Dark energy without dark energy: Average observational quantities

David L. Wiltshire (University of Canterbury, NZ)

DLW: **New J. Phys. 9 (2007) 377**

Phys. Rev. Lett. 99 (2007) 251101

arXiv:0712.3984

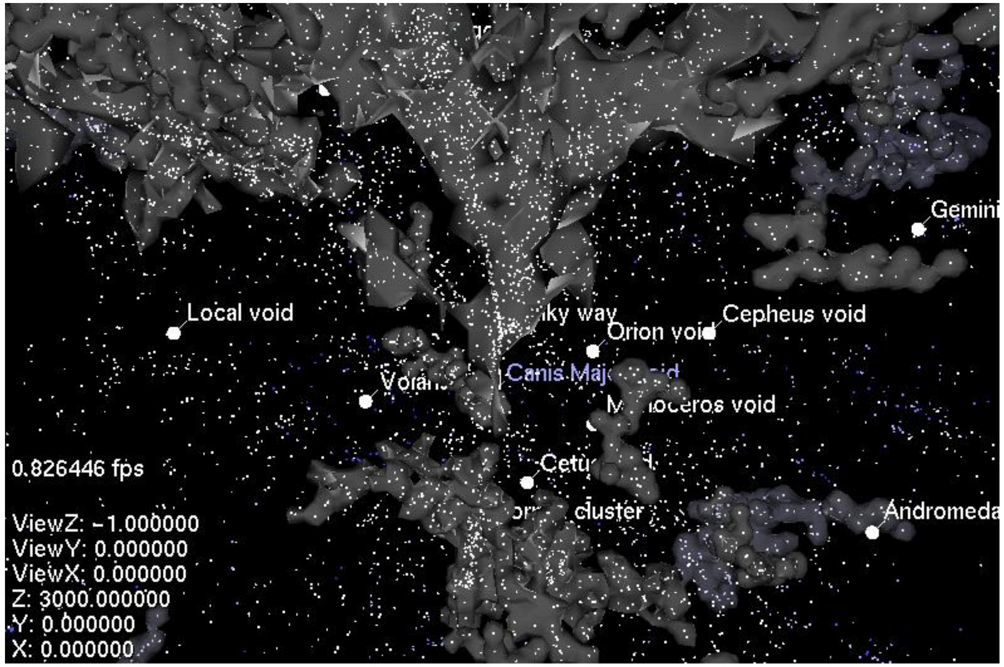
Phys. Rev. D78 (2008) 084032

[arXiv:0809.1183];

new results, to appear

B.M. Leith, S.C.C. Ng and DLW:

ApJ 672 (2008) L91



Is the Universe accelerating?

- Deduction of acceleration relies on assuming geometry of the Universe is described at all epochs by a single homogeneous isotropic geometry

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

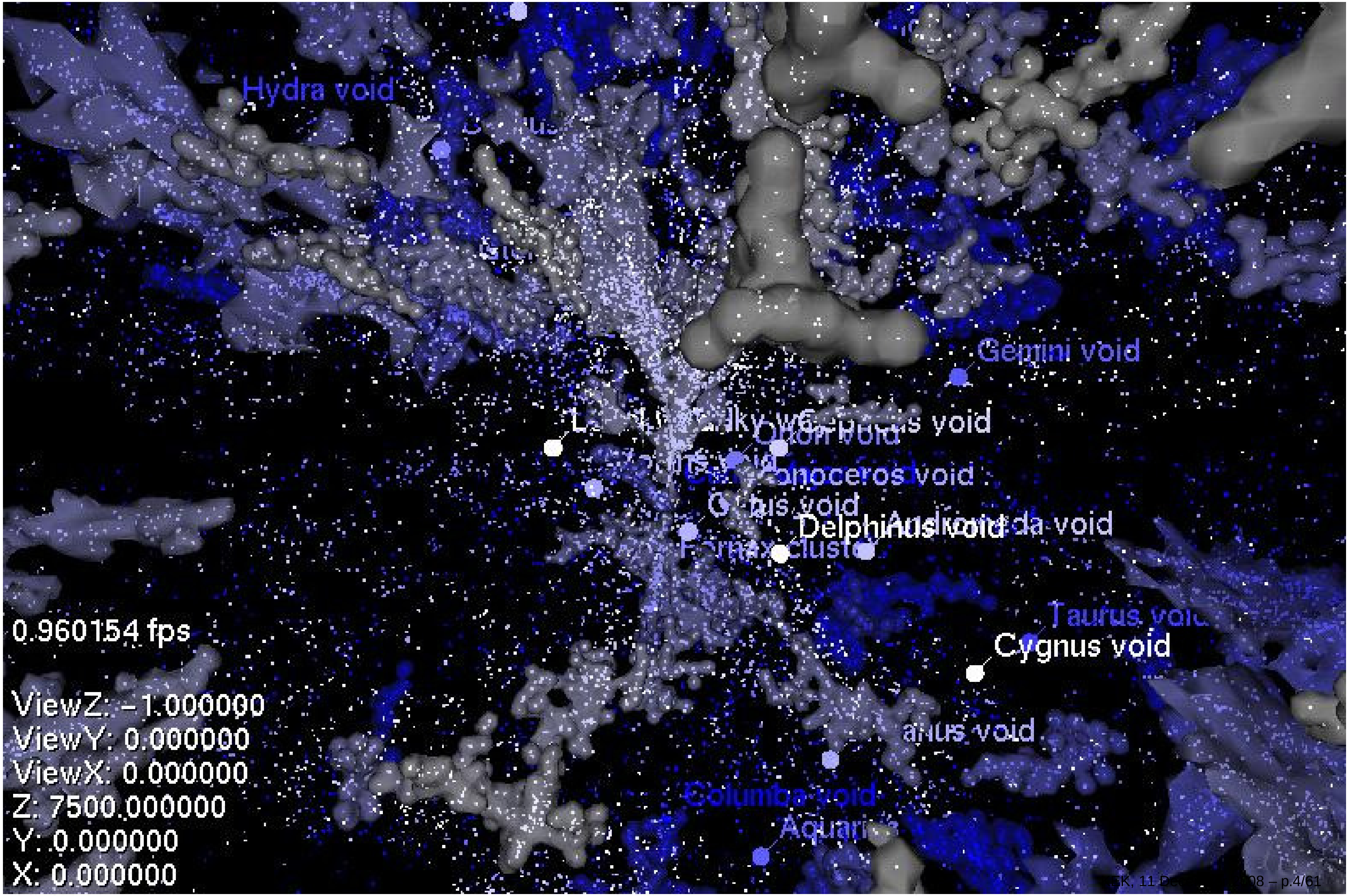
- Leads to a need for dark energy: a smooth component with $P = w\rho$, violating strong energy condition, and $w \simeq -1$ (ΛCDM) matches observations well.
- My hypothesis: in ordinary general relativity, dark energy is a mis-identification in gradients of *cosmological quasilocal gravitational energy* associated with *dynamical gradients* in spatial curvature in a universe as inhomogeneous as the one we observe.

Inhomogeneities, averaging, backreaction

“. . . if one wishes to propose an alternative model, then it is necessary to show that all the predictions of this model are compatible with observations. . . ” (Ishibashi and Wald, 2006)

- Evidence of CMB provides strong constraints on stucture formation. Last scattering density fluctuations $\delta\rho/\rho \sim 10^{-5}$ in photons and baryons, maybe $\sim 10^{-3}$ in non–baryonic dark matter.
- Evidence of galaxy clustering provides strong contraints on structure formation. Universe is void-dominated: density contrasts $\delta\rho/\rho \sim -1$ affect 40-50% present epoch volume on scales $30h^{-1}$Mpc.
- Standard LCDM reproduces much, but not voids as empty as what is observed.

6df: voids & bubble walls (A. Fairall, UCT)



Inhomogeneities, averaging, backreaction

- List of other problems for LCDM is considerable: primordial lithium abundances; coincidence problem; large-angle puzzles in CMB anisotropies (asymmetric power; axis of evil; dark flow. . .); ISW amplitude; Sandage-de Vaucouleurs paradox; tension between WMAP bestfit parameters and direct mass estimates; gravitational lensing time delays; Hubble bubble . . .
- The idea that “a Newtonianly perturbed FLRW metric appears to describe our universe very accurately on all scales” can only be made by ignoring such observational evidence.
- In observational science, some tests will always involve unaccounted systematics. Best we can do is to compare different models on every available test.

Approach 1: Perturbation theory

- Much argument about backreaction from perturbation theory, often near a FLRW background
- Debate is largely about mathematical consistency, and conclusions vary with assumptions made
- Debate shows a potential problem but if backreaction changes the background, then present epoch FLRW model (without Λ) may simply be the wrong background
- Intuition based on weak field approximation being good despite strong density contrasts $\delta\rho/\rho \gg 1$ within galaxies relates to vacuum geometries with well-defined Killing vectors, and non-zero Weyl curvature
- Ricci curvature gradients, with no well-defined Killing vectors are much more nebulous. Observe $\delta\rho/\rho \sim -1$. Expect $|\delta R|/R \sim 1$.

Approach 2: Large voids, LTB models

- Much work has been done on spherically symmetric Lemaître–Tolman–Bondi models
- Unlikely symmetry for whole universe; difficult to reconcile with initial conditions at last scattering and structure formation
- Requires voids ten times larger than those typically observed plus very special position for us
- Violates Copernican principle. But a relaxed Copernican principle based on the fact of statistical homogeneity is possible, even if Friedmann equations are superceded

Approach 3: Non-linear backreaction

- Nonlinear averaging of full inhomogeneous Einstein equations leads to modificatied evolution equations
- Leaving aside mathematical arguments about assumptions of Buchert averaging, there are considerable problems in its interpretation
- Buchert and Carfora recognise that “cosmological parameters are dressed” but most analyses ignore this and try to interpret things in terms of a single average geometry (e.g. most recently Larena, Alimi, Buchert, Kunz & Corasaniti, arXiv:0808.1161; Rosenthal & Flanagan, arXiv:0809.2107).
- When the geometry has large inhomogeneities, variance in geometry must be accounted for. A single average geometry misses the point.

Carfora-Piotrkowska-Buchert-Ehlers -Russ-Soffel-Kasai-Börner equations

For irrotational dust cosmologies, with energy density, $\rho(t,\mathbf{x})$, expansion, $\theta(t,\mathbf{x})$, and shear, $\sigma(t,\mathbf{x})$, on a compact domain, $\mathcal{D}$, of a suitably defined spatial hypersurface of constant average time, t, and spatial 3–metric, average cosmic evolution described by exact equations

$$\begin{aligned}
3\frac{\dot{\bar{a}}^2}{\bar{a}^2} &= 8\pi G\langle\rho\rangle - \tfrac{1}{2}\langle\mathcal{R}\rangle - \tfrac{1}{2}\mathcal{Q} \\
3\frac{\ddot{\bar{a}}}{\bar{a}} &= -4\pi G\langle\rho\rangle + \mathcal{Q} \\
\partial_t\langle\rho\rangle &+ 3\frac{\dot{\bar{a}}}{\bar{a}}\langle\rho\rangle = 0 \\
\partial_t\left(\bar{a}^6\mathcal{Q}\right) &+ \bar{a}^4\partial_t\left(\bar{a}^2\langle\mathcal{R}\rangle\right) = 0. \\
\mathcal{Q} &\equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle
\end{aligned}$$

Back-reaction

Angle brackets denote the spatial volume average, e.g.,

$$\langle \mathcal{R} \rangle \equiv \left(\int_{\mathcal{D}} \mathrm{d}^3x \sqrt{\det {}^3g}\, \mathcal{R}(t, \mathbf{x}) \right) / \mathcal{V}(t)$$

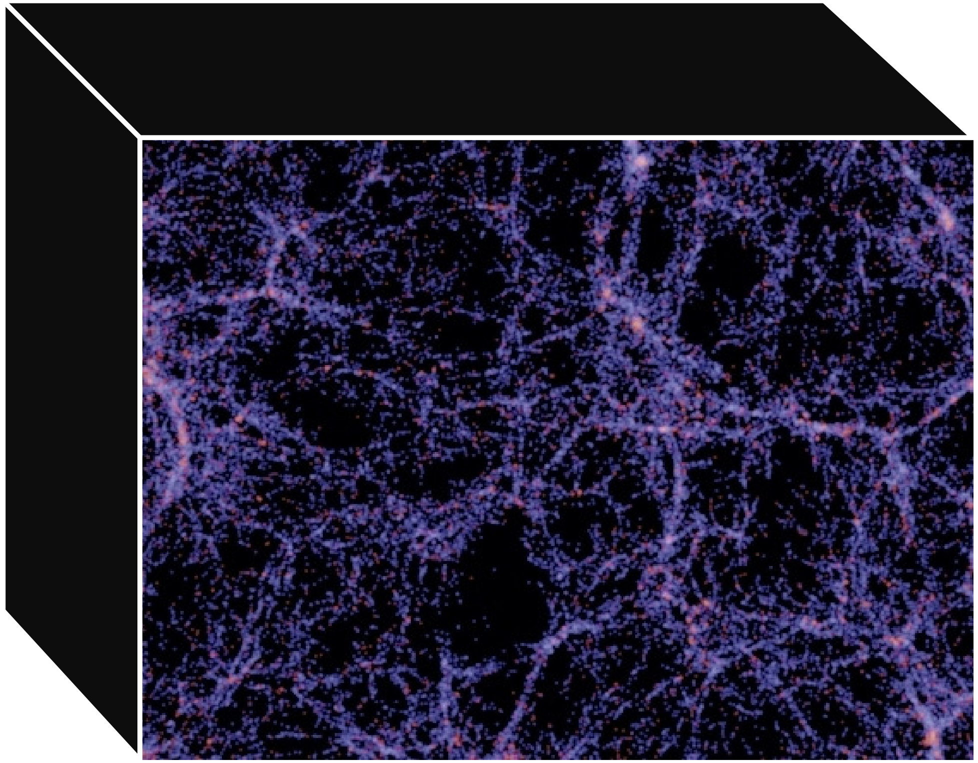
$$\langle \theta \rangle = 3\frac{\dot{\bar{a}}}{\bar{a}}$$

Generally for any scalar Ψ,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \Psi \rangle - \langle \frac{\mathrm{d}\Psi}{\mathrm{d}t} \rangle = \langle \Psi\theta \rangle - \langle \theta \rangle \langle \Psi \rangle$$

- The extent to which the back–reaction, $\mathcal{Q}$, can lead to apparent cosmic acceleration or not has been the subject of much debate.

My synthesis and new approach



- Need to consider nonlinear backreaction, but average quantities must be related to position of observer, accounting for inhomogeneity of observed physical scales within a cell of statistical homogeneity
- Interpret spatial averages with an additional physical hypothesis – Cosmological Equivalence Principle

What is dust?

- In inhomogeneous cosmology "comoving with dust" is not always the same as "comoving with CMB" (isotropic observer)
- Definition of dust (and its "centre of mass") depends on scale of coarse–graining
- I identify the uniform regional Hubble flow slicing as providing a class of isotropic observers below the scale of statistical homogeneity
- An ansatz; but less restrictive than FLRW ansatz; motivated by CEP and taking care of Sandage-de Vaucouleurs paradox.
- Can be understood as relative deceleration of background, typically or order 10^{-10}ms^{-2} of expanding regions of different average density

Two/three scale model

$$\bar{a}^3 = f_{\rm wi} a_{\rm w}{}^3 + f_{\rm vi} a_{\rm v}{}^3$$

- Split as negatively curved void fraction with scale factor $a_{\rm v}$ and spatially flat “wall” fraction with scale factor $a_{\rm w}$.
- $f_{\rm vi} = 1 - f_{\rm wi}$ is the fraction of present epoch horizon volume which was in uncompensated underdense perturbations at last scattering.
- Assume $\delta^2 H_w \equiv \frac{1}{9}\left(\langle\theta^2\rangle_{\rm w} - \langle\theta\rangle_{\rm w}^2\right) = \frac{1}{3}\langle\sigma^2\rangle_w$, $\delta^2 H_v \equiv \frac{1}{9}\left(\langle\theta^2\rangle_{\rm v} - \langle\theta\rangle_{\rm v}^2\right) = \frac{1}{3}\langle\sigma^2\rangle_v$. By assumption, backreaction within voids and walls is neglected, but not in combined statistical average

$$\mathcal{Q} = 6 f_{\rm v}(1 - f_{\rm v})\left(H_{\rm v} - H_{\rm w}\right)^2 = \frac{2\dot{f}_{\rm v}{}^2}{3 f_{\rm v}(1 - f_{\rm v})}\,.$$

Two/three scale model

- Buchert equations for volume averaged observer, with $f_{\rm v}(t) = f_{\rm vi}{a_{\rm v}}^3/\bar{a}^3$ (void volume fraction) and $k_{\rm v} < 0$

$$\bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_{\mathcal{Q}} = 1,$$
$$\bar{a}^{-6}\partial_t\left(\bar{\Omega}_{\mathcal{Q}}\bar{H}^2\bar{a}^6\right) + \bar{a}^{-2}\partial_t\left(\bar{\Omega}_k\bar{H}^2\bar{a}^2\right) = 0\,.$$

where the bare parameters are

$$\bar{\Omega}_M = \frac{8\pi G\bar{\rho}_{M0}\bar{a}_0^3}{3\bar{H}^2\bar{a}^3}, \qquad \bar{\Omega}_k = \frac{-k_{\rm v}{f_{\rm vi}}^{2/3}{f_{\rm v}}^{1/3}}{\bar{a}^2\bar{H}^2},$$
$$\bar{\Omega}_{\mathcal{Q}} = \frac{-\dot{f}_{\rm v}^{\,2}}{9f_{\rm v}(1-f_{\rm v})\bar{H}^2}\,.$$

General exact solution PRL 99, 251101

- Irrespective of physical interpretation the two scale Buchert equation can be solved exactly

$$a_{\rm w} = a_{\rm w0} t^{2/3}$$

$$a_{\rm w0} \equiv \bar{a}_0 \left[\frac{9}{4} {f_{\rm wi}}^{-1} (1-\epsilon_i)\, \bar{\Omega}_{M0} {\bar{H}_0}^2 \right]^{1/3}; \; C_\epsilon \equiv \frac{\epsilon_i \bar{\Omega}_{M0} {f_{\rm v0}}^{1/3}}{\bar{\Omega}_{k0}}$$

$$\sqrt{u(u+C_\epsilon)} - C_\epsilon \ln\left(\left|\frac{u}{C_\epsilon}\right|^{\frac{1}{2}} + \left|1+\frac{u}{C_\epsilon}\right|^{\frac{1}{2}} \right) = \frac{\alpha}{\bar{a}_0}(t+t_\epsilon)$$

where $u \equiv {f_{\rm v}}^{1/3} \bar{a}/\bar{a}_0 = {f_{\rm vi}}^{1/3} a_{\rm v}/\bar{a}_0$, $\alpha = \bar{a}_0 \bar{H}_0 \bar{\Omega}_{k0}^{1/2} / {f_{\rm v0}}^{1/6}$, and constraints relate many constants.

- 4 independent parameters: e.g., $\bar{H}_0$, $f_{\rm v0}$, ϵ_i, $f_{\rm vi}$.

Tracker solution limit

- Parameters ϵ_i and $f_{\rm vi}$ should be restricted by power spectrum at last scattering, e.g.,

$$\left(\frac{\delta\rho}{\rho}\right)_{\mathcal{H}_{\rm i}} = f_{\rm vi}\left(\frac{\delta\rho}{\rho}\right)_{\rm vi} \sim -10^{-6}\ \text{to} - 10^{-5}$$

E.g., $f_{\rm vi} \sim 10^{-3}$, $(\delta\rho/\rho)_{\rm vi} \sim -10^{-3}$; or $f_{\rm vi} \sim -10^{-2}$, $(\delta\rho/\rho)_{\rm vi} \sim -10^{-4}$; or $f_{\rm vi} \sim 3\times 10^{-3}$, $(\delta\rho/\rho)_{\rm vi} \sim -3\times 10^{-3}$.

- Tracker limit, $\epsilon_i = 0$, $t_\epsilon = 0$, with $a_{\rm v} = a_{\rm v0} t$, reached to within 1% by redshift $z \sim 37$ for reasonable priors.

$$\bar{a} = \frac{\bar{a}_0 (3\bar{H}_0 t)^{2/3}}{2+f_{\rm v0}}\left[3f_{\rm v0}\bar{H}_0 t + (1-f_{\rm v0})(2+f_{\rm v0})\right]^{1/3}$$

$$f_{\rm v} = \frac{3f_{\rm v0}\bar{H}_0 t}{3f_{\rm v0}\bar{H}_0 t + (1-f_{\rm v0})(2+f_{\rm v0})}$$

Physical interpretation

- As pointed out by Ishibashi and Wald, need to relate average quantities to physical observables
- My interpretation assumes coarse–graining at scale of statistical homogeneity; average over horizon volume.
- Uniform quasi–local Hubble flow below this scale; volume average time, t, and average curvature, $\langle\mathcal{R}\rangle$, not local observable for all isotropic observers
- Observers within galaxies assumed to be within spatially flat finite infinity regions with geometry

$$\mathrm{d}s^2_{\mathcal{F}_I} = -\mathrm{d}\tau_{\mathrm{w}}^2 + a_{\mathrm{w}}{}^2(\tau_{\mathrm{w}})\left[\mathrm{d}\eta_{\mathrm{w}}^2 + \eta_{\mathrm{w}}^2 \mathrm{d}\Omega^2\right]$$

- Determine dressed cosmological parameters, as if local clocks and rulers were extended to whole universe

Physical interpretation

- Interpret solution of Buchert eqns by radial null cone average

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \bar{a}^2(t)\,\mathrm{d}\bar{\eta}^2 + A(\bar{\eta},t)\,\mathrm{d}\Omega^2,$$

where $\int_0^{\bar{\eta}_{\mathcal{H}}} \mathrm{d}\bar{\eta}\, A(\bar{\eta},t) = \bar{a}^2(t)\mathcal{V}_{\mathrm{i}}(\bar{\eta}_{\mathcal{H}})/(4\pi)$.

- Match radial null geodesics to those of finite infinity geometry with uniform local Hubble flow; since $a_{\mathrm{w}} = {f_{\mathrm{wi}}}^{-1/3}(1-f_{\mathrm{v}})\,\bar{a}$

$$\mathrm{d}\eta_{\mathrm{w}} = \frac{{f_{\mathrm{wi}}}^{1/3}\mathrm{d}\bar{\eta}}{\bar{\gamma}\,(1-f_{\mathrm{v}})^{1/3}}$$

Define η_{w} by integral of above on radial null-geodesics.

Physical interpretation

- Extend spatially flat wall geometry to dressed geometry

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a^2(\tau)\left[\mathrm{d}\bar\eta^2 + r_{\rm w}^2(\bar\eta,\tau)\,\mathrm{d}\Omega^2\right]$$

where $r_{\rm w} \equiv \bar\gamma\,(1-f_{\rm v})^{1/3}\,f_{\rm wi}{}^{-1/3}\eta_{\rm w}(\bar\eta,\tau)$, $a=\bar a/\bar\gamma$.

- Dressed average Hubble parameter

$$H = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau} = \frac{1}{\bar a}\frac{\mathrm{d}\bar a}{\mathrm{d}\tau} - \frac{1}{\bar\gamma}\frac{\mathrm{d}\bar\gamma}{\mathrm{d}\tau}$$

H is greater than wall Hubble rate; smaller than void Hubble rate measured by wall (or any one set of) clocks

$$\bar H(t) = \frac{1}{\bar a}\frac{\mathrm{d}\bar a}{\mathrm{d}t} = \frac{1}{a_{\rm v}}\frac{\mathrm{d}a_{\rm v}}{\mathrm{d}\tau_{\rm v}} = \frac{1}{a_{\rm w}}\frac{\mathrm{d}a_{\rm w}}{\mathrm{d}\tau} < H < \frac{1}{a_{\rm v}}\frac{\mathrm{d}a_{\rm v}}{\mathrm{d}\tau}$$

Physical interpretation

- Dressed average deceleration parameter

$$q = \frac{-1}{H^2 a^2} \frac{\mathrm{d}^2 a}{\mathrm{d}\tau^2}$$

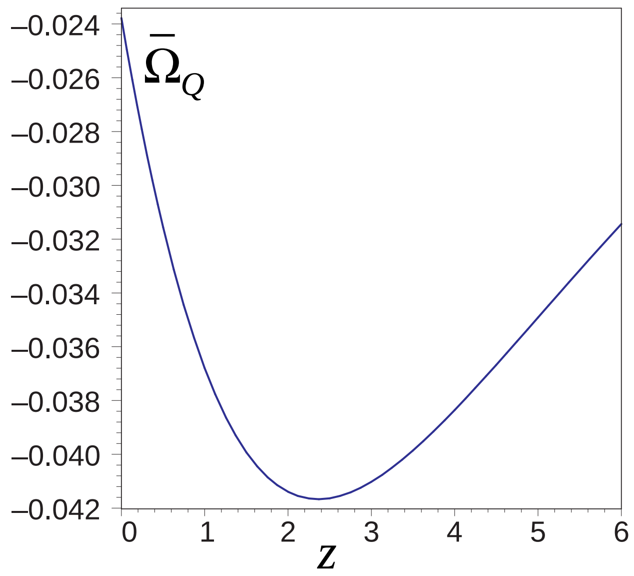
  Can have $q < 0$ even though $\bar{q} = \frac{-1}{\bar{H}^2 \bar{a}^2} \frac{\mathrm{d}^2 \bar{a}}{\mathrm{d}t^2} > 0$; difference of clocks important. For tracker solution

$$\tau = \tfrac{2}{3}t + \frac{2(1-f_{\rm v0})(2+f_{\rm v0})}{27 f_{\rm v0} \bar{H}_0} \ln\left(1 + \frac{9 f_{\rm v0} \bar{H}_0 t}{2(1-f_{\rm v0})(2+f_{\rm v0})}\right)$$

- Dressed matter density parameter numerically close to usual FLRW one

$$\Omega_M = \bar{\gamma}^3 \bar{\Omega}_M \,.$$

Large vs short wavelength viewpoint



- Long wavelength perturbations shift you to a different FLRW model
- Since $|\bar{\Omega}_{\mathcal{Q}}| < 0.042$ typically this is correct; but which FLRW model? $k = -1$ with $\bar{\Omega}_{M0} = 0.127$, $t_0 = 18.6$ Gyr.
- It is the interplay of "short wavelength" inhomogeneities within a nested structure, and relative calibration. Furthermore, perturbations about fixed FLRW background has broken down.

Redshift, luminosity distance

- Cosmological redshift (last term tracker solution)

$$z+1=\frac{a}{a_0}=\frac{\bar{a}_0\bar{\gamma}}{\bar{a}\bar{\gamma}_0}=\frac{(2+f_{\rm v})f_{\rm v}{}^{1/3}}{3f_{\rm v0}{}^{1/3}\bar{H}_0 t}=\frac{2^{4/3}t^{1/3}(t+b)}{f_{\rm v0}{}^{1/3}\bar{H}_0 t(2t+3b)^{4/3}},$$

  where $b=2(1-f_{\rm v0})(2+f_{\rm v0})/[9f_{\rm v0}\bar{H}_0]$

- Dressed luminosity distance relation $d_L=(1+z)D$ where the *effective comoving distance* to a redshift z is $D=a_0 r_{\rm w}$, with

$$r_{\rm w}=\bar{\gamma}\,(1-f_{\rm v})^{1/3}\int_t^{t_0}\frac{{\rm d}t'}{\bar{\gamma}(t')(1-f_{\rm v}(t'))^{1/3}\bar{a}(t')}\,.$$

Redshift, luminosity distance

- Perform integral for tracker solution

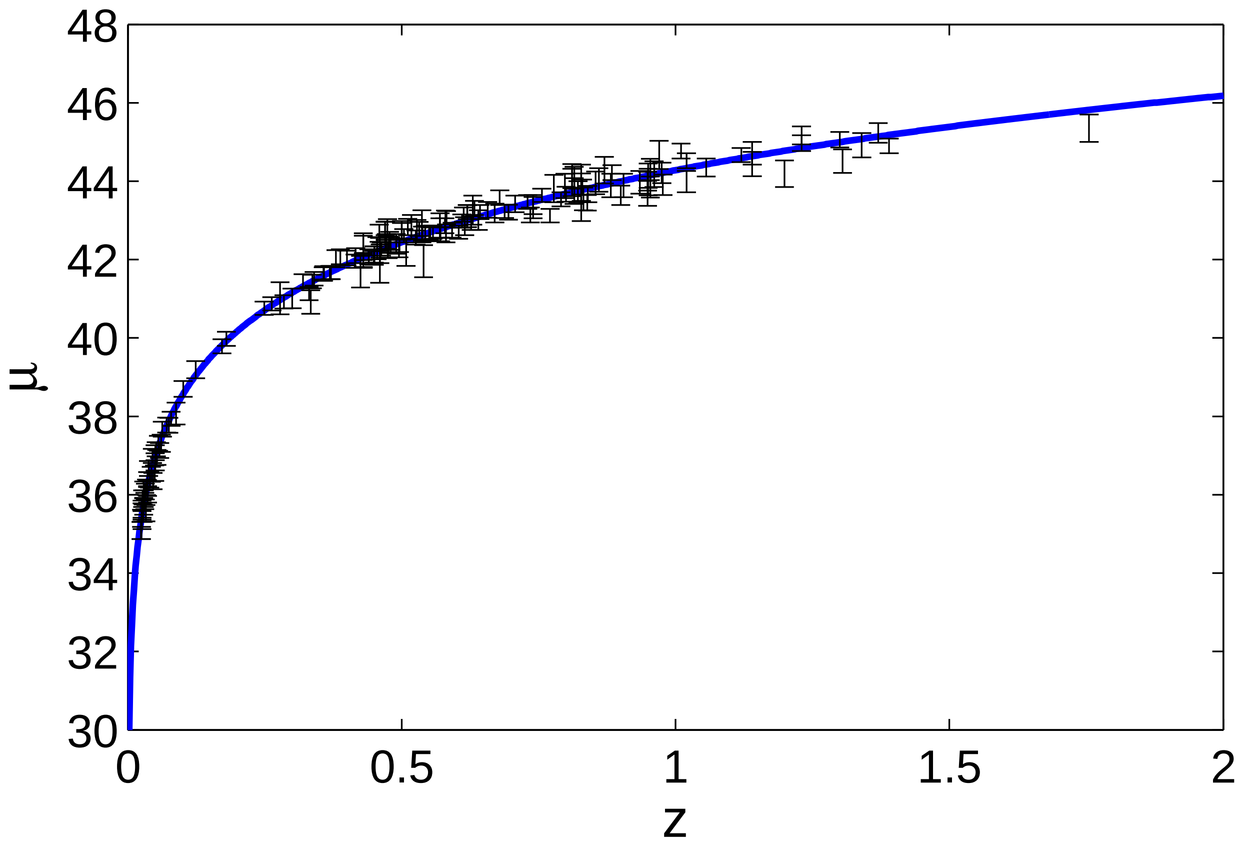
$$\begin{aligned} D_A = \frac{D}{1+z} = \frac{d_L}{(1+z)^2} &= (t)^{\frac{2}{3}} \int_t^{t_0} \frac{2\mathrm{d}t'}{(2+f_{\mathrm{v}}(t'))(t')^{2/3}} \\ &= t^{2/3}(\mathcal{F}(t_0) - \mathcal{F}(t)) \end{aligned}$$

where

$$\begin{aligned} \mathcal{F}(t) &= 2t^{1/3} + \frac{b^{1/3}}{6} \ln\left(\frac{(t^{1/3}+b^{1/3})^2}{t^{2/3} - b^{1/3}t^{1/3} + b^{2/3}} \right) \\ &\quad + \frac{b^{1/3}}{\sqrt{3}} \tan^{-1}\left(\frac{2t^{1/3} - b^{1/3}}{\sqrt{3}\, b^{1/3}} \right). \end{aligned}$$

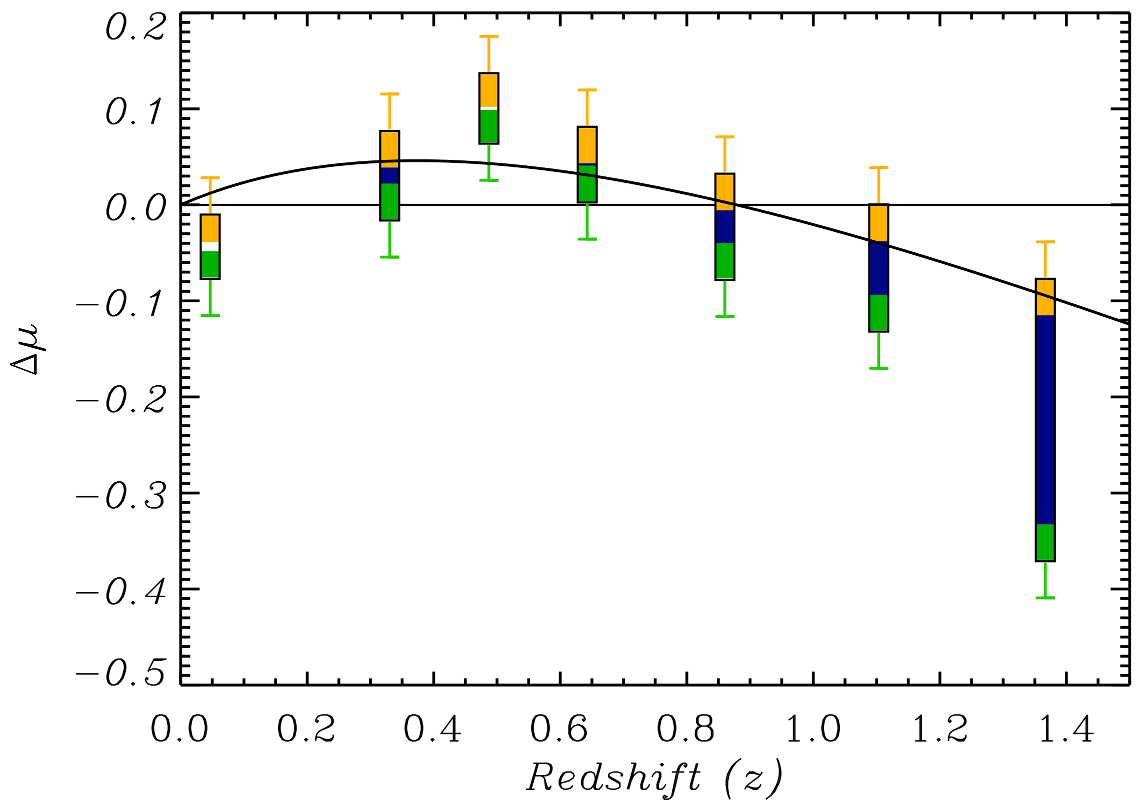
t given implicity in terms of z by previous relation

Test 1: SneIa luminosity distances



- Type Ia supernovae of Riess07 Gold data set fit with χ^2 per degree of freedom $= 0.9$
- With $55 \le H_0 \le 75$ km sec^{-1} Mpc^{-1}, $0.01 \le \Omega_{M0} \le 0.5$, find Bayes factor $\ln B = 0.27$ in favour or FB model (marginally): statistically indistinguishable from ΛCDM.

Test 1: SneIa luminosity distances

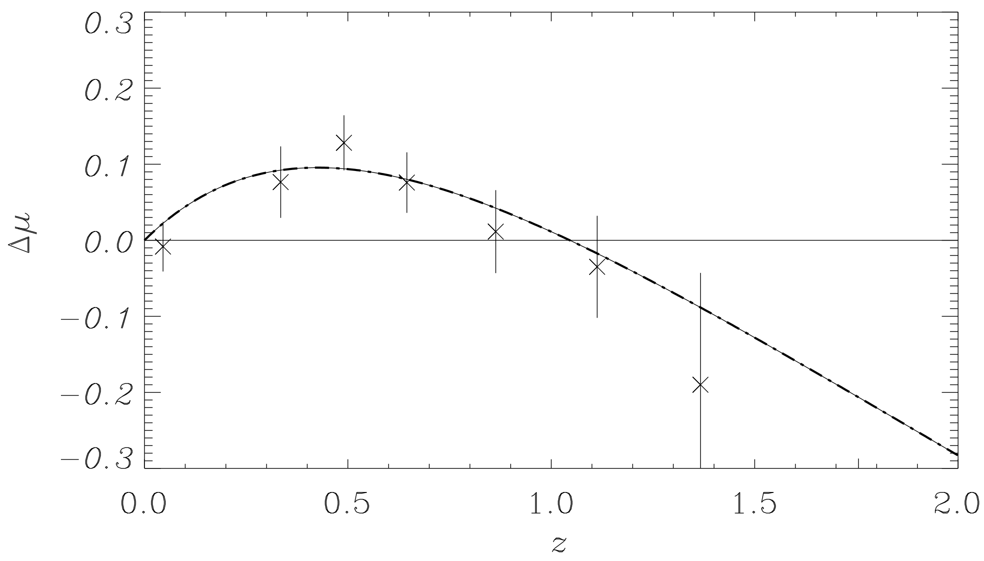


- Plot shows difference of model apparent magnitude and that of an empty Milne universe of same Hubble constant $H_0 = 61.73$ km sec^{-1} Mpc^{-1}. *Note*: residual depends on the expansion rate of the Milne universe subtracted (2σ limits on H_0 indicated by whiskers)

Comparison ΛCDM models

Best-fit spatially flat ΛCDM

$H_0 = 62.7$ km sec^{-1} Mpc^{-1},
$\Omega_{M0} = 0.34, \Omega_{\Lambda 0} = 0.66$



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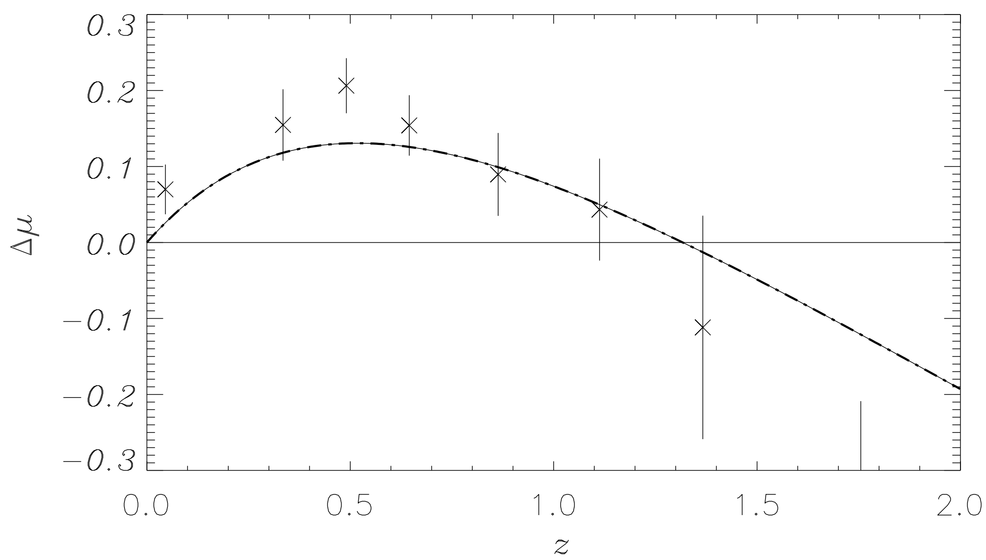
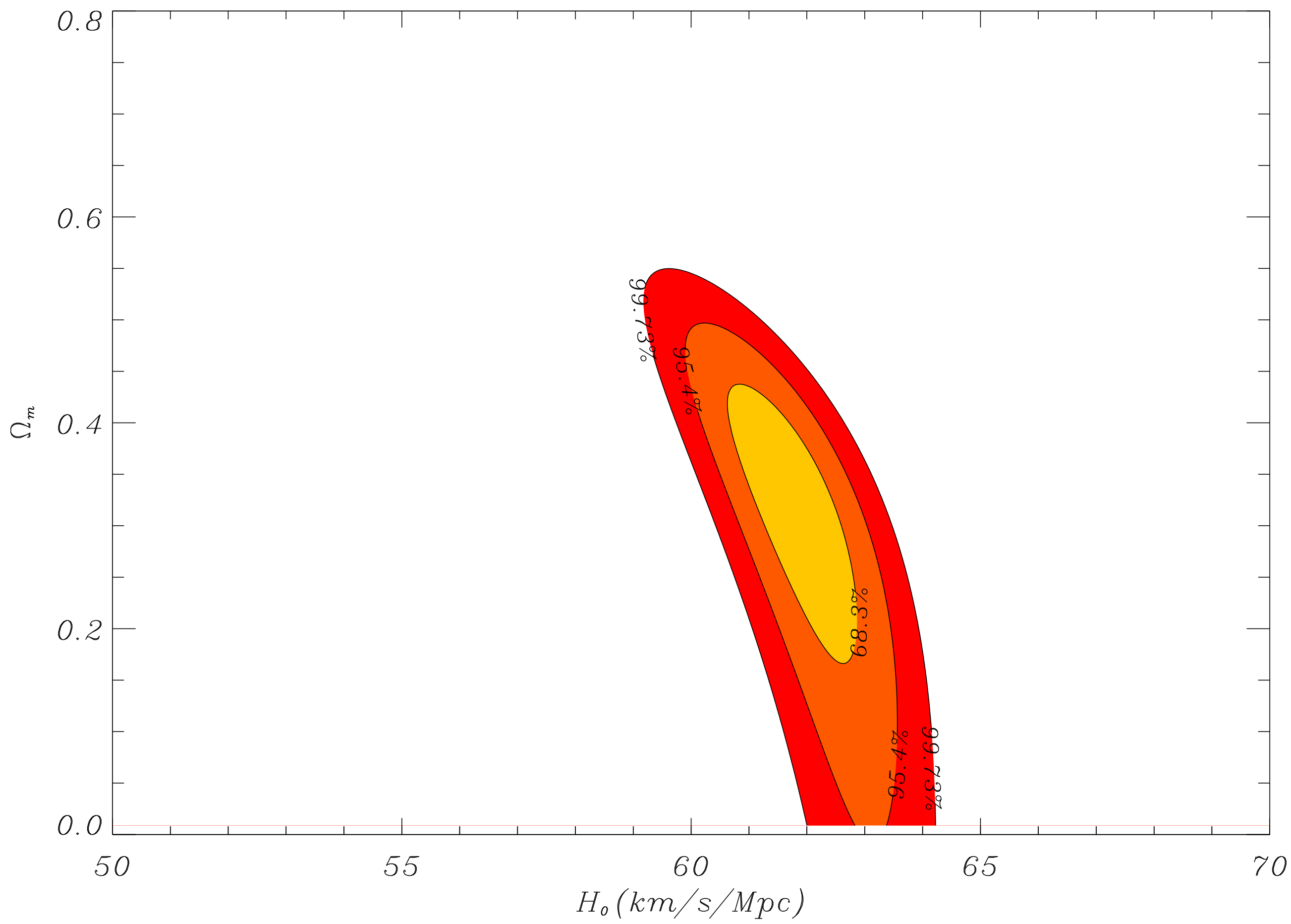
$H_0 = 65$ km sec^{-1} Mpc^{-1},
$\Omega_{M0} = 0.29, \Omega_{\Lambda 0} = 0.71$

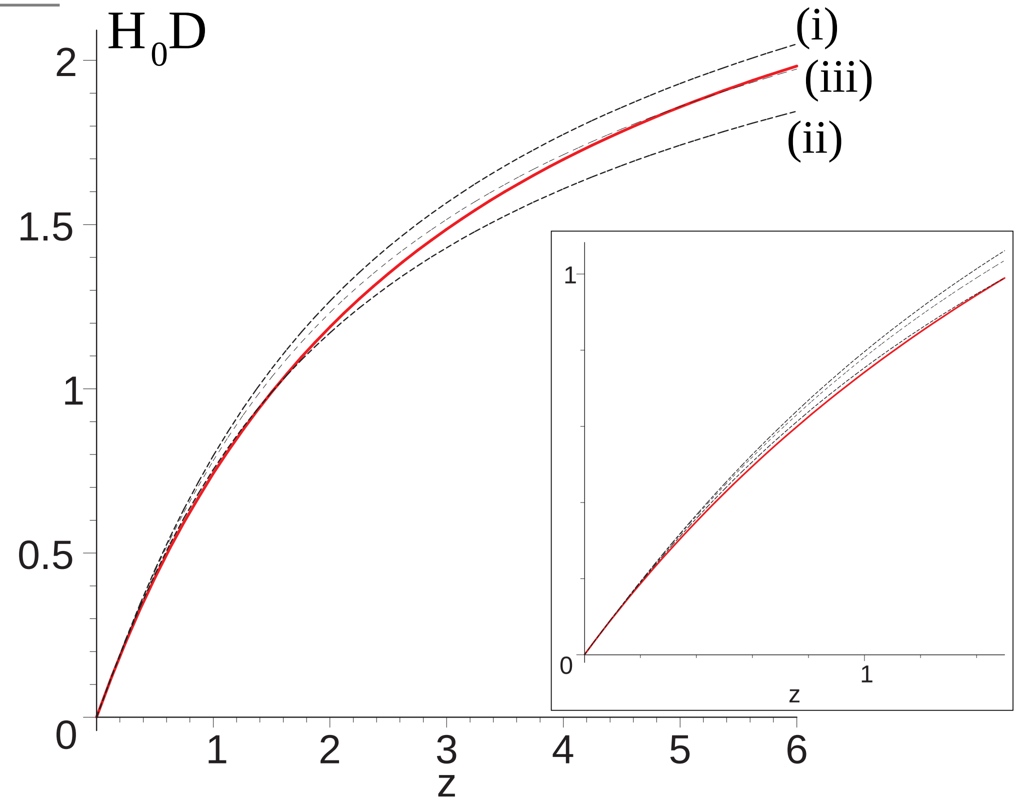


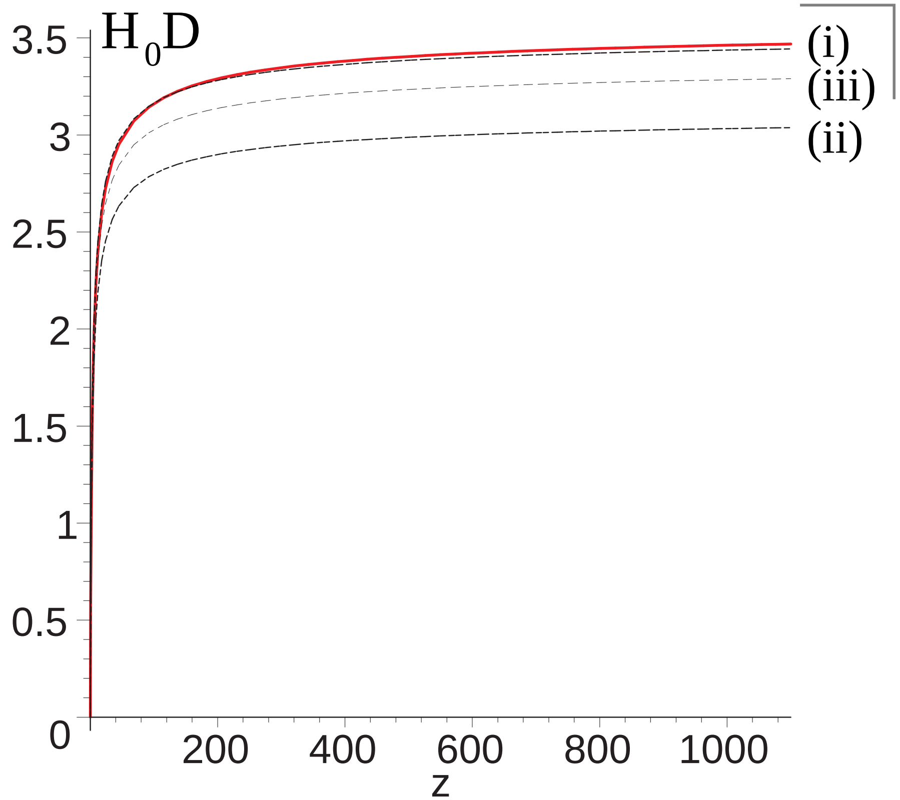
Test 1: SneIa luminosity distances



Best–fit H_0 agrees with HST key team, Sandage et al., $H_0 = 62.3 \pm 1.3$ (stat) $\pm\, 5.0$ (syst) km sec^{-1} Mpc^{-1} [ApJ 653 (2006) 843].

Dressed "comoving distance" $D(z)$



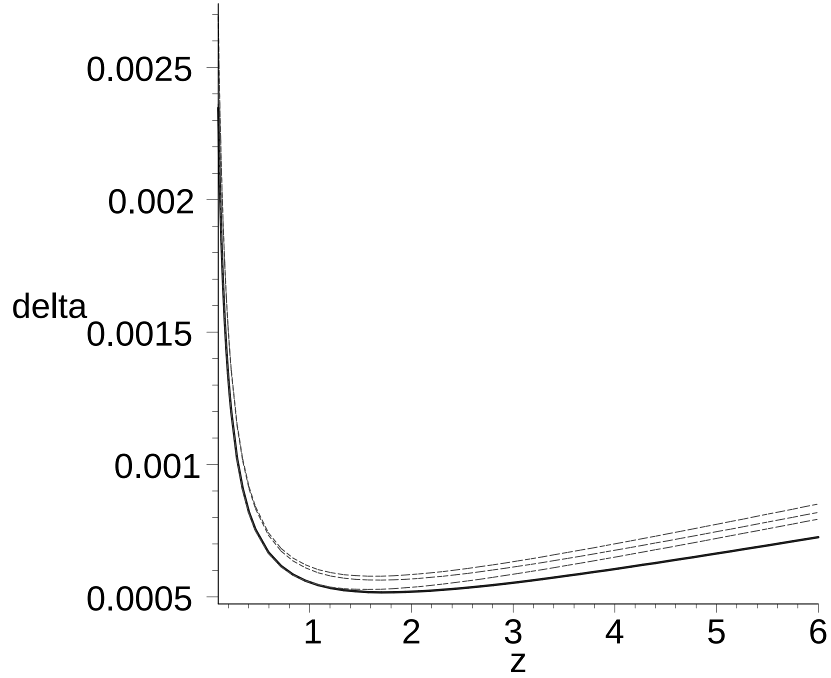


Best-fit FB model **(red line)** compared to 3 spatially flat ΛCDM models: **(i)** best–fit to WMAP5 only ($\Omega_\Lambda = 0.751$); **(ii)** best–fit to (Riess07) SneIa only ($\Omega_\Lambda = 0.66$); **(iii)** joint WMAP5 + BAO + SneIa fit ($\Omega_\Lambda = 0.721$)

Dressed "comoving distance" $D(z)$

- FB model closest to best–fit ΛCDM to *SneIa only* result ($\Omega_{M0} = 0.34$) at *low redshift*. Note also, for $z > 1$ SneIae are "unexpectedly bright as compared to ΛCDM prediction" (Kowalski et al, ApJ 686 (2008) 749). For FB model SneIa this is expected.
- FB model closest to best–fit *WMAP5 only* result, ($\Omega_{M0} = 0.249$) at *high redshift*
- Over the range $1 < z < 6.6$ tested by gamma ray bursters (GRBs) the FB model fits Schaefer's sample of 69 GRBs "better than concordance ΛCDM" (B.E. Schaefer, in preparation)
- Maximum difference in apparent magnitude between FB model and LCDM is only 0.18–0.34 mag at $z = 6$. Will need much data to get statistically significant test.

Angular size of standard objects

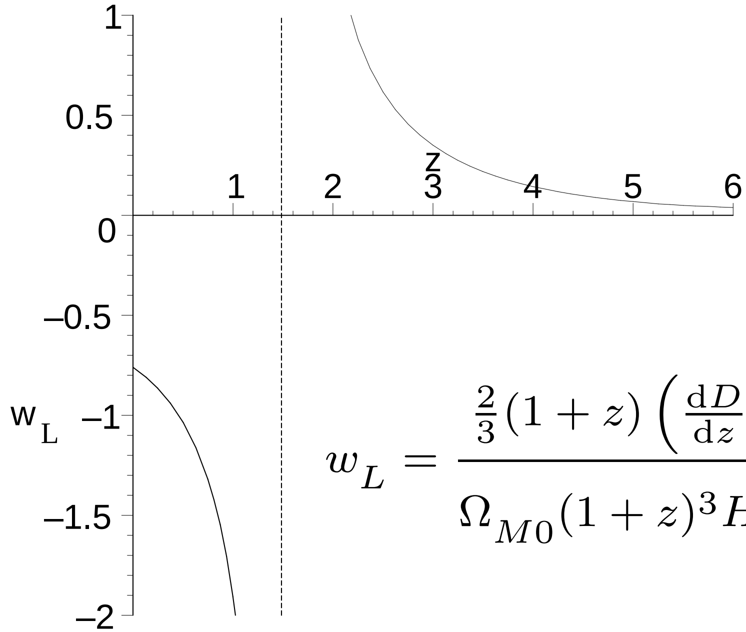


Angular size of 1Mpc standard ruler for the FB model with $f_{\mathrm{v}0} = 0.762$ (solid line) as compared to three spatially flat ΛCDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in previous figures.

- Angular size relation $\delta = D/d_A$ differs only slightly (about 9–15% at most up to $z \lesssim 6$) from ΛCDM
- Minimum δ at $z = 1.74$ for $f_{\mathrm{v}0} = 0.762$, c.f. minimum δ at $z = 1.63$ for spatially flat ΛCDM with $\Omega_{M0} = 0.28$.

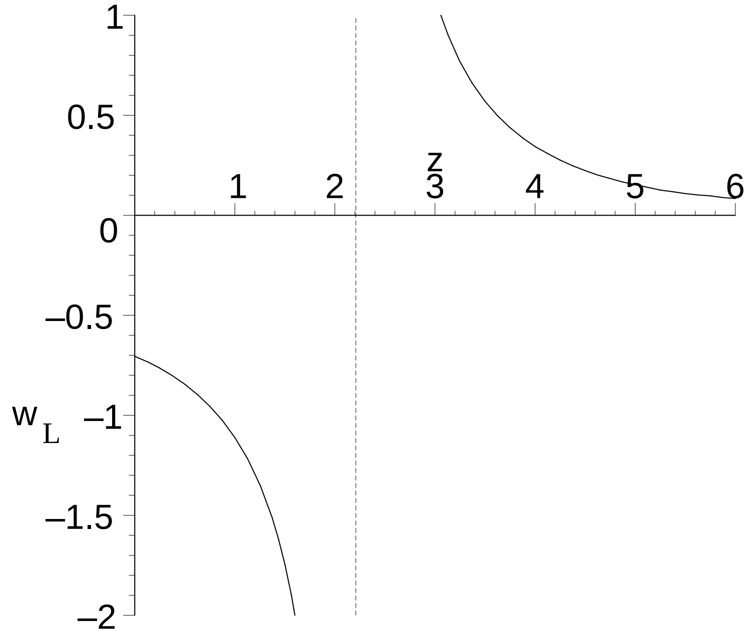
Equivalent “equation of state”?

(i)



$$w_L = \frac{\frac{2}{3}(1+z)\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^{-1}\frac{\mathrm{d}^2D}{\mathrm{d}z^2}+1}{\Omega_{M0}(1+z)^3H_0^2\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^2-1}$$

(ii)



A formal “dark energy equation of state” $w_L(z)$ for the best-fit FB model, $f_{v0} = 0.76$, calculated directly from $r_w(z)$: **(i)** $\Omega_{M0} = 0.33$; **(ii)** $\Omega_{M0} = 0.279$.

- Description by a “dark energy equation of state” makes no sense when there is no physics behind it; but average value $w_L \simeq -1$ for $z < 0.7$ makes empirical sense.

Sahni, Shafieloo and Starobinsky $Om(z)$

- Sahni, Shafieloo and Starobinsky propose a dark energy diagnostic

$$Om(z) = \frac{\frac{H^2(z)}{H_0^2} - 1}{(1+z)^3 - 1},$$

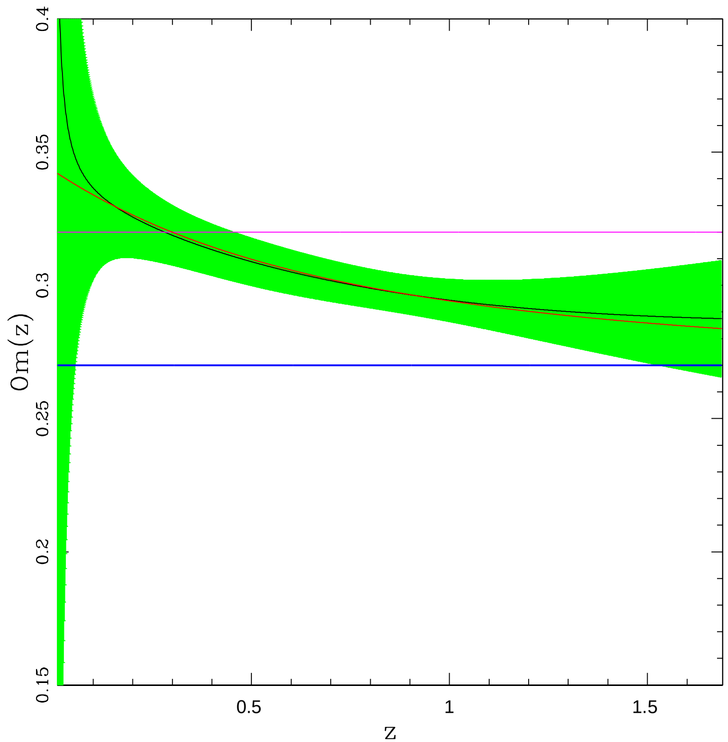
For FLRW models $OM(z) = \Omega_{M0}$ for ΛCDM $\forall z$ since

$$Om(z) = \Omega_{M0} + (1 - \Omega_{M0}) \frac{(1+z)^{3(1+w)} - 1}{(1+z)^3 - 1}.$$

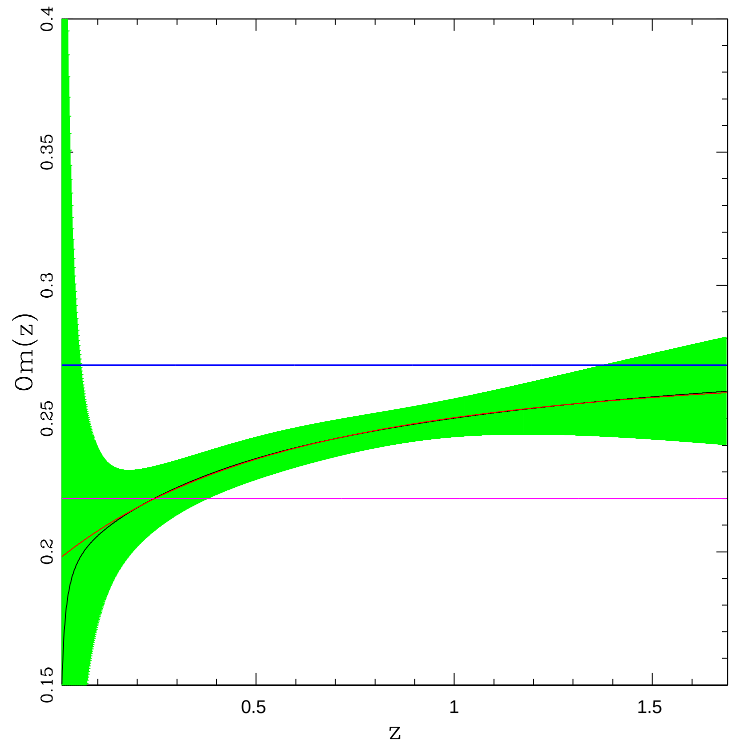
but there are differences for other dark energy models.

- Note: $Om(z) \to \Omega_{M0}$ at large z for all w.

Sahni, Shafieloo and Starobinsky $Om(z)$



(i)

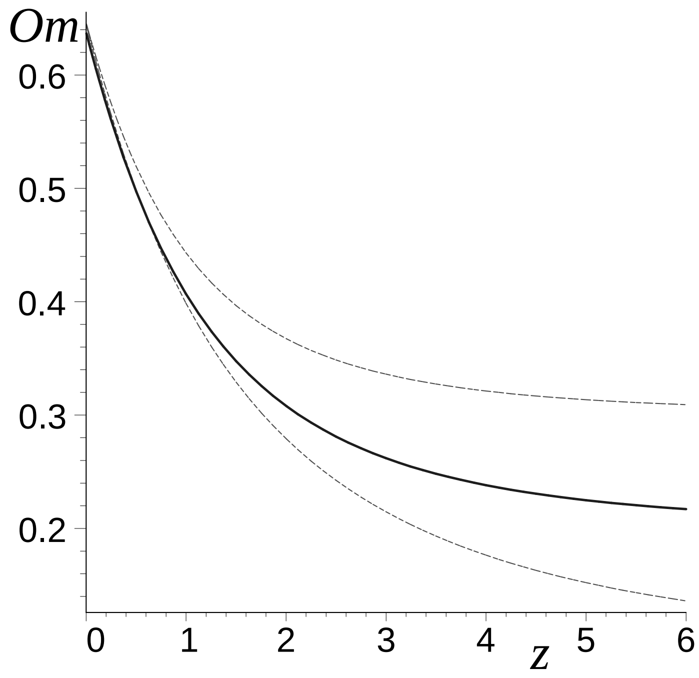


(ii)

(i) Quintessence, $w = -0.9, \Omega_{M0} = 0.27$; **(ii)** Phantom, $w = -1.1, \Omega_{M0} = 0.27$.

- Sample DE models: (i) quintessence; (ii) phantom. [Sahni, Shafieloo and Starobinsky, PR D78, 103502 (2008)]

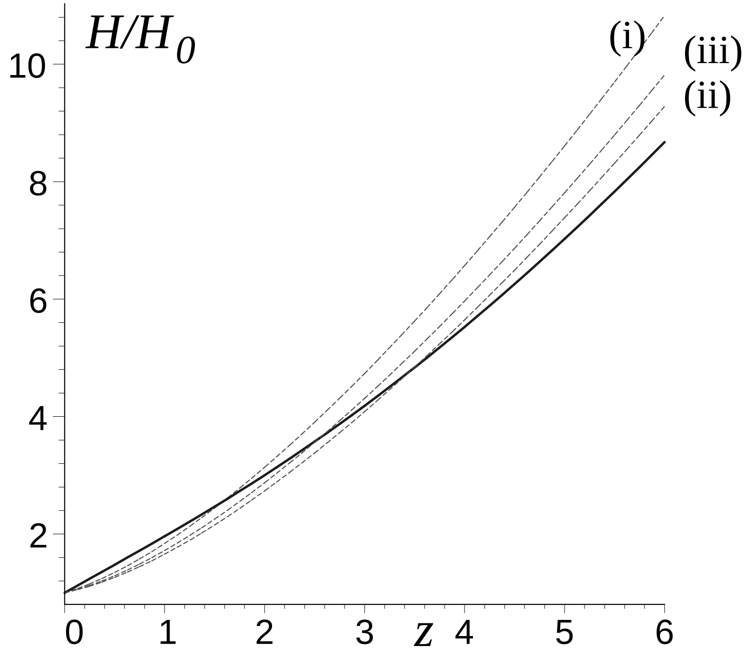
$Om(z)$ test



$Om(z)$ for the tracker solution with best–fit value $f_{\mathrm{v}0} = 0.762$ (solid line), and 1σ limits

- $Om(0) = \frac{2}{3}\, H'|_0 = \frac{2(8f_{\mathrm{v}0}{}^3 - 3f_{\mathrm{v}0}{}^2 + 4)(2 + f_{\mathrm{v}0})}{4f_{\mathrm{v}0}{}^2 + f_{\mathrm{v}0} + 4}$ is larger than for DE models
- For large z, does not asymptote to Ω_{M0} but to $Om(\infty) = \frac{2(1 - f_{\mathrm{v}0})(2 + f_{\mathrm{v}0})^3}{(4f_{\mathrm{v}0}{}^2 + f_{\mathrm{v}0} + 4)^2}) < \Omega_{M0}$ if $f_{\mathrm{v}0} > 0.25$.

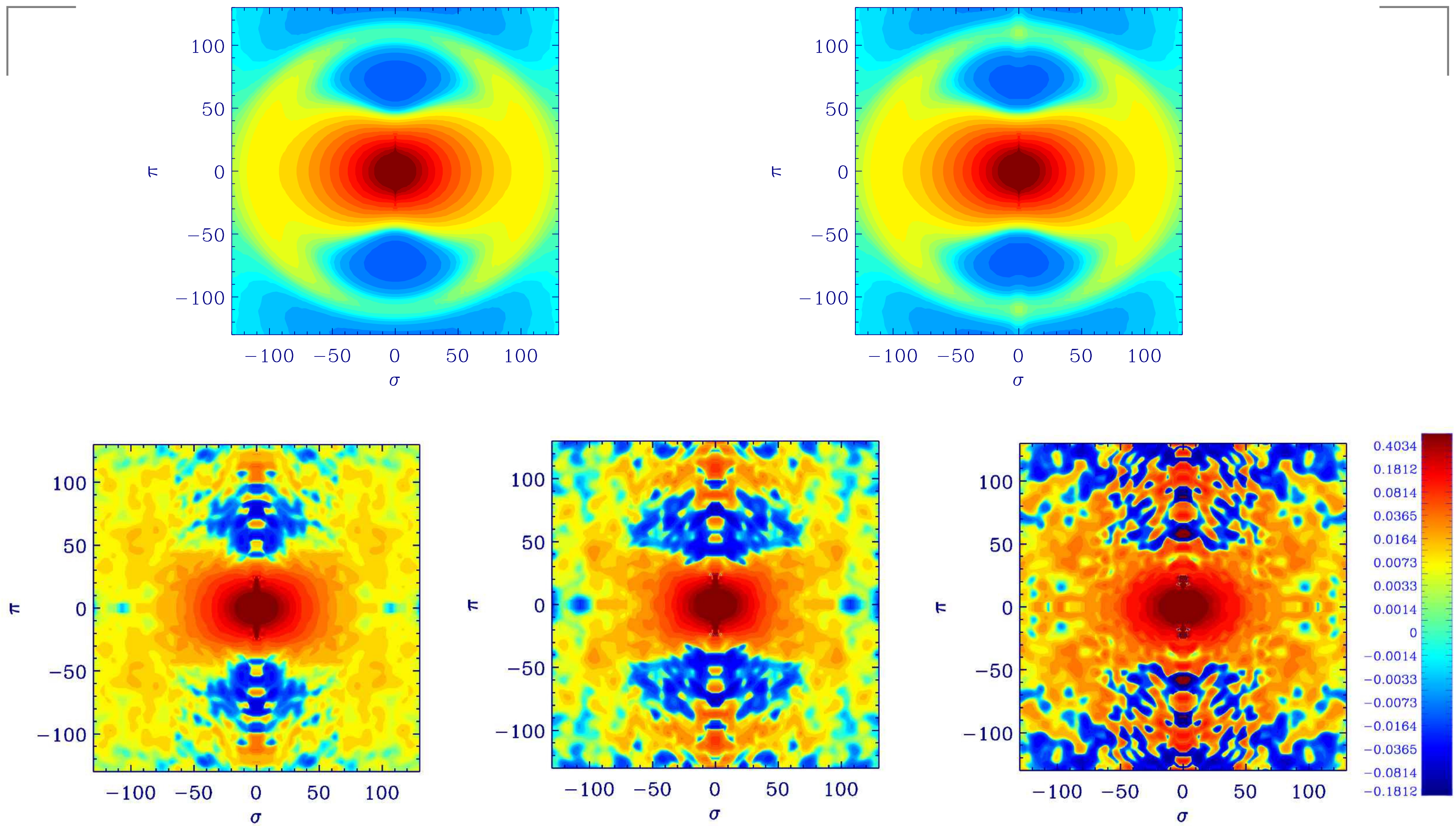
$H(z)/H_0$



$H(z)/H_0$ for $f_{v0} = 0.762$ (solid line) is compared to three spatially flat ΛCDM models: **(i)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.249, 0.751)$; **(ii)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.34, 0.66)$; **(iii)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.279, 0.721)$.

- Function $H(z)/H_0$ displays quite different characteristics
- For $0 < z \lesssim 1.7$, $H(z)/H_0$ is larger for FB model, but value of H_0 assumed also affects $H(z)$ numerical value

Gaztañaga, Cabre and Lui 0807.3551



$z = 0.15\text{-}0.47$ $z = 0.15\text{-}0.30$ $z = 0.40\text{-}0.47$

Gaztañaga, Cabre and Lui 0807.3551

- From redshift space distortions of BAO scale (Kaiser effect) for LCDM model, $H_0 = 71.6$ km sec^{-1} Mpc^{-1},
  $H(0.24) = 79.7 \pm 2.1(\pm 1.0)$ km sec^{-1} Mpc^{-1} and
  $H(0.43) = 86.5 \pm 2.15(\pm 1.0)$ km sec^{-1} Mpc^{-1}
- For FB model, $H_0 = 61.7$ km sec^{-1} Mpc^{-1},
  $H(0.24) = 75.9^{+1.7}_{-1.3}$ km sec^{-1} Mpc^{-1};
  $H(0.43) = 87.0^{+1.6}_{-1.5}$ km sec^{-1} Mpc^{-1}
  for Riess07 gold data.
- Need to reanalyse BAO data reduction to compare H(z); but Kaiser analysis uses perturbation theory about FLRW, which also has to be revisited.

Redshift time drift (Sandage–Loeb test)

- $H(z)/H_0$ measurements are model dependent.
  However,

$$\frac{1}{H_0}\frac{\mathrm{d}z}{\mathrm{d}\tau} = 1 + z - \frac{H}{H_0}$$

- For ΛCDM

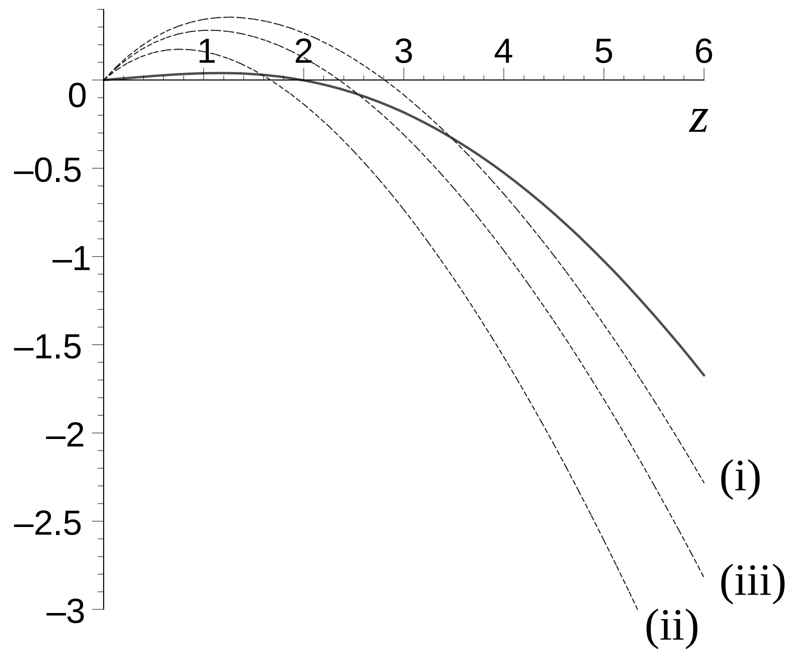
$$\frac{1}{H_0}\frac{\mathrm{d}z}{\mathrm{d}t} = (1+z) - \sqrt{\Omega_{M0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{k0}(1+z)^2}.$$

- For FB model

$$\frac{1}{H_0}\frac{\mathrm{d}z}{\mathrm{d}\tau} = 1 + z - \frac{3\left(2t^2 + 3bt + 2b^2\right)}{H_0 t\,(2t+3b)^2}\,,$$

where t is given implicitly in terms of z.

Redshift time drift (Sandage–Loeb test)



$H_0^{-1}\,\frac{\mathrm{d}z}{\mathrm{d}\tau}$ for the FB model with $f_{\mathrm{v}0} = 0.762$ (solid line) is compared to three spatially flat ΛCDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in previous figures.

- Measurement is extremely challenging. May be feasible over a 10–20 year period by precision measurements of the Lyman-α forest over redshift $2 < z < 5$ with next generation of Extremely Large Telescopes

Clarkson, Bassett and Lu homogeneity test

- For FLRW equations, irrespective of dark energy model

$$\Omega_{k0} = \frac{\mathcal{B}}{[H_0 D(z)]^2} = \text{const}$$

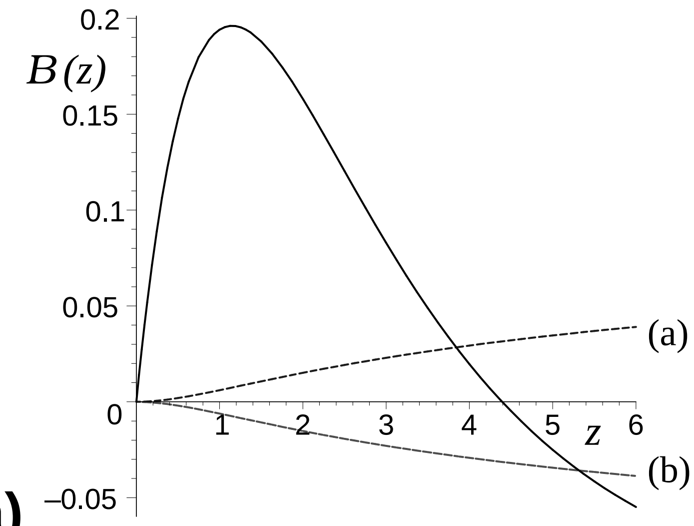
where $\mathcal{B}(z) \equiv [H(z)D'(z)]^2 - 1$. Thus

$$\mathcal{C}(z) \equiv 1 + H^2(DD'' - D'^2) + HH'DD' = 0$$

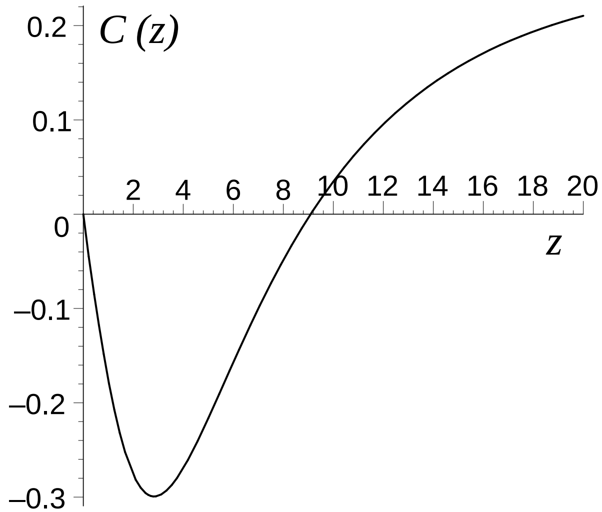
for any homogeneous isotropic cosmology, irrespective of DE.

- Clarkson, Bassett and Lu [PRL 101 (2008) 011301] call this a “test of the Copernican principle”. However, it is merely a test of (in)homogeneity.

Clarkson, Bassett and Lu homogeneity test



(a)

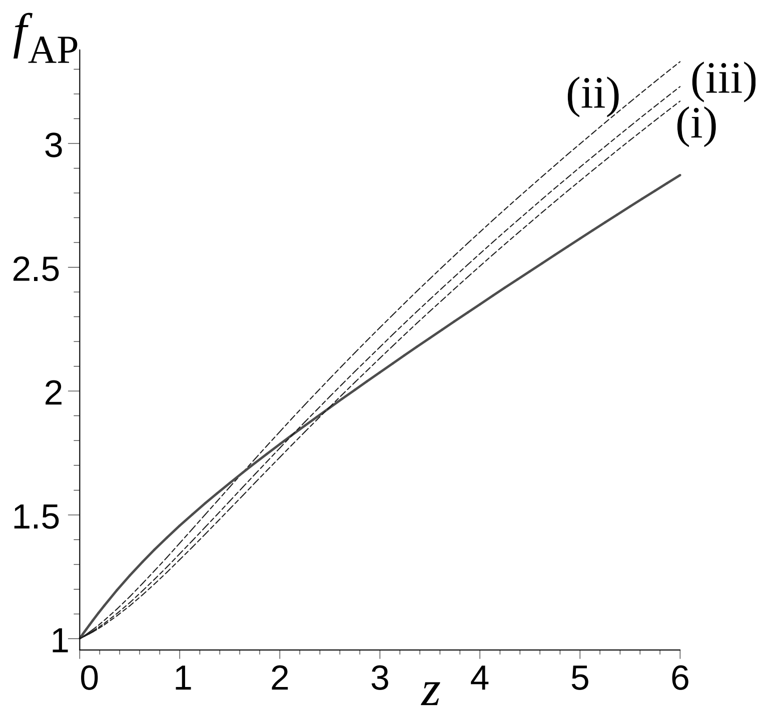


(b)

(a) $\mathcal{B} \equiv [H(z)D'(z)]^2 - 1$ for FB model with $f_{v0} = 0.762$ (solid line) and two ΛCDM models (dashed lines): **(i)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda 0} = 0.71$, $\Omega_{k0} = 0.01$; **(ii)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda 0} = 0.73$, $\Omega_{k0} = -0.01$; **(b)** $\mathcal{C}(z)$.

- Will give a powerful test of FLRW assumption in future, with quantitative different prediction for FB model.

Alcock–Paczyński test

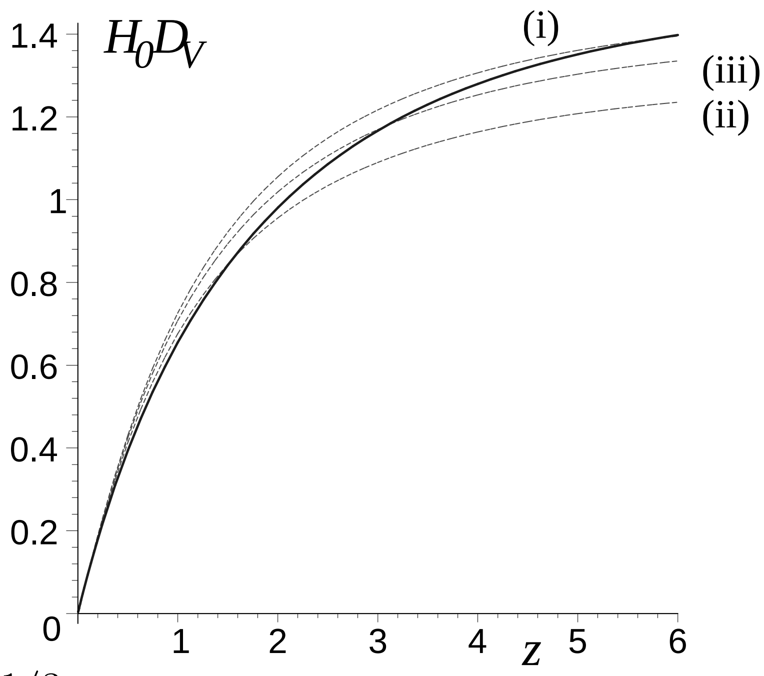


$f_{\rm AP} = \frac{1}{z}\left|\frac{\delta\theta}{\delta z}\right|$ for the tracker solution with $f_{\rm v0} = 0.762$ (solid line) is compared to three spatially flat ΛCDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in earlier figures

- For a comoving standard ruler subtending and angle θ,

$$f_{\rm AP} = \frac{1}{z}\left|\frac{\delta\theta}{\delta z}\right| = \frac{HD}{z} = \frac{3\left(2t^2+3bt+2b^2\right)(1+z)D_A}{t\,(2t+3b)^2\,z}$$

Baryon Acoustic Oscillation test function

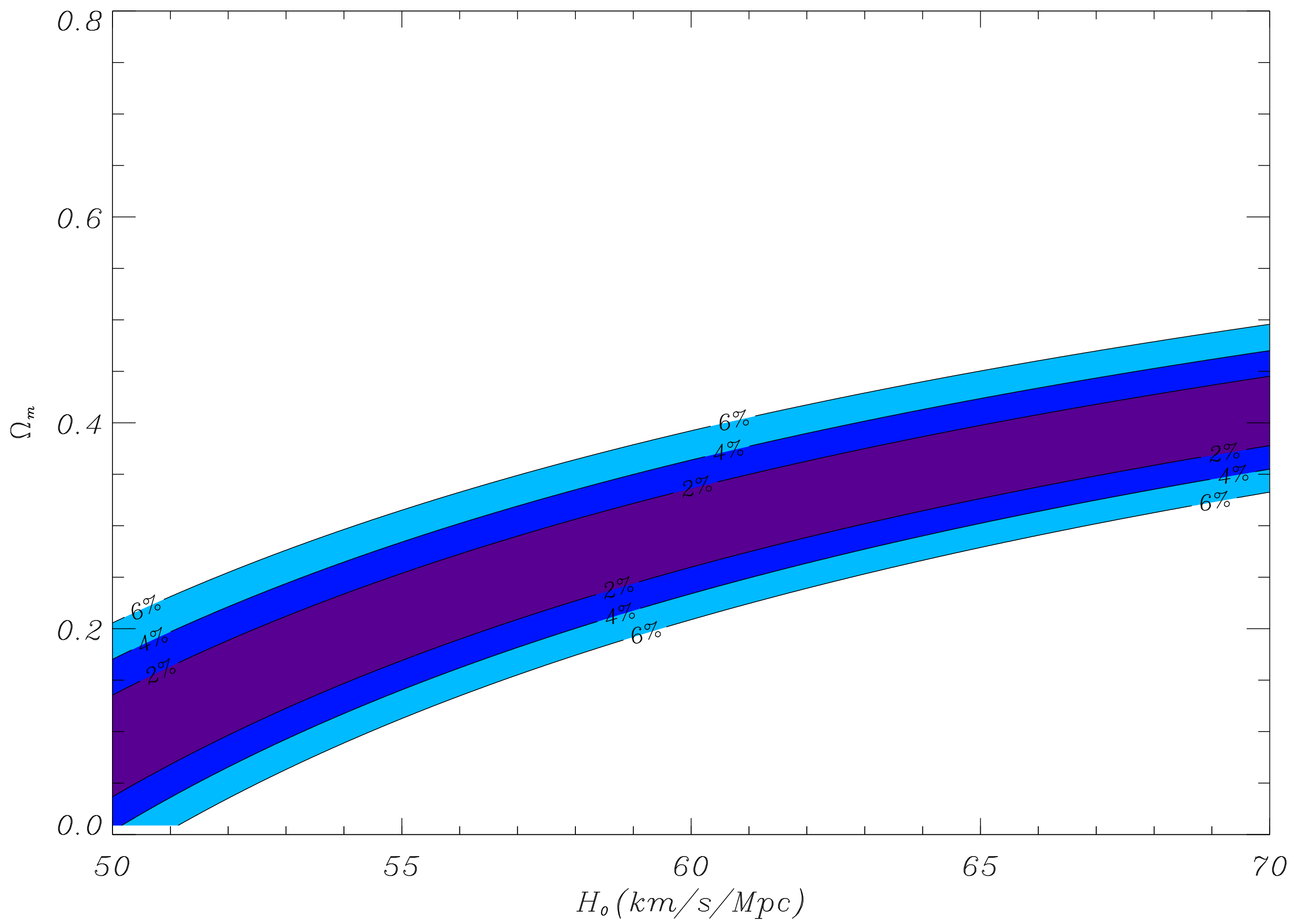


$H_0 D_V = H_0 D f_{\rm AP}^{-1/3}$ for the tracker solution with $f_{\rm v0} = 0.762$ (solid line) is compared to three spatially flat ΛCDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in earlier figures

- BAO tests of galaxy clustering typically consider

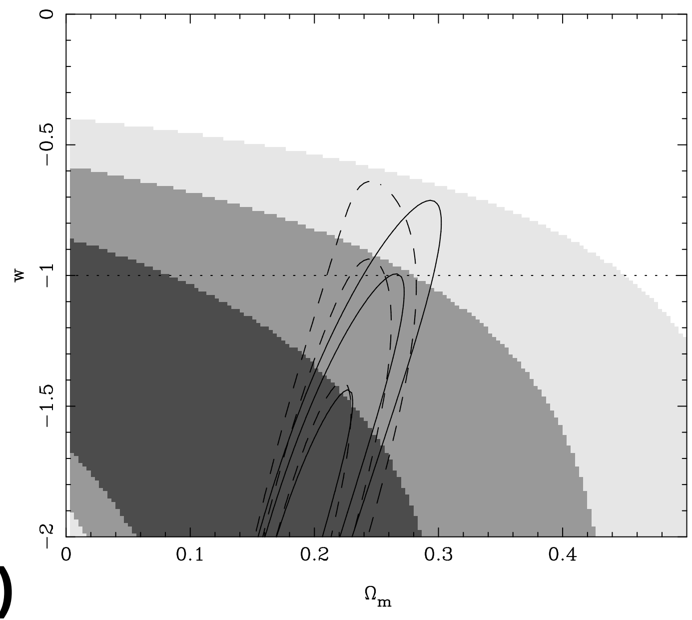
$$D_V = \left[\frac{zD^2}{H(z)}\right]^{1/3} = D f_{\rm AP}^{-1/3}.$$

Test 2: Baryon acoustic oscillation scale

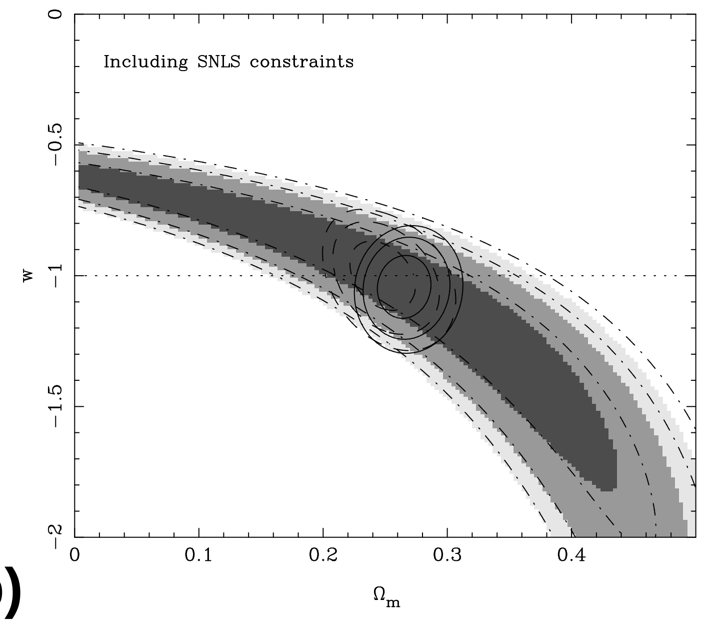


Parameters within the (Ω_m,H_0) plane which fit the effective comoving baryon acoustic oscillation scale of $104h^{-1}$ Mpc, as seen in 2dF and SDSS.

$D_V(0.35)/D_V(0.2)$



(a)



(b)

ΛCDM constraints on $w(z)$ of Percival et al. (2007): **(a)** Likelihood contours from $D_V(0.35)/D_V(0.2)$; **(b)** As for (a) with SNLS constraints added.

- With ΛCDM Percival et al estimate $D_V(0.35)/D_V(0.2) = 1.812 \pm 0.060$ from galaxy clustering compared to estimate of 1.666 ± 0.010 from SNLS: 2.4σ discrepancy
- For FB model $D_V(0.35)/D_V(0.2) = 1.632^{+0.005}_{-0.007}$ estimate from Riess07 gold data BUT clustering data reduction has to be revisited

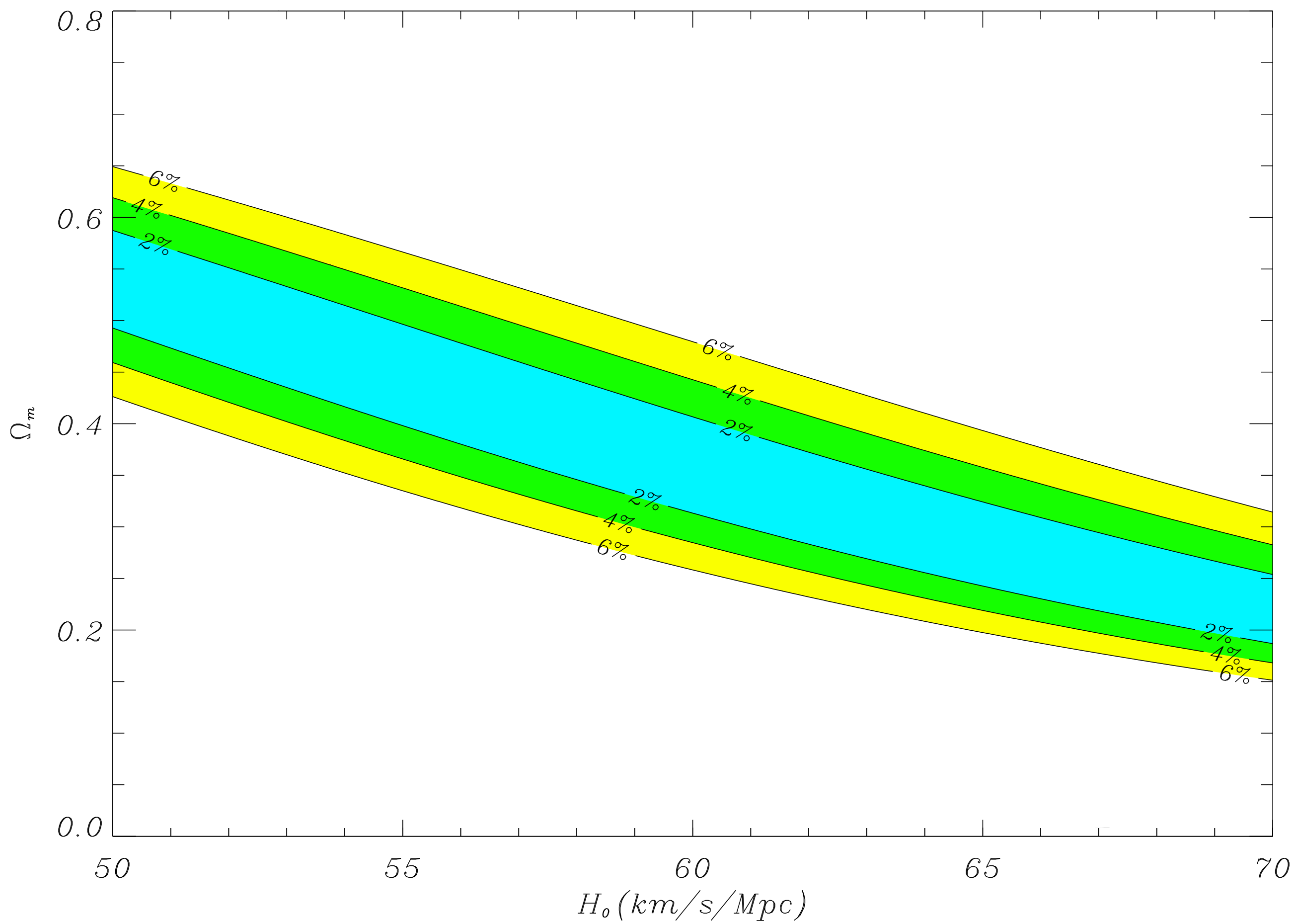
CMB – calibration of sound horizon

- Physics at last–scattering same as perturbed Einstein–de Sitter model. What is changed is relative calibration between now and then.
- Estimated proper distance to comoving scale of the sound horizon at any epoch for volume–average observer [$\bar{x} = \bar{a}/\bar{a}_0$, so $\bar{x}_{\rm dec} = \bar{\gamma}_0^{-1}(1+z_{\rm dec})^{-1}$]

$$\bar{D}_s = \frac{\bar{a}(t)}{\bar{a}_0}\frac{c}{\sqrt{3}\,\bar{H}_0}\int_0^{\bar{x}_{\rm dec}} \frac{{\rm d}\bar{x}}{\sqrt{(1+0.75\,\bar{\Omega}_{B0}\bar{x}/\bar{\Omega}_{\gamma 0})(\bar{\Omega}_{M0}\bar{x}+\bar{\Omega}_{R0})}},$$

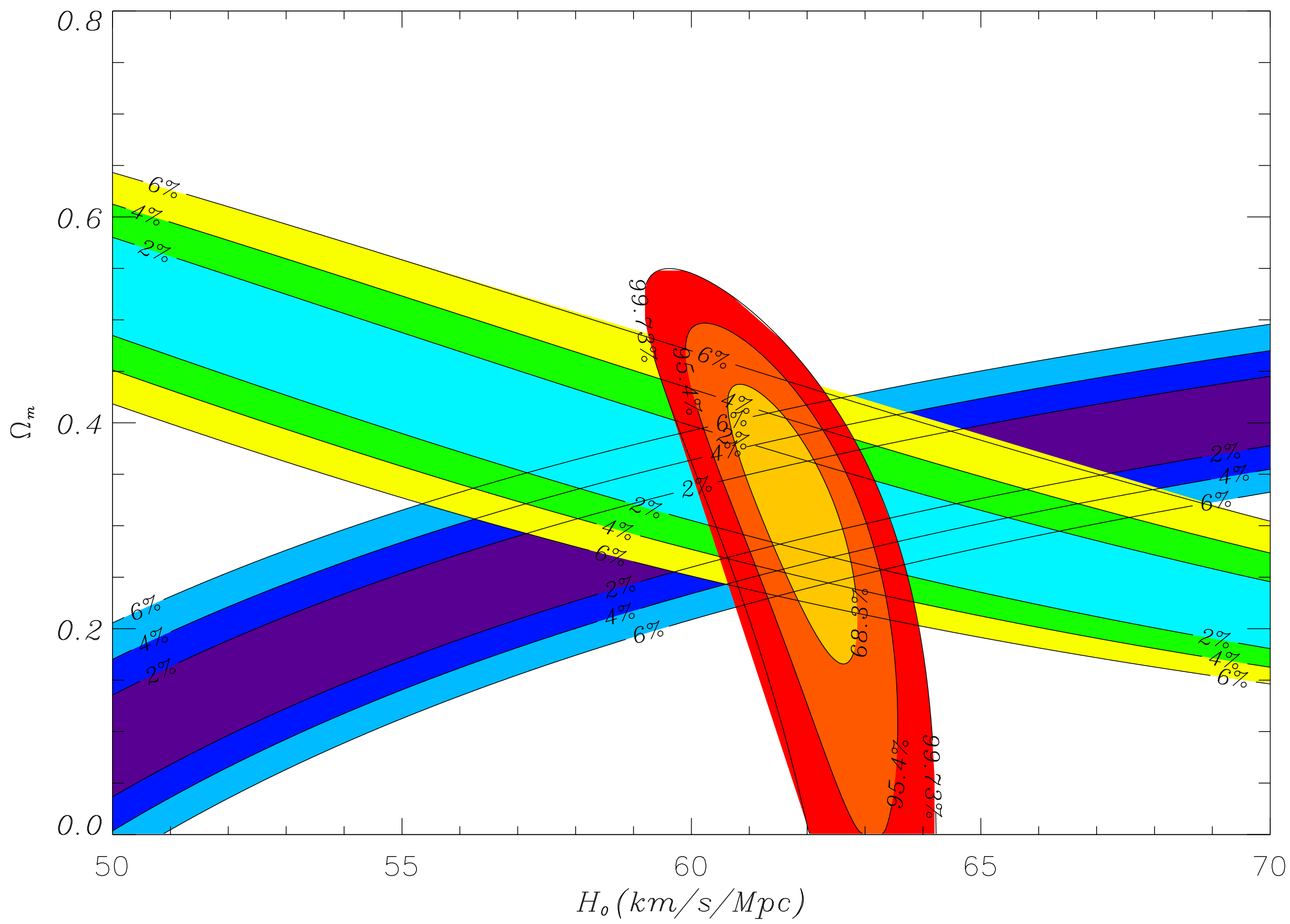
- For wall observer $D_s(\tau) = \bar{\gamma}^{-1} D_s$
- Volume–average observer measures lower mean CMB temperature ($\bar{T}_0 = T_0/\bar{\gamma}_0 \sim 1.98$ K, c.f. $T_0 \sim 2.73$ K in walls)

Test 3: Angular scale of CMB Doppler peaks



Parameters within the (Ω_m, H_0) plane which fit the angular scale of the sound horizon $\delta = 0.01$ rad deduced for WMAP, to within 2%, 4% and 6%.

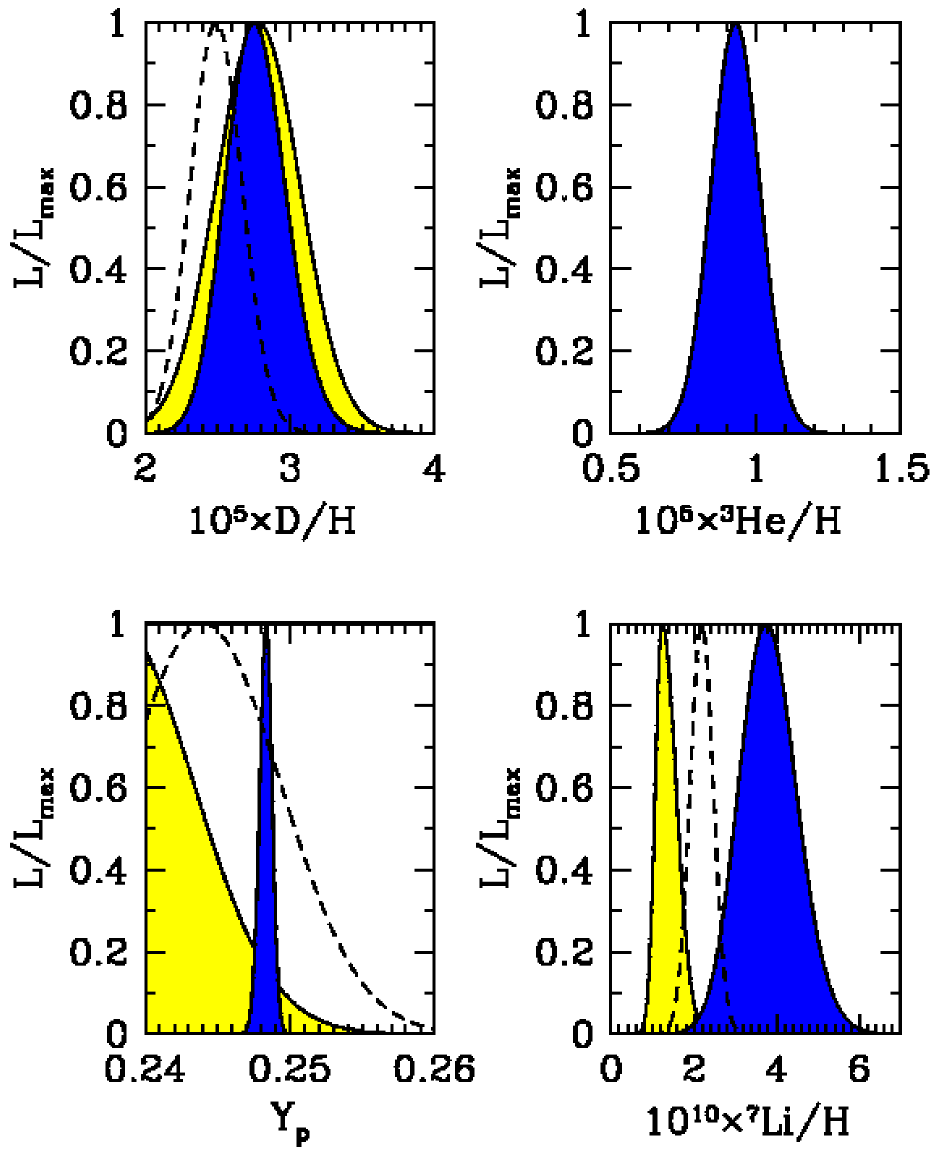
Agreement of independent tests



Best–fit parameters: $H_0 = 61.7^{+1.2}_{-1.1}$ km sec^{-1} Mpc^{-1}, $\Omega_m = 0.33^{+0.11}_{-0.16}$ (1σ errors for SneIa only) [Leith, Ng & Wiltshire, ApJ 672 (2008) L91]

Li abundance anomaly

Big-bang nucleosynthesis, light element abundances and WMAP with ΛCDM cosmology.



Primordial lithium abundance

- Recalibration of baryon/photon ratio means lithium abundance problem can be potentially eliminated
- Use local baryon–to–photon ratio $\eta_{B\gamma} = 4.6$–5.6×10^{-10} admitting concordance with lithium abundances favoured prior to WMAP in 2003
- Conventional dressed parameter $\Omega_{M0} = 0.33$ for wall observer means $\bar{\Omega}_{M0} = 0.127$ for the volume–average.
- Conventional theory predicts the *volume–average baryon fraction*. With old BBN favoured $\eta_{B\gamma}$:

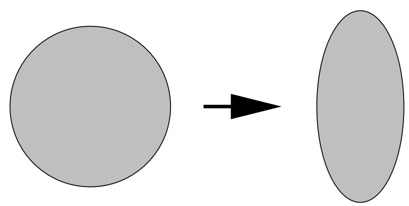
  $\bar{\Omega}_{B0} \simeq 0.027$–$0.033$; but this translates to a conventional dressed baryon fraction parameter $\Omega_{B0} \simeq 0.072$–0.088
- Detailed analysis of CMB peaks is a huge task; ISW effect has to be revisited from first principles

ISW amplitude

- Relative focussing between voids and walls
- Integrated Sachs–Wolfe effect needs recomputation
- Correlation of radio-galaxies, voids and superclusters etc with CMB positively detected and well established (Boughn and Crittenden 2004, … Granett, Neyrinck and Szapudi, 2008)
- Amplitude of effect consistently of order 2σ greater than LCDM prediction
- Exact amplitude needs to be determined. However, since $|\bar{\Omega}_{\mathcal{Q}}| \lesssim 0.42$ maybe rough order of magnitude similar to that of an open $k = -1$ universe with $\bar{\Omega}_{M0} = 0.127$, $t_0 = 18.6$ Gyr?

Spatial curvature: ellipticity puzzle

- Negative spatial curvature should manifest itself in other ways than angular–diameter distance of sound horizon
- Claim: greater geodesic mixing from negative spatial curvature registers ellipticity in the CMB anisotropy spectrum



- Ellipticity has been detected since COBE, and statistical significance increases with each data release (Gurzadyan et al., Phys. Lett. **A 363** (2007) 121; Mod. Phys. Lett. **A 20** (2005) 813,. . .)
- Theoretical derivation needs to be better understood; directly from Sachs optical equation

Other CMB puzzles

- Axis–of–evil alignment of large angle multipoles; Power asymmetry; Dark flow (Kashlinsky et al); Cold spot;
- Freeman *et al*., ApJ 638 (2006) 1, find from an analysis of several possible systematic errors that only one: a 1-2% increase of our peculiar velocity to recalibrate our CMB dipole could account for axis–of–evil anomaly
- A random alignment of foreground voids below scale of homogeneity with Rees–Sciama effect could account for this. (C.f., e.g. Inoue and Silk, 2006.) A Rees–Sciama dipole of order $\Delta T/T \sim 10^{-5}$ would come with Rees–Sciama quadrupole of same order etc
- Recalibration of dipole would fundamentally change assessment of puzzles such as cold spot, dark flow etc

Alleviation of age problem

- Old structures seen at large redshifts are a problem for ΛCDM.
- Problem alleviated here; expansion age is increased, by an increasingly larger relative fraction at larger redshifts, e.g., for best–fit values
  ΛCDM $\tau = 0.85$ Gyr at $z = 6.42$, $\tau = 0.365$ Gyr at $z = 11$
  FB $\tau = 1.14$ Gyr at $z = 6.42$, $\tau = 0.563$ Gyr at $z = 11$
- Present age of universe for best-fit is $\tau_0 \simeq 14.7$ Gyr for wall observer; $t_0 \simeq 18.6$ Gyr for volume–average observer.
- Suggests problems of under–emptiness of voids in Newtonian N-body simulations may be an issue of using volume–average time?? The simulations need to carefully reconsidered.

Variance of Hubble flow

- Relative to “wall clocks” the global average Hubble parameter $H_{\rm av} > \bar H$
- $\bar H$ is nonetheless also the locally measurable Hubble parameter within walls
- TESTABLE PREDICTION:

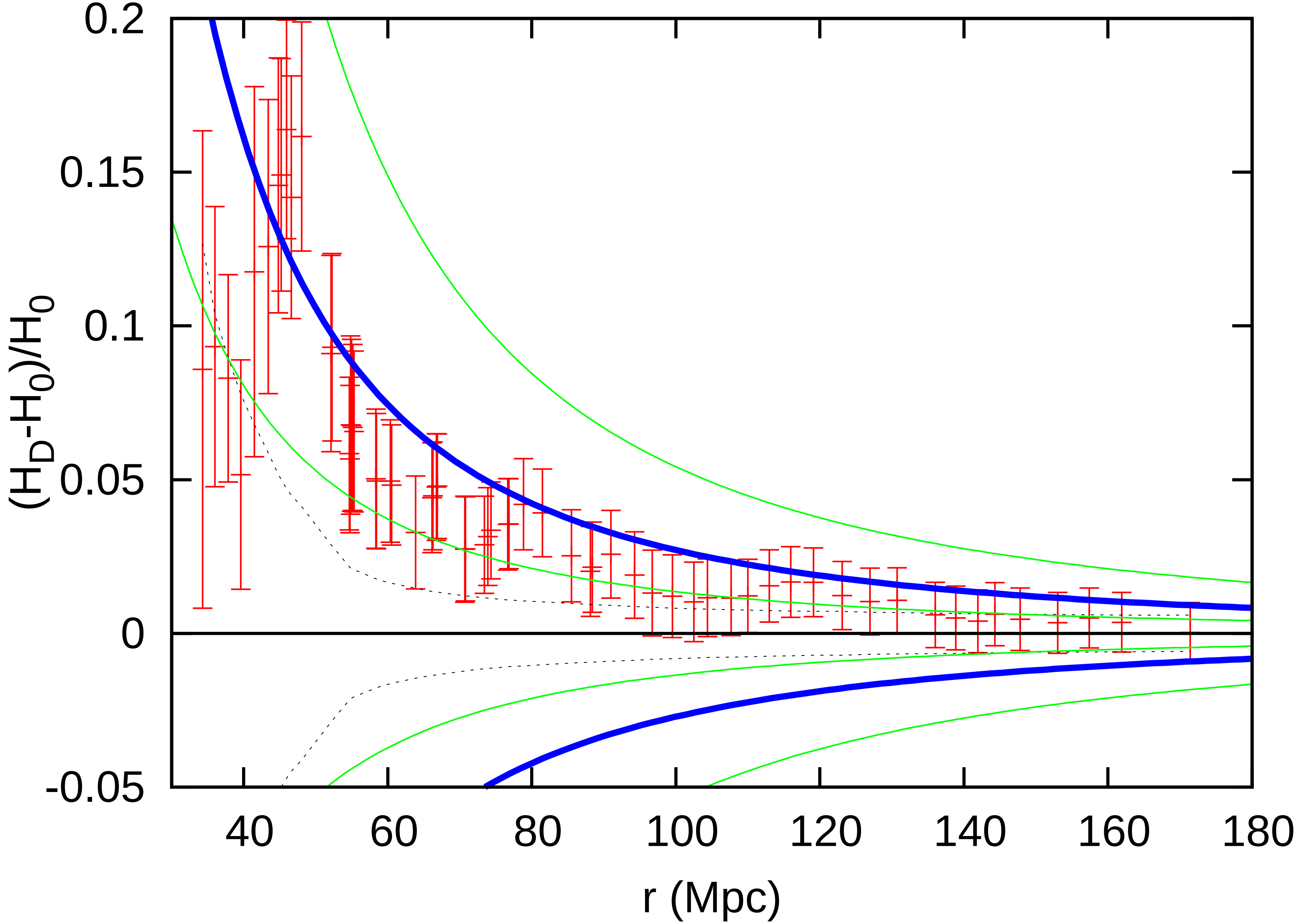
$$H_{\rm av} = \bar\gamma_{\rm w}\bar H - \bar\gamma_{\rm w}^{-1}\bar\gamma_{\rm w}'$$

- With $H_0 = 62\,$km sec^{-1} Mpc^{-1}, expect according to our measurements:
  $\bar H_0 = 48\,$km sec^{-1} Mpc^{-1} within ideal walls (e.g., around Virgo cluster?); and
  $\bar H_{\rm v0} = 76\,$km sec^{-1} Mpc^{-1} across local voids (scale $\sim$ 45 Mpc)

Explanation for Hubble bubble

- As voids occupy largest volume of space expect to measure higher average Hubble constant locally until the global average relative volumes of walls and voids are sampled at scale of homogeneity; thus expect maximum H_0 value for isotropic average on scale of dominant void diameter, $30h^{-1}$Mpc, then decreasing til levelling out by $100h^{-1}$Mpc.
- Consistent with observed Hubble bubble feature (Jha, Riess, Kirshner ApJ 659, 122 (2007)), which is unexplained (and problem for) ΛCDM.
- Intrinsic variance in apparent Hubble flow exposes a local scale dependence which may partly explain difficulties astronomers have had in converging on a value for H_0.

N. Li and D. Schwarz, arxiv:0710.5073v1–2



Best fit parameters

- Hubble constant $H_0 = 61.7^{+1.2}_{-1.1}$ km sec^{-1} Mpc^{-1}
- present void volume fraction $f_{\rm v0} = 0.76^{+0.12}_{-0.09}$
- bare density parameter $\bar{\Omega}_{M0} = 0.125^{+0.060}_{-0.069}$
- dressed density parameter $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$
- non–baryonic dark matter / baryonic matter mass ratio $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0})/\bar{\Omega}_{B0} = 3.1^{+2.5}_{-2.4}$
- bare Hubble constant $\bar{H}_0 = 48.2^{+2.0}_{-2.4}$ km sec^{-1} Mpc^{-1}
- mean lapse function $\bar{\gamma}_0 = 1.381^{+0.061}_{-0.046}$
- deceleration parameter $q_0 = -0.0428^{+0.0120}_{-0.0002}$
- wall age universe $\tau_0 = 14.7^{+0.7}_{-0.5}$ Gyr

Model comparison

	ΛCDM	FB scenario
SneIa luminosity distances	Yes	Yes
BAO scale (clustering)	Yes	Yes
Sound horizon scale (CMB)	Yes	Yes
Doppler peak fine structure	Yes	[still to calculate]
Integrated Sachs–Wolfe effect	Yes	[still to calculate]
Primordial ^{7}Li abundances	No	Yes?
CMB ellipticity	No	[Maybe]
CMB low multipole anomalies	No	[Foreground void: Rees–Sciama dipole]
Hubble bubble	No	Yes
Nucleochronology dates of old globular clusters	Tension	Yes
X-ray cluster abundances	Marginal	Yes
Emptiness of voids	No	[Maybe]
Sandage-de Vaucouleurs paradox	No	Yes
Coincidence problem	No	Yes

Discussion

- Much work remains to be done!
- Do minivoids have a separate curvature scale?
- Averaging scheme should be revisited using, e.g., constant mean (extrinsic) curvature (CMC) hypersurfaces below scale of apparent homogeneity; or other generalization of Bičak, Katz and Lynden–Bell perturbative gauge.
- For such a foliation one must excise finite infinity regions; and treat them different. Necessary also in Buchert approach since vorticity significant in bound structures
- Physical/mathematical formulation in terms of quasilocal energy needs to be fully explored. Sussman has begun looking at spherical symmetric case.

Discussion

- Possible relation to Ricci flow (Buchert and Carfora 2003, 2008) needs to be explored
- Rethink Newtonian approximation and N-body simulations
- Precision cosmology is entering a regime in which LCDM can be tested, against other forms of dark energy and against the “timescape” model
- Tests presented here will help distinguish models
- Prediction of scale dependence of Hubble flow within $100h^{-1}$Mpc is a very distinctive prediction
- No old-style “new” physics here, but much new physics left unanswered in the old physics of GR.