(Received 29 May 1992)

The understanding of our Universe is based on the working hypothesis that the homogeneous and isotropic models give a successful description on a very large scale, despite the nonlinear inhomogeneity of the matter distribution in the present Universe. We consider the compatibility problem between the overall homogeneity and isotropy and the local inhomogeneity. A scheme to construct inhomogeneous irrotational dust universes which are homogeneous and isotropic on average is shown in the framework of general relativity; they represent "relativistic pancake solutions" analogous to those in Newtonian cosmology.

$$\rho_b = \langle \rho \rangle \equiv \lim_{V \rightarrow \Sigma_t} \frac{1}{\int_V [\det(g_{ij})]^{1/2} d^3 x} \int_V \rho [\det(g_{ij})]^{1/2} d^3 x, \quad V \subset \Sigma_t. \quad (6)$$

3-invariant spatial averaging of the 3-scalar has been introduced.

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Age of the universe: Influence of the inhomogeneities on the global expansion factorHeinz Russ,¹ Michael H. Soffel,² Masumi Kasai,^{3,4} and Gerhard Börner⁴¹*Institut für Astronomie und Astrophysik, Universität Tübingen, Auf der Morgenstelle 10, D-72076 Tübingen, Germany***spatial averaging, time derivative, and commutation rule in General Relativity**

We introduce the averaging procedure [22]

$$\langle A \rangle = \frac{1}{V} \int_V A \sqrt{g} d^3x, \quad (2.10)$$

APPENDIX A: COMMUTATION RULE

The time derivative of an averaged quantity reads

$$\frac{d}{dt} \langle A \rangle = -\frac{\dot{V}}{V} \langle A \rangle + \frac{1}{V} \int_V (\dot{A} \sqrt{g} + A \dot{\sqrt{g}}) d^3x. \quad (A1)$$

This leads to the commutation rule [22,16,17]

$$\frac{d}{dt} \langle A \rangle - \langle \dot{A} \rangle = -\langle \theta \rangle \langle A \rangle + \langle A \theta \rangle, \quad (A2)$$

**Averaging formalism,
developed mainly by
Thomas Buchert?**

**Anyway,
Let us clear up
the confusion.**

General Setup**The metric**

$$ds^2 = -(N dt)^2 + \gamma_{ij} dx^i dx^j$$

(gauge not yet fixed)

The extrinsic curvature

$$K^i_j = \frac{1}{2N} \gamma^{ik} \dot{\gamma}_{kj}$$

(represents the 3-dim. deformation)

General Setup

3-dim. volume V

$$V = \int_D \sqrt{\det(\gamma_{ij})} d^3x$$

(D: a compact domain on $t=\text{const.}$ slice)

the scale factor $a(t)$

$$3 \frac{\dot{a}}{a} \equiv \frac{\dot{V}}{V}$$

(defined from the volume expansion rate)

101

General Setup

The averaging procedure

$$\langle A \rangle \equiv \frac{1}{V} \int_D A \sqrt{\gamma} d^3x$$

\Downarrow

$$3 \frac{\dot{a}}{a} = \langle NK^i_i \rangle$$

The deviation from a uniform Hubble flow

$$V^i_j \equiv NK^i_j - \frac{\dot{a}}{a} \delta^i_j$$

102

General Setup

The averaged Einstein eq.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \langle T_{00} \rangle - \frac{1}{6} \langle N^2 {}^{(3)}R \rangle - \frac{1}{6} \langle (V^i_i)^2 - V^i_j V^j_i \rangle$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \langle T_{00} + N^2 T_{ii} \rangle + \frac{1}{3} \langle (V^i_i)^2 - V^i_j V^j_i \rangle + \frac{1}{3} \langle NN^{|l}_i + \dot{N}K^i_i \rangle$$

103

**Up to this point,
the treatment is fully general.
(Gauge is not yet fixed.
cf. “Buchert formalism”)**

How to evaluate it?

104

Solving by iteration

Putting the linearized solution (in the Newtonian gauge)

$$ds^2 = -(1 + 2\phi(x))dt^2 + a^2(1 - 2\phi(x))\delta_{ij} dx^i dx^j$$

into the R.H.S. ... \Downarrow

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\langle T_{00}\rangle + \frac{1}{a^2}\langle\phi_{,i}\phi_{,i}\rangle$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\langle T_{00} + \rho_b a^2 v^2\rangle - \frac{1}{3a^2}\langle\phi_{,i}\phi_{,i}\rangle$$

105

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\langle T_{00}\rangle + \frac{1}{a^2}\langle\phi_{,i}\phi_{,i}\rangle > \frac{8\pi G}{3}\langle T_{00}\rangle$$

The backreaction

increases

the expansion rate?

106

No.

Not necessarily.

107

Check the average density $\bar{\rho}$
should obey

$$\bar{\rho} a^3 = \text{const.}$$

(Otherwise, the averaged spacetime is not compatible with Friedmann.)

Clearly, $\langle T_{00}\rangle \neq \bar{\rho}$

108

In order to guarantee

$$\dot{\bar{\rho}} + 3\frac{\dot{a}}{a}\bar{\rho} = 0,$$

it is uniquely determined

$$\bar{\rho} \equiv \langle T_{00} + \rho_b a^2 v^2 \rangle + \frac{1}{4\pi G a^2} \langle \phi_{,i} \phi_{,i} \rangle$$

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The averaged Einstein equation
should be written in terms of

$$\dots = \bar{\rho} + \boxed{\text{additional contributions}}$$



“the backreaction”

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Summary

111

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\bar{\rho} - \frac{1}{9a^2}\langle \phi_{,i} \phi_{,i} \rangle$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\bar{\rho}$$

- The backreaction does not change the acceleration \ddot{a} .

112

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\bar{\rho} - \frac{1}{9a^2}\langle\phi_{,i}\phi_{,i}\rangle < \frac{8\pi G}{3}\bar{\rho}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\bar{\rho}$$

- The backreaction decreases \dot{a}/a .

113

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\bar{\rho} - \frac{1}{9a^2}\langle\phi_{,i}\phi_{,i}\rangle$$

- The backreaction term behaves as a (small) positive curvature term.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_b - \frac{k}{a^2}$$

(cf. Friedmann equation)

114

Furthermore, the results are

- consistent with other (comoving) gauge calculations.
- not dependent on the definition of the averaging.

115

No Go Theorem

116

Assumption 1:

**The universe after
decoupling was slightly
perturbed Friedmann.
(Supported by CMB obs.)**

117

Assumption 2:

**Perturbation theory well
describes
the inhomogeneous metric.
(Even for $\delta > 1$)**

cf. Futamase's approximation scheme,
the relativistic Zeldovich approximation (Kasai), etc.

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Then...

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**Nonlinear backreaction
neither **accelerate**
nor **decelerate**
the cosmic expansion.**

120

arXiv.org > astro-ph > astro-ph/0602506 Search for (Help | Advan
All paper

Astrophysics, abstract astro-ph/0602506

From: Masumi Kasai [[view email](#)]
Date: Thu, 23 Feb 2006 07:51:14 GMT (31kb)

Toward a No-go Theorem for Accelerating Universe by Nonlinear Backreaction

Authors: [Masumi Kasai](#), [Hideki Asada](#), [Toshifumi Futamase](#)
Comments: 6 pages (PTPTeX); accepted for publication in Prog. Theor. Phys

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**One more thing
about
averaging**

122

**Light feels
local metric,
not the
averaged one.**

123

**Average of light
propagation in
inhomogeneous spacetime
is not equal to
light propagation in
the averaged spacetime.**

124

**Attempt to explain
apparent
acceleration
without
Dark Energy**

125

Inhomogeneous approach

- **Tomita (2000a, 2000b, 2001, ...)**
local void model
- **Iguchi, Nakamura, Nakao (2002)**
Lemaitre-Tolman-Bondi
- **Alnes et al. (2006)**
Lemaitre-Tolman-Bondi

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**Simplified
toy models
to represent
the actual universe.**

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Enough about toy models

128

What's the observed data telling us about inhomogeneity?

129

The standard analysis in the Friedmann model

130

Summary of SNe Ia $m-z$ relation by Perlmutter et al. (1999)

131

THE ASTROPHYSICAL JOURNAL, 517:565–586, 1999 June 1

MEASUREMENTS OF Ω AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE

S. PERLMUTTER,¹ G. ALDERING, G. GOLDBABER,¹ R. A. KNOP, P. NUGENT, P. G. CASTRO,² S. DEUSTUA, S. FABBRO,³
A. GOOBAR,⁴ D. E. GROOM, I. M. HOOK,⁵ A. G. KIM,^{1,6} M. Y. KIM, J. C. LEE,⁷ N. J. NUNES,² R. PAIN,³
C. R. PENNYPACKER,⁸ AND R. QUIMBY
Institute for Nuclear and Particle Astrophysics, E. O. Lawrence Berkeley National Laboratory, Berkeley, CA 94720

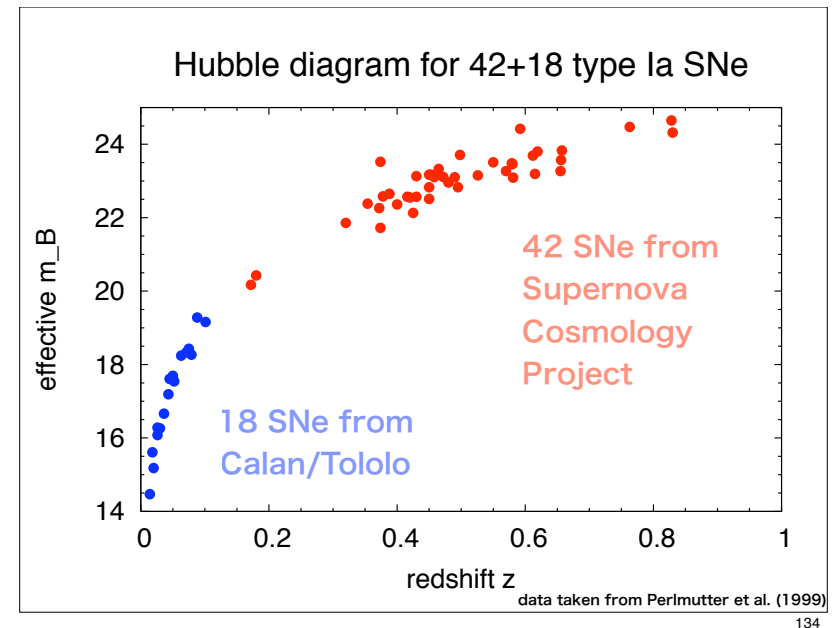
ABSTRACT

We report measurements of the mass density, Ω_M , and cosmological-constant energy density, Ω_Λ , of the universe based on the analysis of 42 type Ia supernovae discovered by the Supernova Cosmology Project. The magnitude-redshift data for these supernovae, at redshifts between 0.18 and 0.83, are fitted jointly with a set of supernovae from the Calán/Tololo Supernova Survey, at redshifts below 0.1, to yield values for the cosmological parameters. All supernova peak magnitudes are standardized using a SN Ia light-curve width-luminosity relation. The measurement yields a joint probability distribution of the cosmological parameters that is approximated by the relation $0.8\Omega_M - 0.6\Omega_\Lambda \approx -0.2 \pm 0.1$ in the region of interest ($\Omega_M \lesssim 1.5$). For a flat ($\Omega_M + \Omega_\Lambda = 1$) cosmology we find $\Omega_M^{\text{flat}} = 0.28^{+0.09}_{-0.08}$ (1 σ statistical) $^{+0.05}_{-0.04}$ (identified systematics). The data are strongly inconsistent with a $\Lambda = 0$ flat cosmology, the simplest

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Observational data

133



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m-z relation to fit the data

135

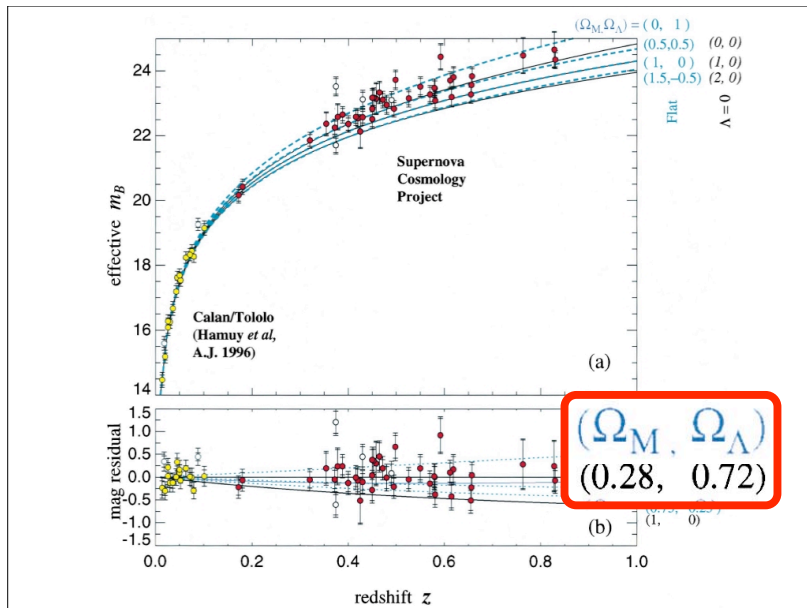
$$m = M - 5 + 5 \log_{10} D_L(z)$$

luminosity distance $D_L(z)$

$$D_L(z) = \frac{c(1+z)}{H_0 \sqrt{1-\Omega_m-\Omega_\Lambda}} \times \sinh \left(\sqrt{1-\Omega_m-\Omega_\Lambda} \int_0^z \frac{dz'}{\sqrt{(1+\Omega_m z')(1+z')^3 - z'(2+z')\Omega_\Lambda}} \right)$$

$D_L(z)$ is a (bit complicated) function of z with the constant parameters H_0 , Ω_m , Ω_Λ .

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Fit Results of Perlmutter et al. (1999)

TABLE 3
FIT RESULTS

Fit	N	χ^2	DOF	Ω_M^{flat}	$P(\Omega_\Lambda > 0)$	Best Fit (Ω_M, Ω_Λ)	Fit Description
Inclusive Fits:							
A	60	98	56	$0.29^{+0.09}_{-0.08}$	0.9984	0.83, 1.42	All supernovae
B	56	60	52	$0.26^{+0.09}_{-0.08}$	0.9992	0.85, 1.54	Fit A, but excluding two residual outliers and two stretch outliers
Primary fit:							
C	54	56	50	$0.28^{+0.09}_{-0.08}$	0.9979	0.73, 1.32	Fit B, but also excluding two likely reddened
Comparison Analysis Techniques:							
D	54	53	51	$0.25^{+0.10}_{-0.09}$	0.9972	0.76, 1.48	No stretch correction ^a
E	53	62	49	$0.29^{+0.12}_{-0.10}$	0.9894	0.35, 0.76	Bayesian one-sided extinction corrected ^b
Effect of Reddest Supernovae:							
F	51	59	47	$0.26^{+0.09}_{-0.08}$	0.9991	0.85, 1.54	Fit B supernovae with colors measured
G	49	56	45	$0.28^{+0.09}_{-0.08}$	0.9974	0.73, 1.32	Fit C supernovae with colors measured
H	40	33	36	$0.31^{+0.11}_{-0.09}$	0.9857	0.16, 0.50	Fit G, but excluding seven next reddest and two next faintest high-redshift supernovae
Systematic Uncertainty Limits:							
I	54	56	50	$0.24^{+0.09}_{-0.08}$	0.9994	0.80, 1.52	Fit C with +0.03 mag systematic offset
J	54	57	50	$0.33^{+0.10}_{-0.09}$	0.9912	0.72, 1.20	Fit C with -0.04 mag systematic offset
Clumped Matter Metrics:							
K	54	57	50	$0.35^{+0.12}_{-0.10}$	0.9984	2.90, 2.64	Empty beam metric ^c
L	54	56	50	$0.34^{+0.10}_{-0.09}$	0.9974	0.94, 1.46	Partially filled beam metric

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TABLE 3
FIT RESULTS

Ω_M^{flat}	$P(\Omega_\Lambda > 0)$	Best Fit (Ω_M, Ω_Λ)	
$0.29^{+0.09}_{-0.08}$	0.9984	0.83, 1.42	All supernovae
$0.26^{+0.09}_{-0.08}$	0.9992	0.85, 1.54	Fit A, but excluding two
$0.28^{+0.09}_{-0.08}$	0.9979	0.73, 1.32	Fit B, but also excluding

$\Omega_m = 0.28, \Omega_\Lambda = 0.72$ is Not the Best Fit...

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Cosmic Acceleration and Dark Energy

140

Results of Perlmutter et al. (1999)

$$\Omega_m = 0.28, \Omega_\Lambda = 0.72$$

(Best Fit: $\Omega_m = 0.73, \Omega_\Lambda = 1.32$)

\Downarrow

$$\frac{\ddot{a}}{a}\bigg|_{t_0} = H_0^2 \left(-\frac{1}{2}\Omega_m + \Omega_\Lambda \right)$$

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Menu

I. Averaging

II. Backreaction

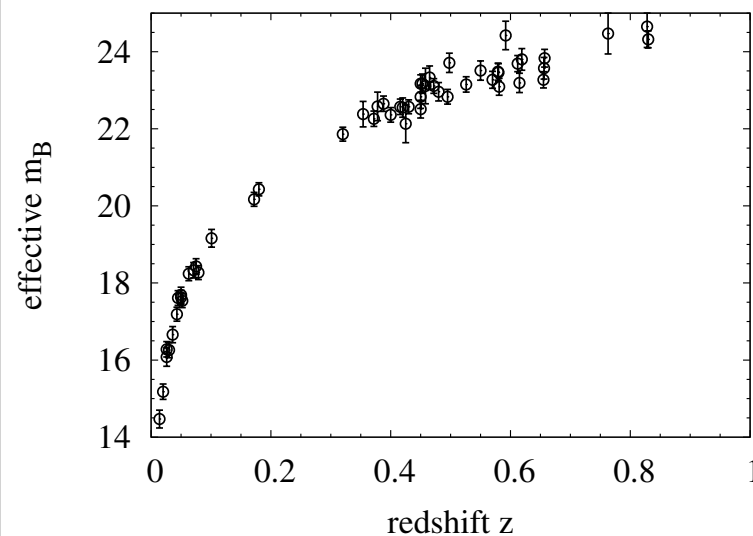
III. Inhomogeneous viewpoint

142

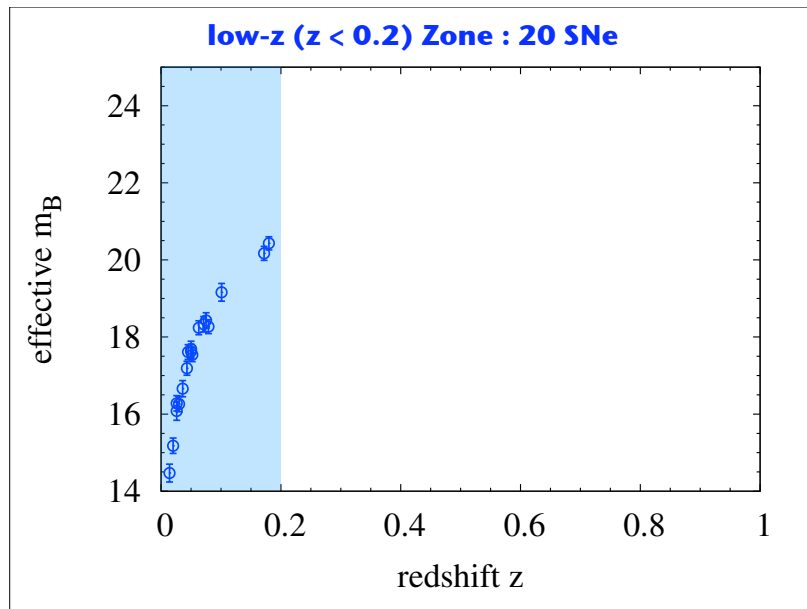
Re-analysis by two-zone model

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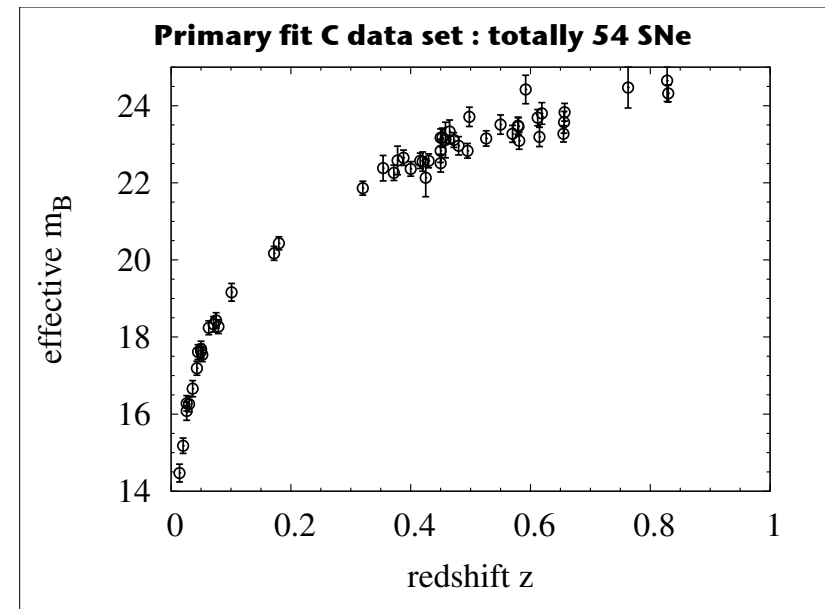
Primary fit C data set : totally 54 SNe



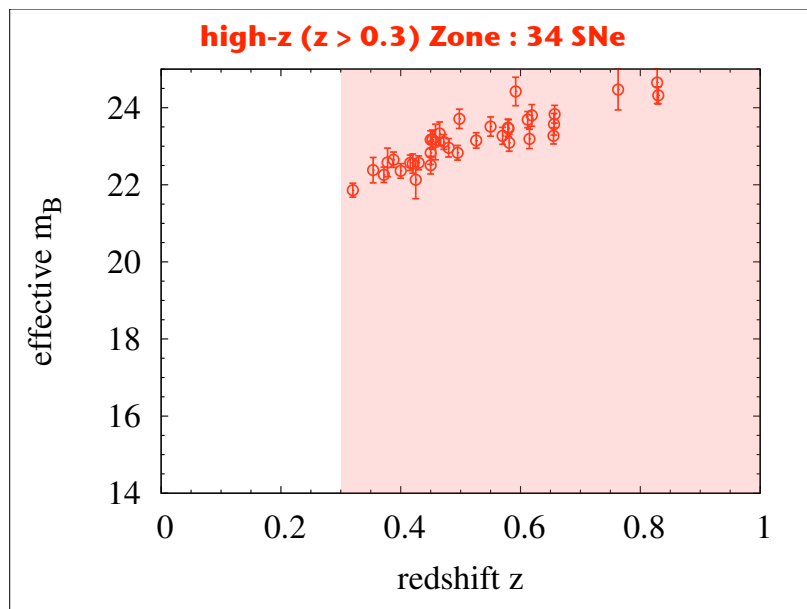
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**The m - z relation
to fit the data**

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$$D_L(z) \equiv \frac{c}{H_0} d(z)$$

$$m = M_{\text{abs}} - 5 + 5 \log_{10} D_L(z) \\ = \mathcal{M} + 5 \log_{10} d(z)$$

$$\mathcal{M} \equiv M - 5 + 5 \log_{10} \frac{c}{H_0}$$

\mathcal{M} : “the magnitude zero-point” or
“ H_0 -free absolute magnitude”

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The m - z relation

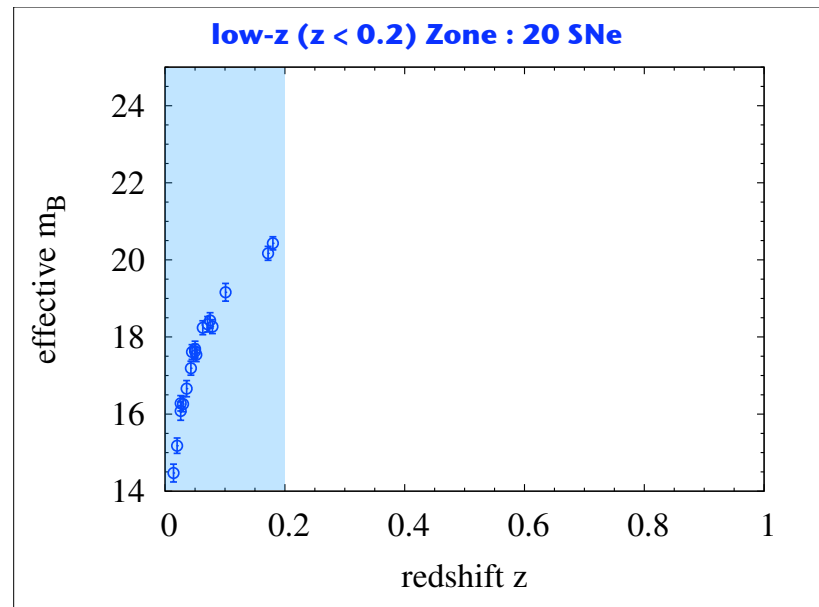
$$m = \mathcal{M} + 5 \log_{10} d(z, \Omega_m)$$

fitting parameters are \mathcal{M}, Ω_m

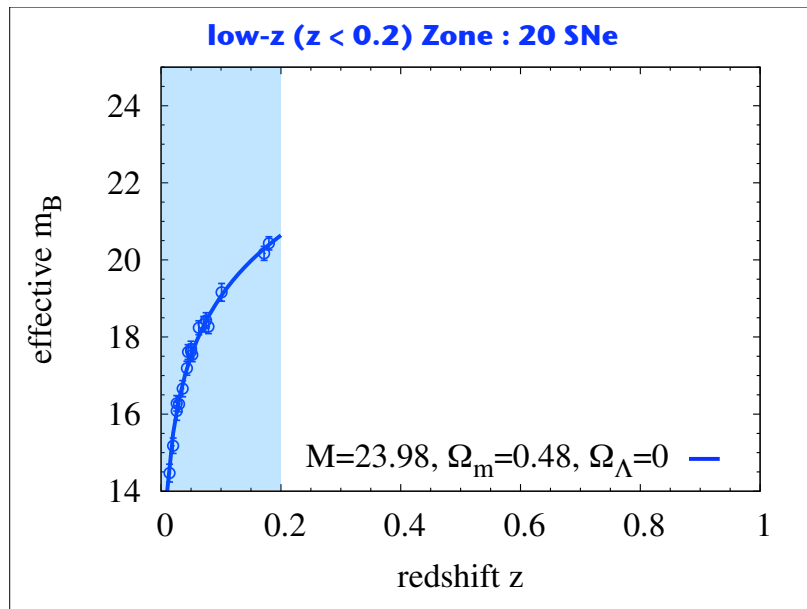
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**low- z ($z < 0.2$)
zone fitting
without Λ**

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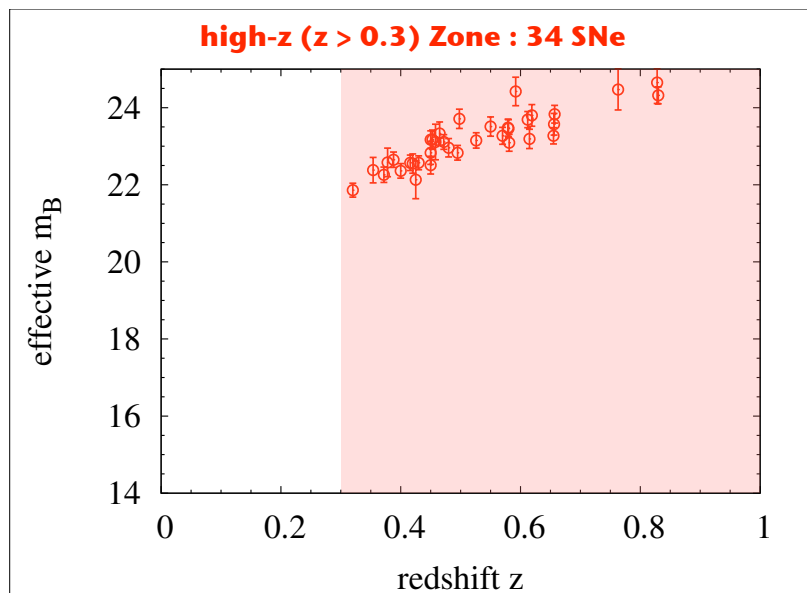
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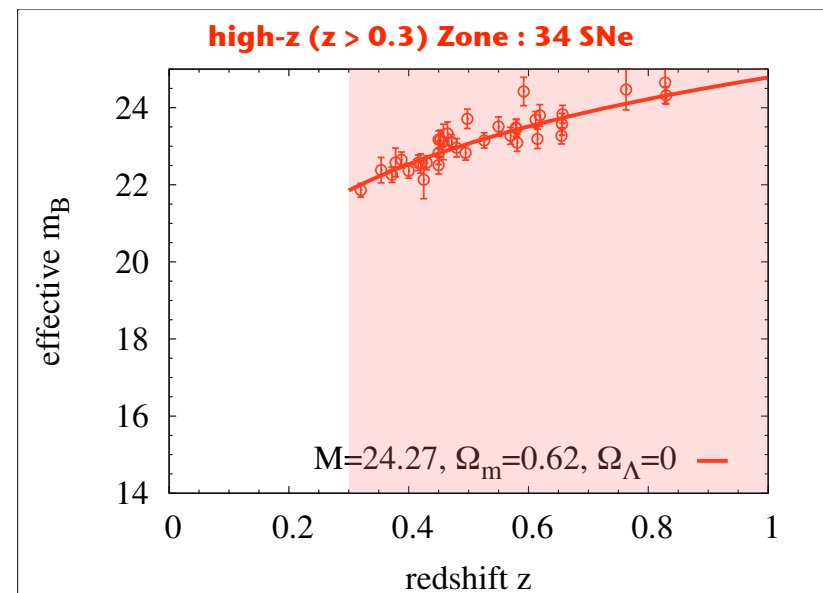
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**high-z ($z > 0.3$)
zone fitting
without Λ**

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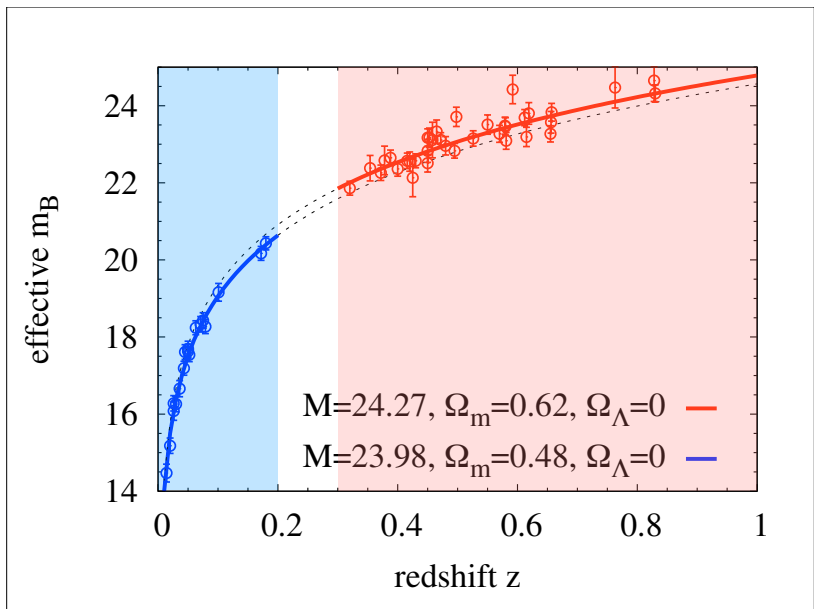
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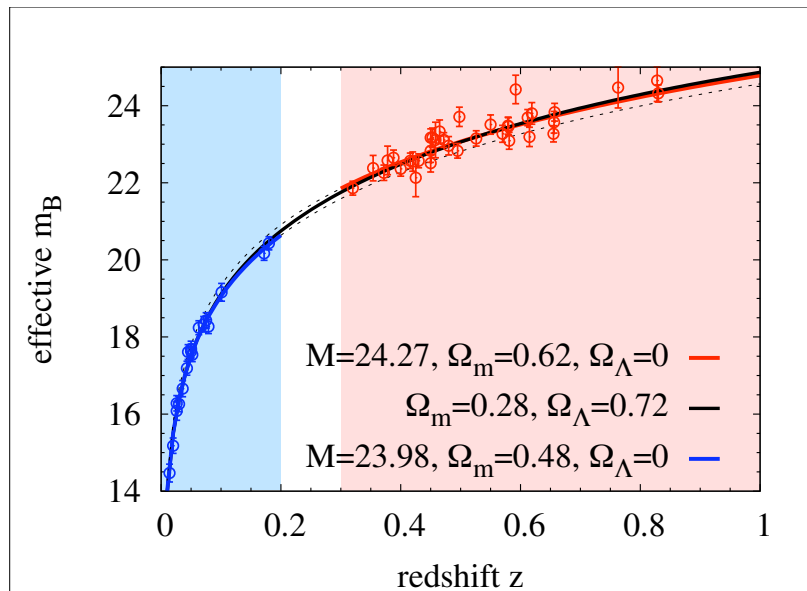
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Plot all fittings...

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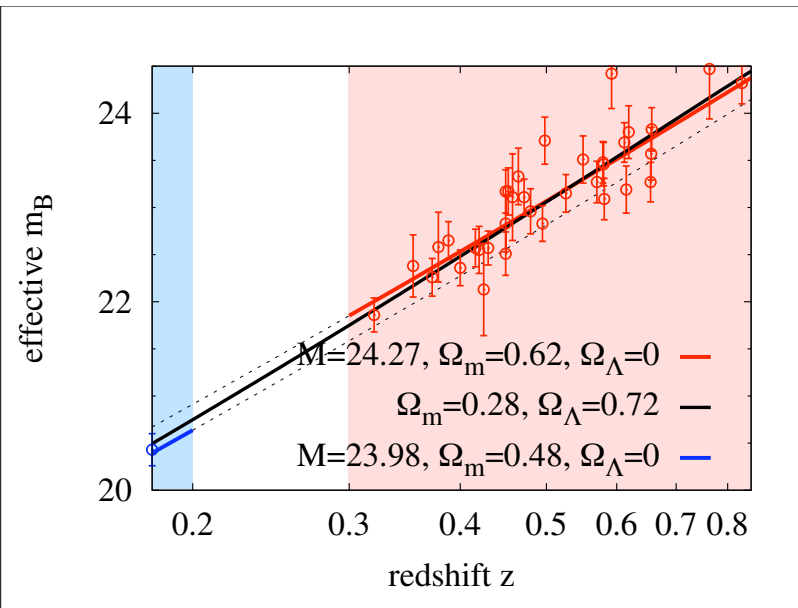
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Close up...

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Determination of the cosmological parameters from the data fitting

162

Determination of the cosmological parameters from the data fitting

- \mathcal{M} (hence H_0) is mainly from low- z data fitting

163

Determination of the cosmological parameters from the data fitting

- \mathcal{M} (hence H_0) is mainly from low- z data fitting
- Assuming the \mathcal{M} is constant, and fitting high- z data, Ω_Λ may be necessary

164

Determination of the cosmological parameters from the data fitting

- \mathcal{M} (hence H_0) is mainly from low- z data fitting
- Assuming the \mathcal{M} is constant, and fitting high- z data, Ω_Λ may be necessary
- If the constancy of \mathcal{M} are relaxed, we don't need Ω_Λ

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Inhomogeneous Interpretation?

166

$\mathcal{M}(\text{low-}z) \neq \mathcal{M}(\text{high-}z) \dots$ Implication?

$$\mathcal{M} = M - 5 + 5 \log_{10} \frac{c}{H_0}$$

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$\mathcal{M}(\text{low-}z) \neq \mathcal{M}(\text{high-}z) \dots$ Implication?

$$\mathcal{M} = \textcircled{M} - 5 + 5 \log_{10} \frac{c}{H_0}$$

In low- z and high- z regions,

1. the absolute luminosity M is different

168

$\mathcal{M}(\text{low-}z) \neq \mathcal{M}(\text{high-}z) \dots$ Implication?

$$\mathcal{M} = M - 5 + 5 \log_{10} \frac{c}{H_0}$$

In **low- z** and **high- z** regions,

1. the absolute luminosity M is different
2. the speed of light c is different

169

$\mathcal{M}(\text{low-}z) \neq \mathcal{M}(\text{high-}z) \dots$ Implication?

$$\mathcal{M} = M - 5 + 5 \log_{10} \frac{c}{H_0}$$

In **low- z** and **high- z** regions,

1. the absolute luminosity M is different
2. the speed of light c is different
3. H_0 is different

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$$\mathcal{M} = \textcircled{M} - 5 + 5 \log_{10} \frac{c}{H_0}$$

In **low- z** and **high- z** regions,

1. the absolute luminosity M is different
 - Astronomically likely...
 - Calibration done only for nearby SNe
 - Difficulty in estimating M of high- z (different environment) SNe

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$$\mathcal{M} = M - 5 + 5 \log_{10} \frac{\textcircled{c}}{H_0}$$

In **low- z** and **high- z** regions,

2. the speed of light c is different
 - modified gravity theories...

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$$\mathcal{M} = M - 5 + 5 \log_{10} \frac{c}{H_0}$$

In **low- z** and **high- z** regions,

3. H_0 is different

- **Inhomogeneous viewpoint**
- **effects of large-scale inhomogeneities**

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SNe m - z relation implies
the existence of Λ ,

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**The meaning:
inhomogeneities
in H_0 and Ω_m**

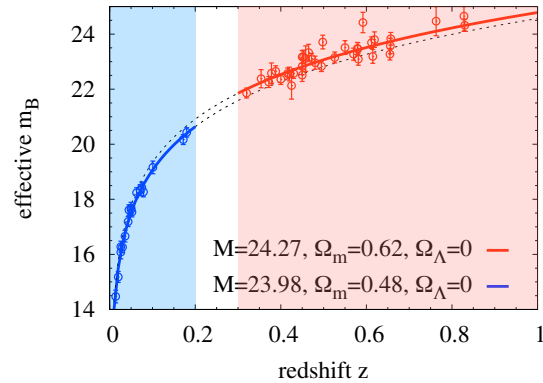
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**The meaning:
**our universe is
inhomogeneous****

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How large is the inhomogeneity in H_0 ?

$$\mathcal{M}(\text{low-}z) - \mathcal{M}(\text{high-}z) = 5 \log_{10} \frac{H_0(\text{high-}z)}{H_0(\text{low-}z)}$$



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How large is the inhomogeneity in H_0 ?

$$23.98 - 24.27 = 5 \log_{10} \frac{H_0(\text{high-}z)}{H_0(\text{low-}z)}$$

↓

$$\frac{H_0(\text{high-}z)}{H_0(\text{low-}z)} = 0.87$$

13% smaller $H_0(\text{high-}z)$ can explain the data
without Dark Energy

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Plausible?

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χ^2 test of Two-Zone model fitting

model	χ^2	# of param.	reduced χ^2
Flat Perlmutter (1999)	57.71	Ω_m, M	1.11

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χ^2 test of Two-Zone model fitting

model	χ^2	# of param.	reduced χ^2
Flat Perlmutter (1999)	57.71	Ω_m, M	1.11
Two-Zone	55.93	$\Omega_m^{\text{low}}, \Omega_m^{\text{high}}$ $M^{\text{low}}, M^{\text{high}}$	1.12

181

χ^2 test of Two-Zone model fitting

model	χ^2	# of param.	reduced χ^2
Flat Perlmutter (1999)	57.71	Ω_m, M	1.11
Two-Zone	55.93	$\Omega_m^{\text{low}}, \Omega_m^{\text{high}}$ $M^{\text{low}}, M^{\text{high}}$	1.12
One- Ω_m Two-Zone	55.95	Ω_m $M^{\text{low}}, M^{\text{high}}$	1.10

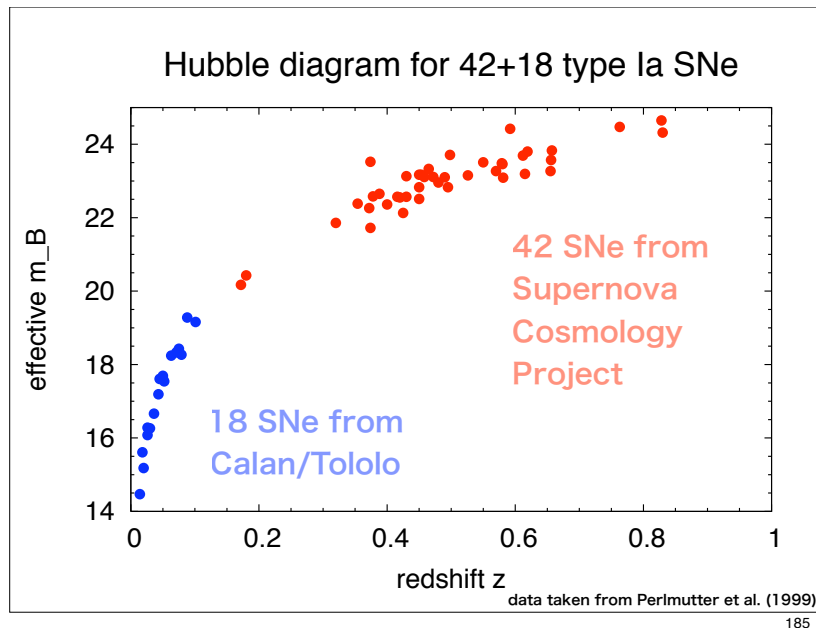
182

**How to incorporate
the **post-Friedmannian** effects
of inhomogeneity
into
 $D_L(z)$?**

183

**an illustration
for $z < 1$ case**

184



185

$$m = M - 5 + 5 \log_{10} D_L(z)$$

luminosity distance $D_L(z)$

$$D_L(z) = \frac{c (1+z)}{H_0 \sqrt{1-\Omega_m-\Omega_\Lambda}} \times \sinh \left(\sqrt{1-\Omega_m-\Omega_\Lambda} \int_0^z \frac{dz'}{\sqrt{(1+\Omega_m z')(1+z')^3 - z' (2+z') \Omega_\Lambda}} \right)$$

$D_L(z)$ is a (bit complicated) function of z with the constant parameters $H_0, \Omega_m, \Omega_\Lambda$.

186

$D_L(z)$ is a (bit complicated) function of z with the constant parameters $H_0, \Omega_m, \Omega_\Lambda$.

Yes, I know. But...

All SNe in the data are $z < 1$, therefore...

the Taylor expansion works.

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The Taylor expansion of $D_L(z)$

$$D_L(z) = \frac{c}{H_0} \left(z + d_2 z^2 + d_3 z^3 + \dots \right)$$

$$d_2 = \frac{1}{4} (2 - \Omega_m + 2 \Omega_\Lambda)$$

$$d_3 = \frac{1}{8} (\Omega_m^2 + 4 \Omega_\Lambda^2 - 4 \Omega_m \Omega_\Lambda - 2 \Omega_m - 4 \Omega_\Lambda)$$

188

$$D_L(z) = \frac{c}{H_0} \left(z + d_2 z^2 + d_3 z^3 + \dots \right)$$

$$m = M - 5 + 5 \log_{10} D_L(z)$$

189

m - z relation for $z < 1$ SNe Ia

$$m = \mathcal{M} + 5 \log_{10} z + 5 \log_{10} \left\{ 1 + d_2 z + d_3 z^2 + \dots \right\}$$

190

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- Fitting parameters are d_2 , d_3 , \mathcal{M} (not H_0 itself)

191

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- For $z \ll 1$,

$$m \simeq \mathcal{M} + 5 \log_{10} z \Rightarrow \mathcal{M} \text{ is obtained}$$

192

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- For $z \ll 1$,

$$m \simeq \mathcal{M} + 5 \log_{10} z \Rightarrow \mathcal{M} \text{ is obtained}$$

- Once d_2, d_3 are obtained,

$$\Omega_m = 2(1 - d_2)(1 - 2d_2) - 2d_3$$

$$\Omega_\Lambda = d_2(2d_2 - 1) - d_3$$

determine Ω_m, Ω_Λ

193

In the **homogeneous (Friedmann)** model,

$$H_0 \equiv \frac{1}{3} \overset{\text{Volume expansion}}{\theta(t_0)}$$

$$\Omega_m \equiv \frac{8\pi G \rho(t_0)}{3 H_0^2}$$

194

In the **inhomogeneous** universe,
 H_0 and Ω_m generally become z -dependent,
 due to the inhomogeneity in $\theta(t, x^i)$ and $\rho(t, x^i)$.

$$H_0(z) = \bar{H}_0 \left(1 + h_1 z + h_2 z^2 + \dots \right)$$

$$\Omega_m(z) = \bar{\Omega}_m \left(1 + \omega_1 z + \omega_2 z^2 + \dots \right)$$

$h_1, h_2, \omega_1, \omega_2, \dots$ are
 the **post-Friedmannian** corrections.

195

Any model which has the Friedmann limit
 (including LTB model)
 can be expressed in this way

$$H_0(z) = \bar{H}_0 \left(1 + h_1 z + h_2 z^2 + \dots \right)$$

$$\Omega_m(z) = \bar{\Omega}_m \left(1 + \omega_1 z + \omega_2 z^2 + \dots \right)$$

in the region $z < 1$

196

**The luminosity distance
with the **post-Friedmannian** corrections**

$$D_L(z) = \frac{c}{\bar{H}_0} \left(z + \tilde{d}_2 z^2 + \tilde{d}_3 z^3 + \dots \right)$$

$$\tilde{d}_2 = \frac{1}{4} \left(2 - \bar{\Omega}_m + 2 \Omega_\Lambda - 4 h_1 \right)$$

$$\tilde{d}_3 = \frac{1}{8} \left(\bar{\Omega}_m^2 + 4 \Omega_\Lambda^2 - 4 \bar{\Omega}_m \Omega_\Lambda - 2 \bar{\Omega}_m - 4 \Omega_\Lambda \right) \\ + (\text{terms with } h_1, h_2, \omega_1)$$

197

From \tilde{d}_2, \tilde{d}_3 obtained by the data fitting,
assuming the Friedmannian $D_L(z)$,
calculate the cosmological parameters...

$$\Omega_\Lambda^{\text{eff}} \equiv \tilde{d}_2 (2 \tilde{d}_2 - 1) - \tilde{d}_3$$

198

From \tilde{d}_2, \tilde{d}_3 obtained by the data fitting,
assuming the Friedmannian $D_L(z)$,
calculate the cosmological parameters...
even if the “bare” value is $\Omega_\Lambda = 0$,

$$\Omega_\Lambda^{\text{eff}} \equiv \tilde{d}_2 (2 \tilde{d}_2 - 1) - \tilde{d}_3 \\ = \frac{1}{8} \left\{ 6 h_1 \bar{\Omega}_m + \frac{4}{3} \omega_1 \bar{\Omega}_m - 4 h_1 + 8 (h_1)^2 + 8 h_2 \right\}$$

$$H_0(z) = \bar{H}_0 (1 + h_1 z + h_2 z^2 + \dots)$$

$$\Omega_m(z) = \bar{\Omega}_m (1 + \omega_1 z + \omega_2 z^2 + \dots)$$

199

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The post-Friedmannian corrections behave as Ω_Λ !

200

Ω_m is also dressed
in the inhomogeneous corrections

$$\begin{aligned}\Omega_m^{\text{eff}} &\equiv 2(1 - \tilde{d}_2)(1 - 2\tilde{d}_2) - 2\tilde{d}_3 \\ &= \left(1 + \frac{3}{2}h_1 + \frac{1}{3}\omega_1\right)\bar{\Omega}_m + 3h_1 + 2(h_1)^2 + 2h_2.\end{aligned}$$

$$\begin{aligned}\bar{\Omega}_m &= \Omega_m(z=0) \\ H_0(z) &= \bar{H}_0(1 + h_1 z + h_2 z^2 + \dots) \\ \Omega_m(z) &= \bar{\Omega}_m(1 + \omega_1 z + \omega_2 z^2 + \dots)\end{aligned}$$

201

**So far,
the data is
Perlmutter et al. (1999)
The Supernova
Cosmology Project**

202

**What about
the new data in
the 21st century?**

203

**Supernova
Legacy Survey
(2006)**

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The Supernova Legacy Survey: measurement of Ω_M , Ω_Λ and w from the first year data set^{★,★★}

P. Astier¹, J. Guy¹, N. Regnault¹, R. Pain¹, E. Aubourg^{2,3}, D. Balam⁴, S. Basa⁵, R. G. Carlberg⁶, S. Fabbro⁷, D. Fouchez⁸, I. M. Hook⁹, D. A. Howell⁶, H. Lafoux³, J. D. Neill⁴, N. Palanque-Delabrouille³, K. Perrett⁶, C. J. Pritchett⁴, J. Rich³, M. Sullivan⁶, R. Taillat^{1,10}, G. Aldering¹¹, P. Antilogus¹, V. Arsenijević⁷, C. Balland^{1,2}, S. Baumont^{1,12}, J. Bronder⁹, H. Courtois¹³, R. S. Ellis¹⁴, M. Filiol⁵, A. C. Gonçalves¹⁵, A. Goobar¹⁶, D. Guide¹, D. Hardin¹, V. Lisset⁴, C. Lidman¹², R. McMahon¹⁷, M. Mouchet^{15,2}, A. Mourao⁷, S. Perlmutter^{11,18}, P. Ripoche⁵, C. Tao⁸, and N. Walton¹⁷

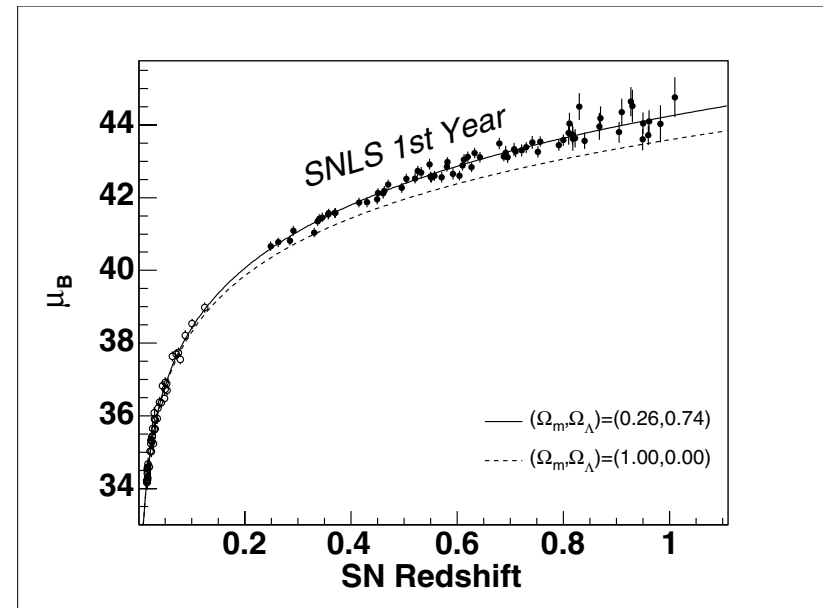
(Affiliations can be found after the references)

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ABSTRACT

We present distance measurements to 71 high redshift type Ia supernovae discovered during the first year of the 5-year Supernova Legacy Survey (SNLS). These events were detected and their multi-color light-curves measured using the MegaPrime/MegaCam instrument at the Canada-France-Hawaii Telescope (CFHT), by repeatedly imaging four one-square degree fields in four bands, as part of the CFHT Legacy Survey (CFHTLS). Follow-up spectroscopy was performed at the VLT, Gemini and Keck telescopes to confirm the nature of the supernovae and to measure their redshift. With this data set, we have built a Hubble diagram extending to $z = 1$, with all distance measurements involving at least two bands. Systematic uncertainties are evaluated making use of the multi-band photometry obtained at CFHT. Cosmological fits to this first year SNLS Hubble diagram give the following results: $\Omega_M = 0.263 \pm 0.042$ (stat) ± 0.032 (sys) for a flat Λ CDM model; and $w = -1.023 \pm 0.090$ (stat) ± 0.054 (sys) for a flat cosmology with constant equation of state w when combined with the constraint from the recent Sloan Digital Sky Survey measurement of baryon acoustic oscillations.

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Preliminary results

- zone-fittings converge only in the range: $z < 0.1$, $0.5 < z$
- multi-zone inhomogeneity?
- transient zone?
- z-dependence of H_0 , Ω_m explicitly shown?

207

Our results give constraints on the inhomogeneous models.

208

**Any models which (try to)
explain SN m-z relation
without **Dark Energy**,
shoud have the following
properties:**

209

**1.
shoud have
Friedmann limit**

210

**2.
behaves as if
Friedmann
in **low-z region**
(**$z < 0.1 \sim 0.2$**)**

211

**3.
behaves as if
Friedmann
in **high-z region**
(**$0.3 \sim 0.5 < z$**)**

212

**4.
mild
inhomogeneity
in H_0 , Ω_m**

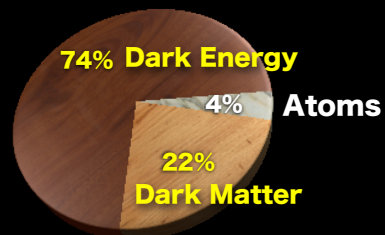
$\Delta H_0 \sim -13\%$, $\Delta \Omega_m \sim 29\%$

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**Still you prefer
homogeneity?**

214

**If you stick to
the homogeneity,**



**you also need
the **Dark Side** of Energy.**

215

**Vader was seduced
by the dark side of
the Force...**

216

**Don't be seduced by
the Dark Side of energy!**

217

**May the Force be with
the inhomogeneous
cosmologists.**

218

**May the Force be with
us.**

219

kasai@phys.hirosaki-u.ac.jp

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