Structure and Production of *p*-shell Ξ-Hypernuclei

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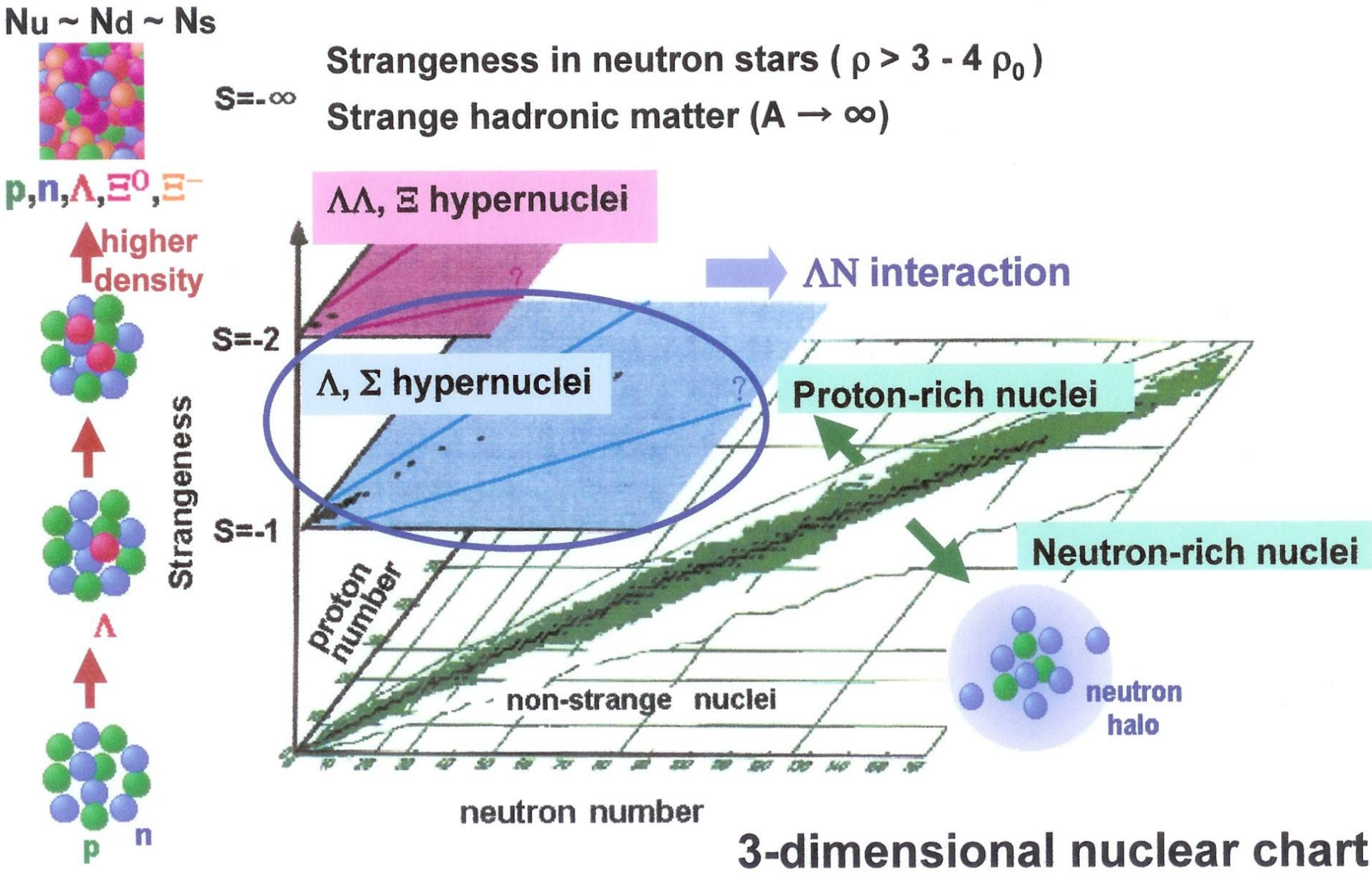
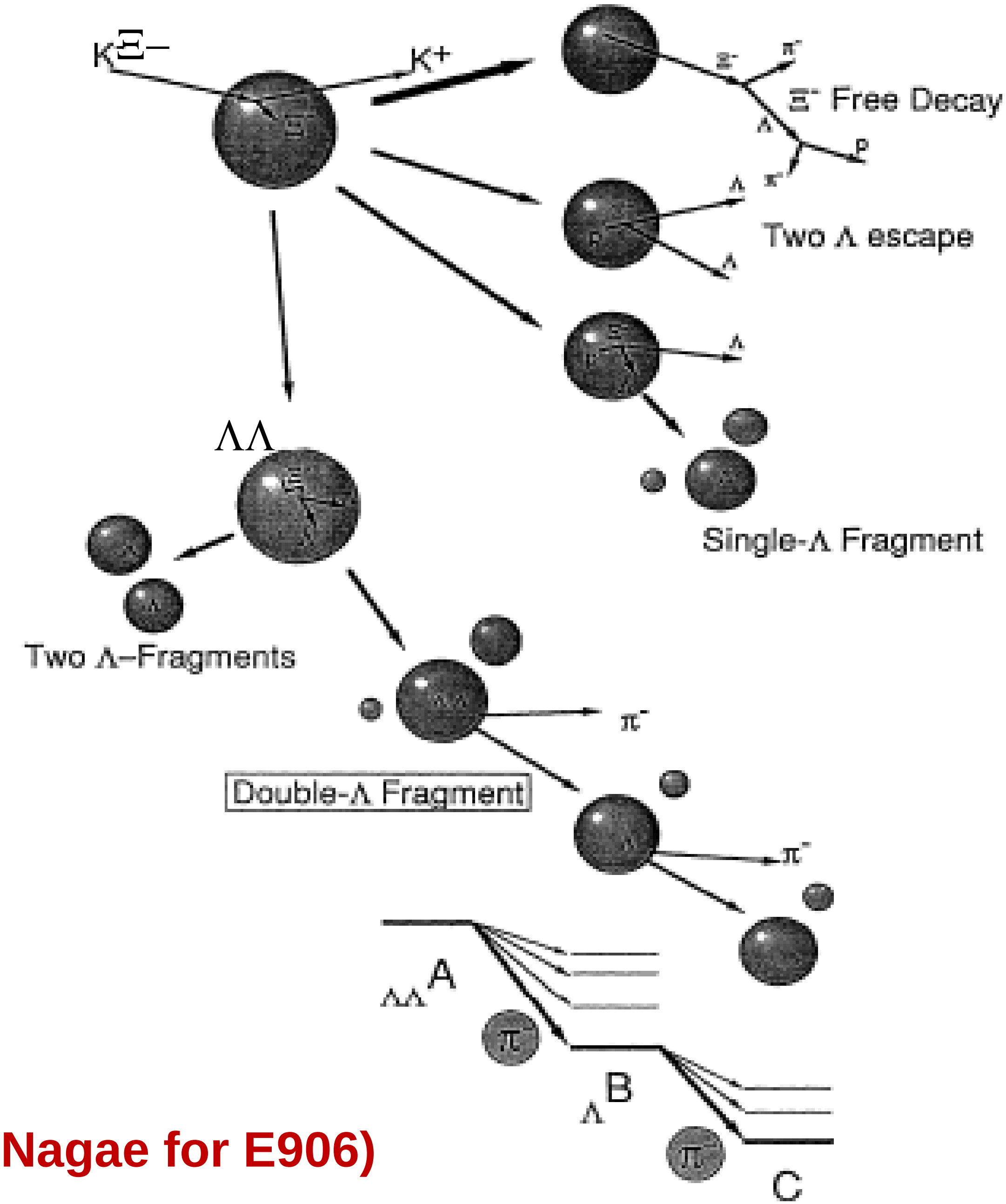
Hyp-X
Sept. 13-18, 2009, Tokaii

keV



(from H. Tamura)

Ξ-hypernuc. Formation is an entrance to the S=-2 world

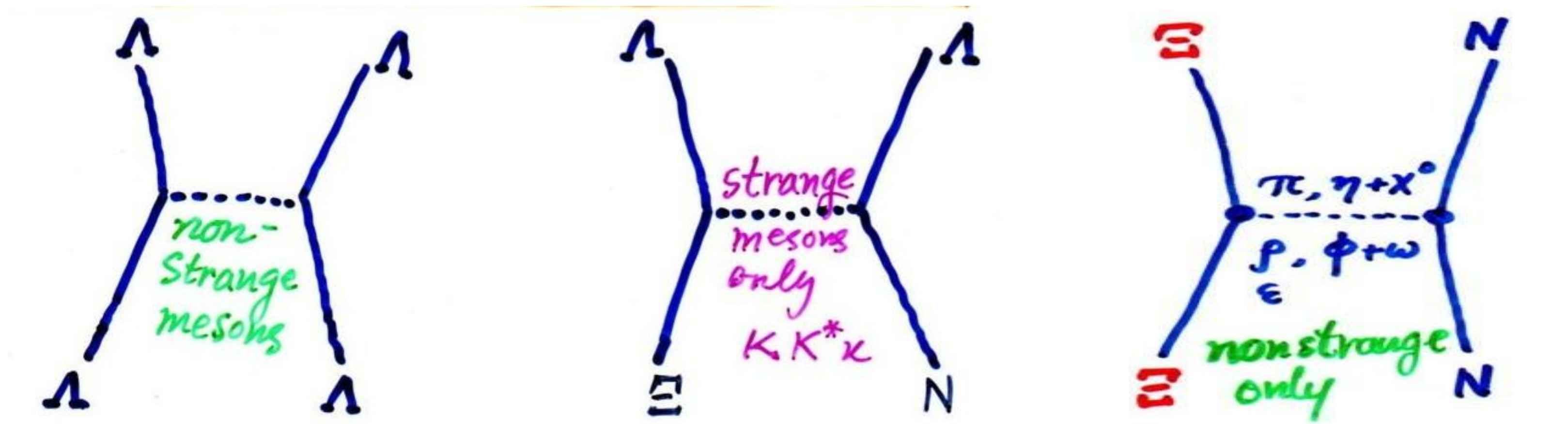


(Original illustration by Nagae for E906)

Fig. 1. A sketch for the whole process of formation and decays of $S=-2$ states in nuclei.

Why Ξ-hypernuclei ?

1) They provide unique information on the *S*=-2 B-B interactions inaccessible otherwise.



2) High-priority experiment at J-PARC（2009－

E-05: “Spectroscopic study of Ξ-hypernucleus via the $^{12}C(K^-,K^+)\ _{\Xi}^{12}Be$ reaction” by T. Nagae et al.

→ Realistic Calculations are required.

Focus on the theoretical status of Ξ-hypernuclear productions

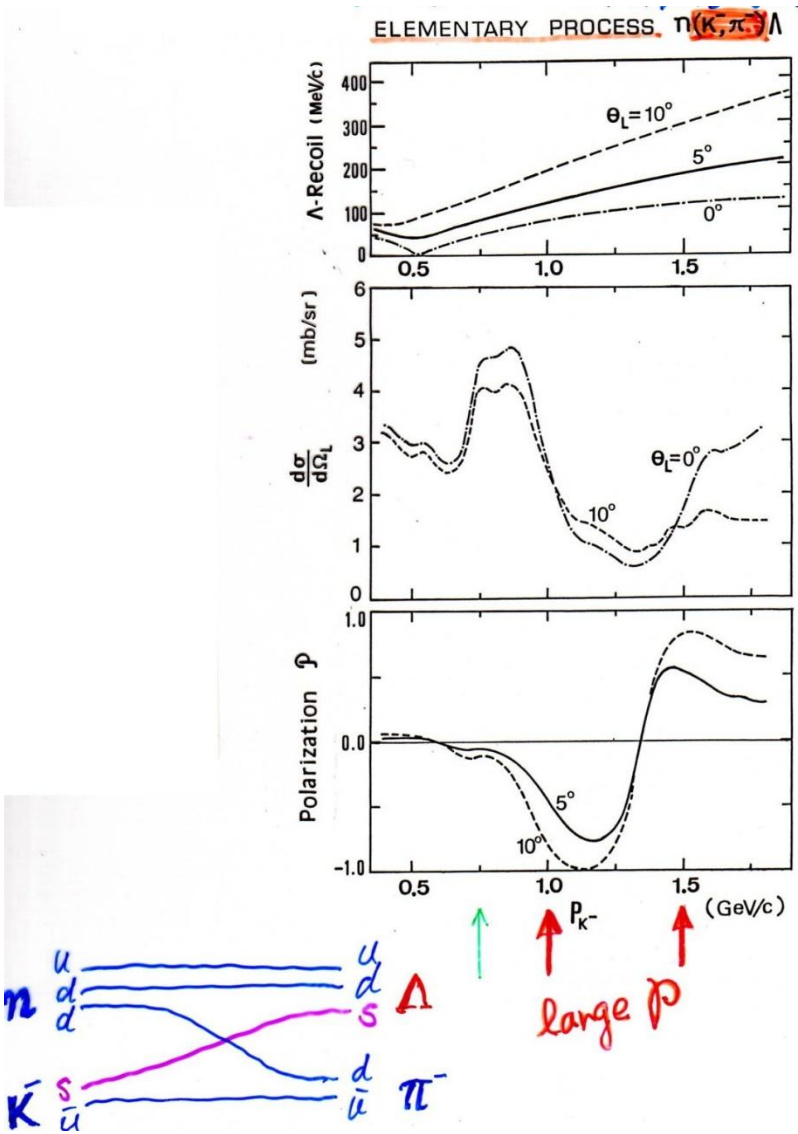
1. DWIA based on the one-body motion of Ξ in an average potential (WS)

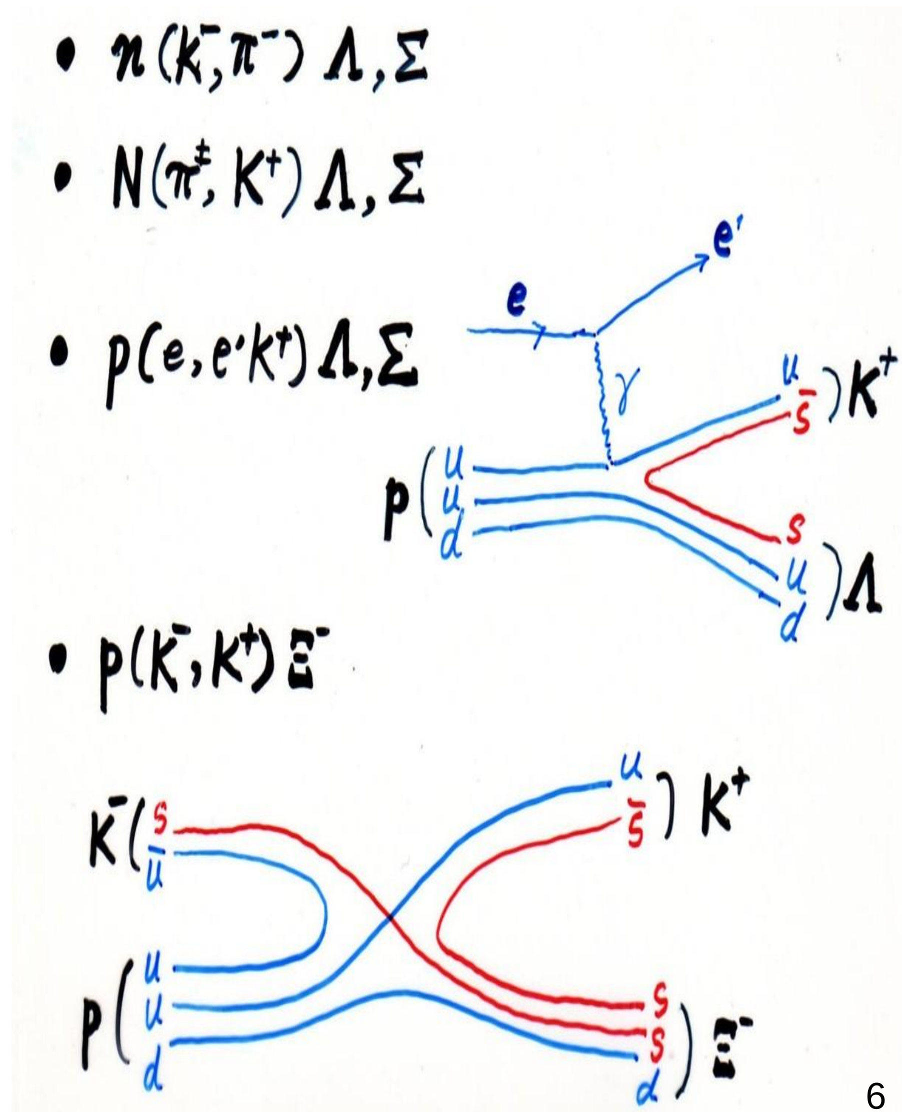
2. DWIA with use of many-body Ξ hypernuclear W.F. based on the available Ξ-N interactions

3. Possible few other targets next to ^{12}C

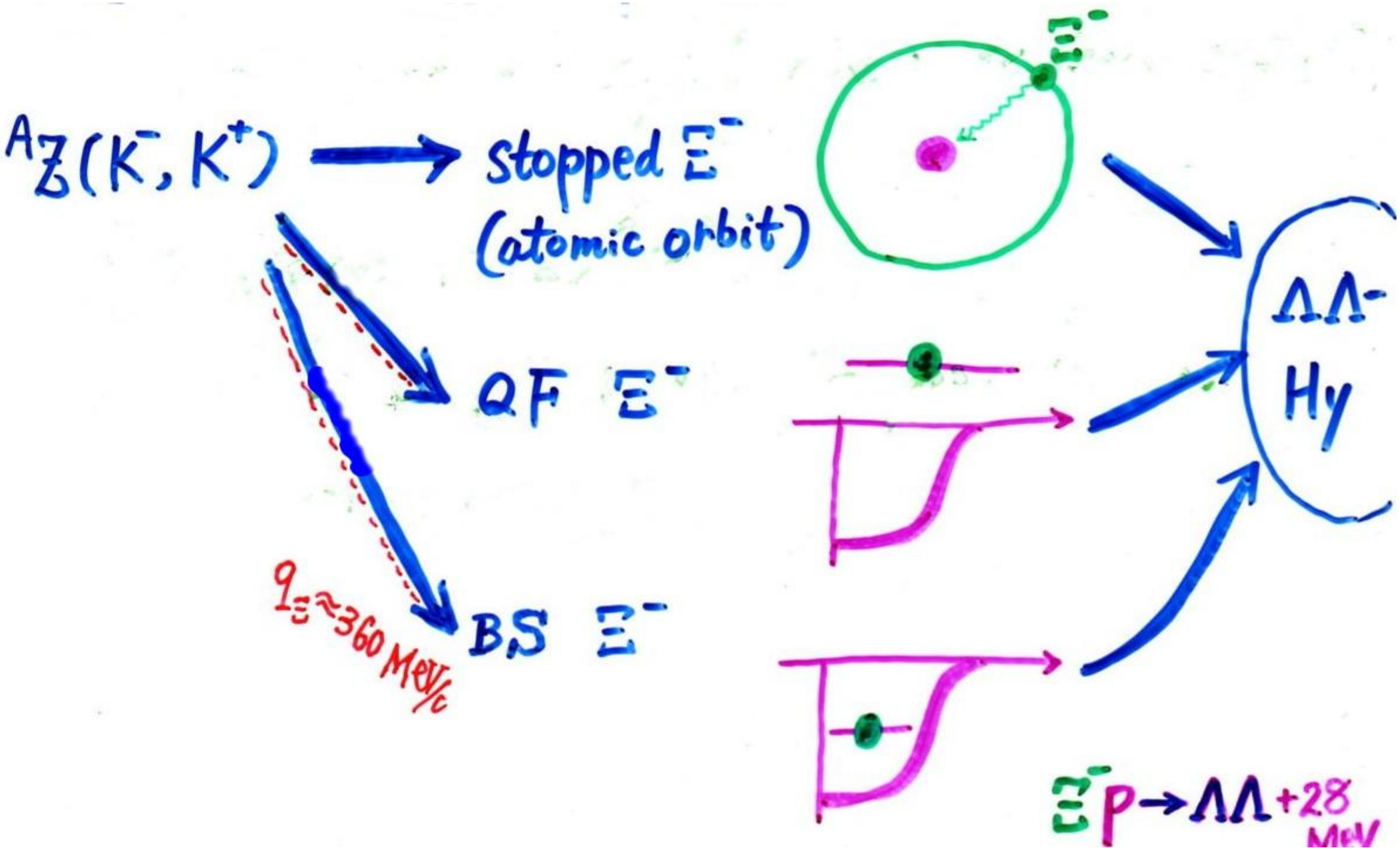
4. Summary

How to produce Ξ hyperon: p(K-,K+) Ξ





Three cases of Ξ production on **nuclear targets:**



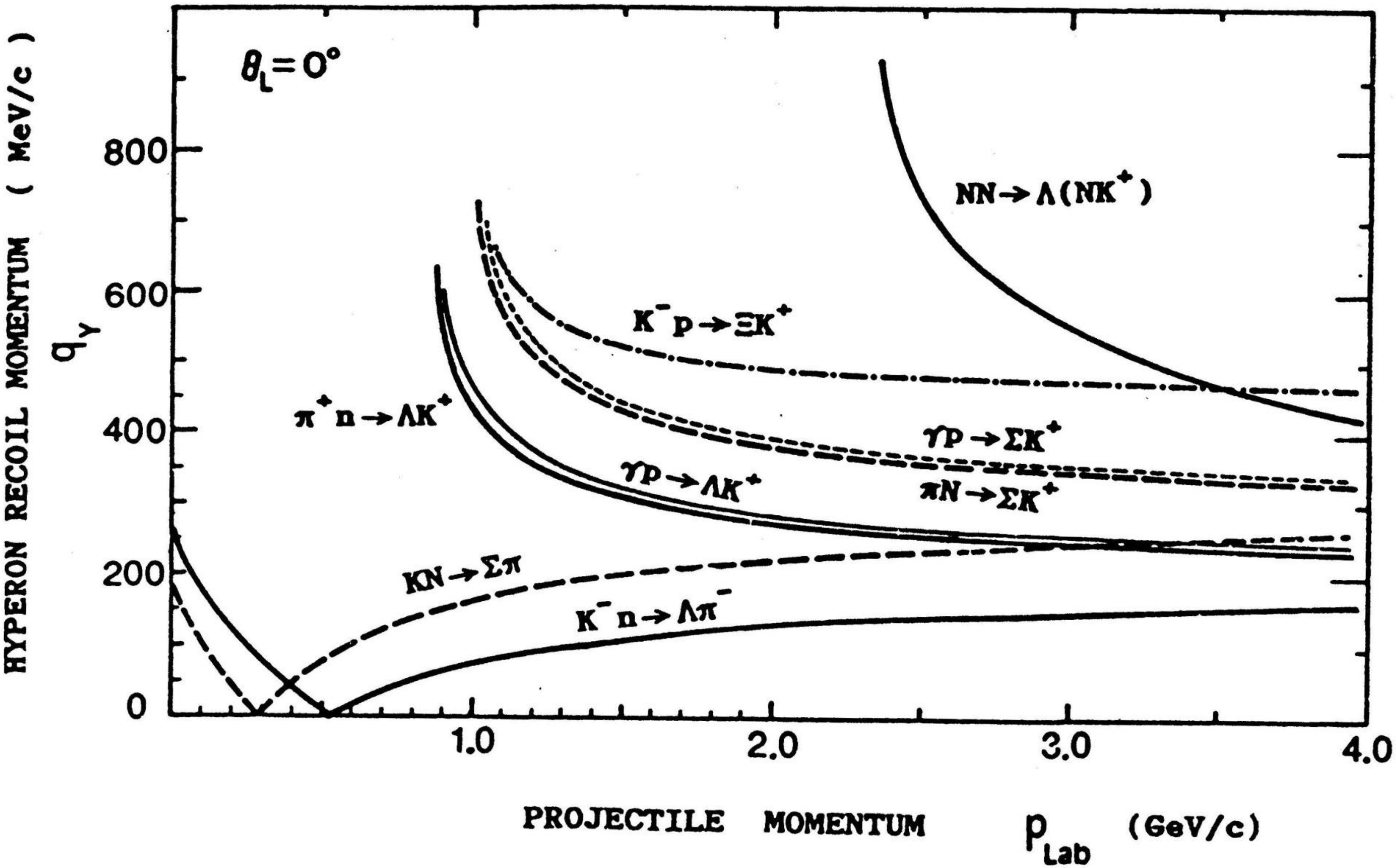
1. DWIA cross sections with Ξ one-body treatment

(1) Ξ one-body motion in an average nuclear potential such as W-S and/or some folding potentials.

(2) Nuclear core-excitations are not taken into account.

(3) K- and K+ distorted waves are obtained by solving the Klein-Gordan Eq. (OR one may take the eikonal approximation.)

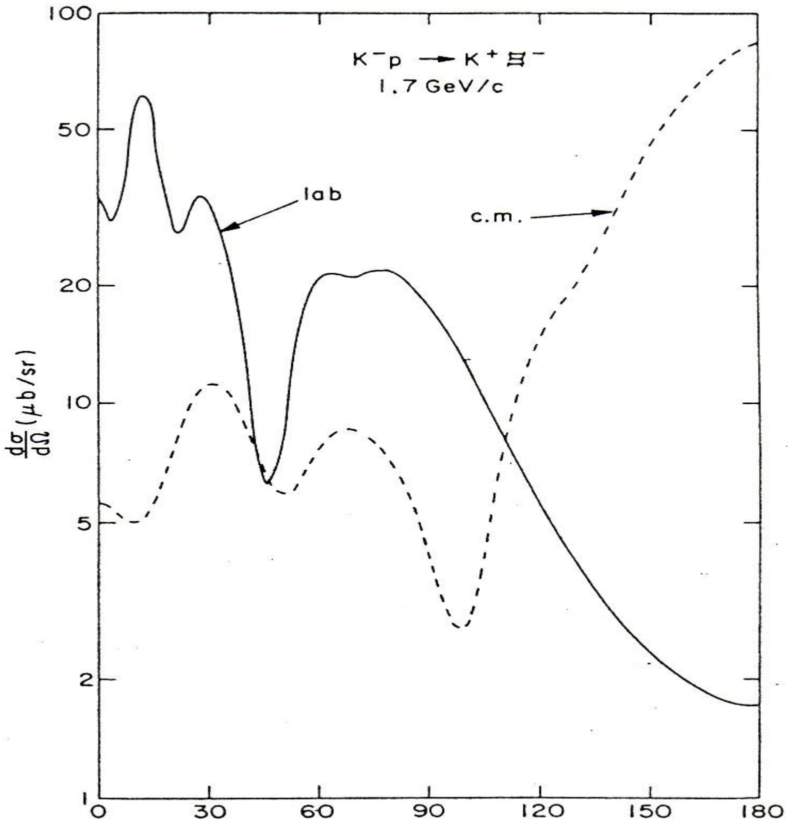
Hyperon recoil momentum and the transition operator determine the reaction characteristics

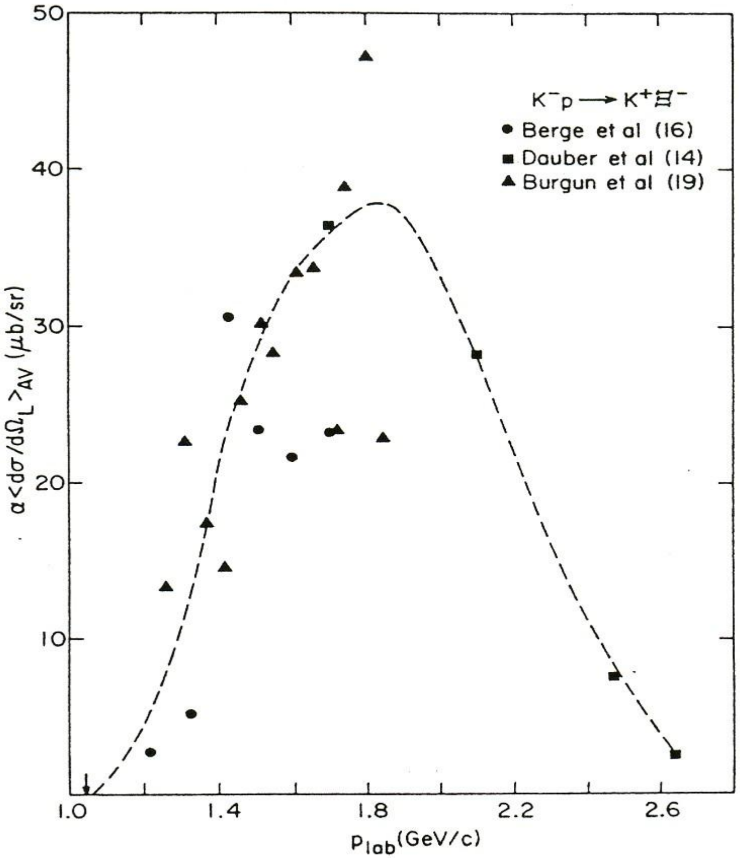


Use the empirical Ξ-production cross section, angular distribution and p_K-dependence

Data from V.Flamino et a, CERN-HERA Report 79-02 (1979)

Pioneering work by C.B.Dover and A.Gal, Annals of Phys. **146** (1983)





DWIA Treatment within Kapur-Peierls method for ^{A}Z(K-,K^{+}) $^{A}_{\Xi}$Z' reaction cross section

$$\frac{d^2\sigma}{d\Omega dE_Y} = \xi \left[\frac{d\sigma(\theta)}{d\Omega} \right]_{\rm elem} S(E_Y, \theta),$$

$$S(E_Y, \theta) = -\frac{1}{\pi} \sum_f {\rm Im} \left[\frac{N_f(E_Y, \theta)}{E_Y - \epsilon_f(E_Y)} \right].$$

ξ = kinematical factor for 2-body to A-body frame transformation

$$N_f(E_Y, \theta) = \langle \Phi_0 | \hat{O}^\dagger(\theta) | \Psi_f(E_Y) \rangle \langle \tilde{\Psi}_f(E_Y) | \hat{O}(\theta) | \Phi_0 \rangle.$$

$$\hat{O}^{(K^-,K^+)} = \int d^3\boldsymbol{r}\, \chi^*_{K^+}(\boldsymbol{k}_f, a\boldsymbol{r}) \chi_{K^-}(\boldsymbol{k}_i, \boldsymbol{r}) \sum_{\nu=1}^{A} V_-^{\,2}(\nu) \delta\left(\boldsymbol{r} - \frac{M_c}{M_A} \boldsymbol{r}_\nu \right),$$

$$\chi^*_{K^+}(\boldsymbol{k}_f, a\boldsymbol{r}) \chi_{K^-}(\boldsymbol{k}_i, \boldsymbol{r}) = \sum_{LM} i^L \sqrt{4\pi[L]}\, \tilde{j}_{LM}(k_i, k_f, \theta; a, r) Y_{LM}(\hat{\boldsymbol{r}}).$$

$\sigma_{K^-p} = 32.5$ mb, $\sigma_{K^-n} = 25.5$ mb, $\sigma_{K^+p} = 19.6$ mb and $\sigma_{K^+n} = 20.1$ mb,

ε (E)= solutions of the following Hamiltonian, depending on the final hypernuclear excitation energy E(given)

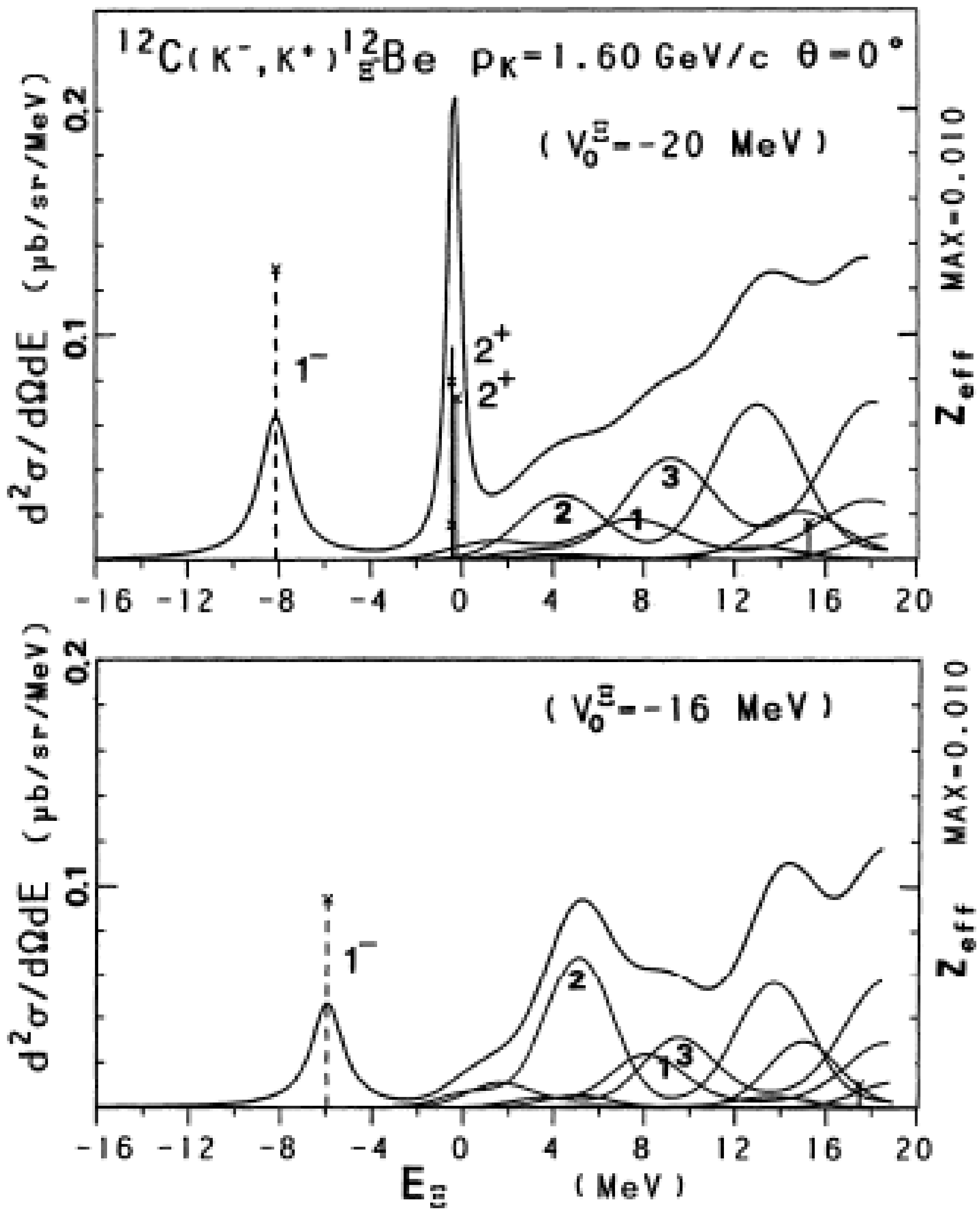
$$\mathcal{H}=(H_N+T_Y+U_Y(r)+\sum v_{YN})+\frac{\hbar^2}{2m_Y}\delta(r-r_c)\left(\frac{d}{dr}-\frac{b}{r_c}\right),$$

$$b=\left.\frac{d\phi_l(k_Y;r)/dr}{\phi_l(k_Y;r)/r}\right|_{r=r_c},\quad \phi_l(k_Y;r)\equiv r\{j_l(k_Yr)+in_l(k_Yr)\}.$$

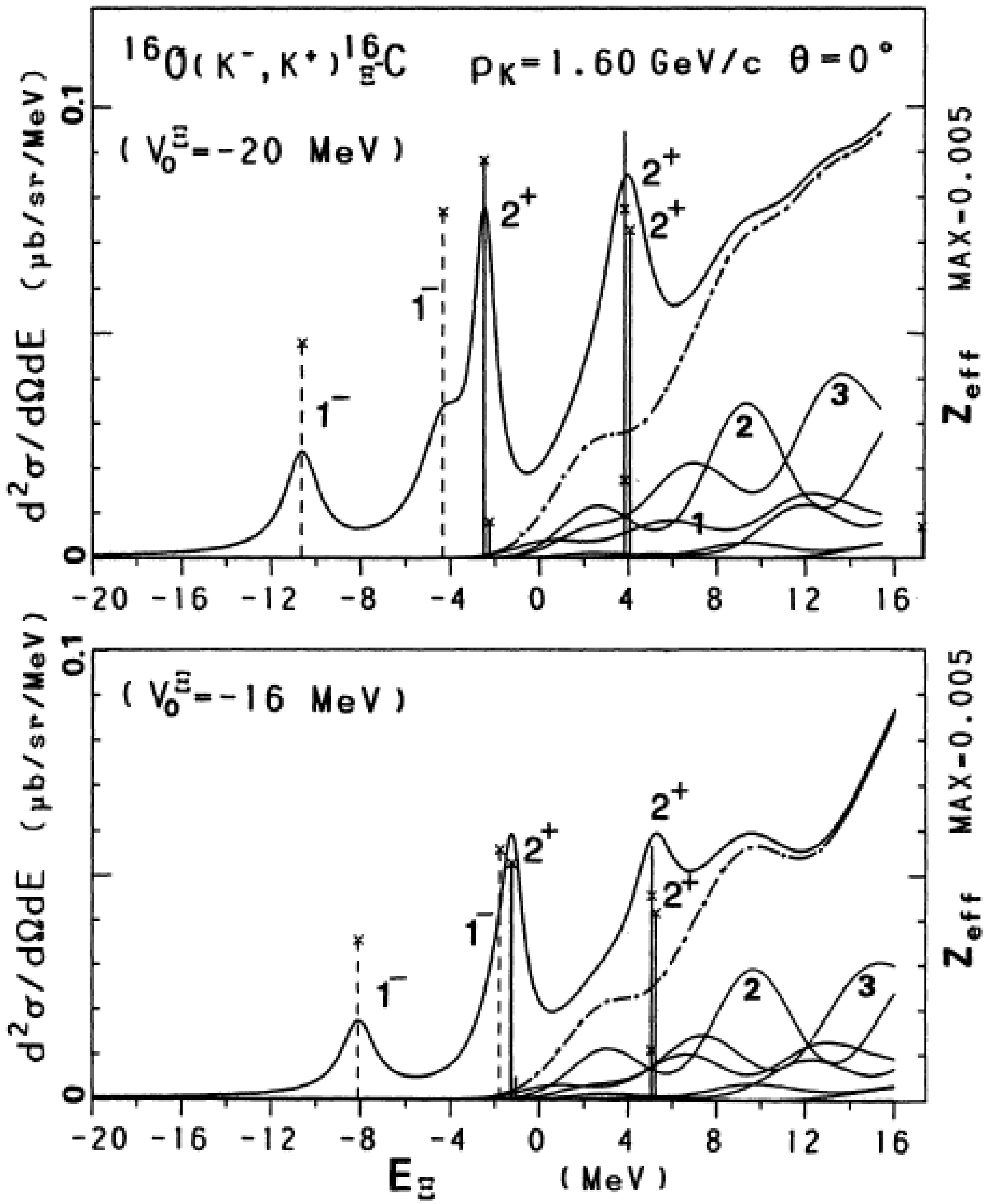
Continuum: Boundary condition for each *l* at channel radius *r*c,

For the case of bound states (*E*<0), if any, *S*(E_Y;) tends to the effective number *Z*eff.

$$\frac{d\sigma(\theta)}{d\Omega_L}=\xi\left[\frac{d\sigma(\theta)}{d\Omega_L}\right]_{K^-p\to\Xi^-K^+}Z_{\rm eff}(i\to f;\theta).$$



Arbitrary smearing widths are used in these figures



- DWIA

Continuum with Kapur-Peierls cal.

Thus the average potential depth could be deduced if clear bound state peaks are identified.

$E_{(0s)}$ & $d\sigma/d\Omega$(0s)

$^{12}_{\Xi^-}$Be			
	V_0(MeV)	E_{0s}	E_{0p}
Ξ^-	−24	−10.74	−1.49
Ξ^-	−18	−7.18	——
Ξ^-	−12	−4.08	——
Ξ^-	−8	−2.42	——
Λ	−32	−11.38	−0.16
p	−53	−35.0	−11.2

- 0.438 μb/sr
- 0.258
- 0.087
- 0.049

Numerical

$^{16}_{\Xi^-}$C		
V_0(MeV)	E_{0s}	E_{0p}
−24	−13.65	−4.07
−18	−9.62	−1.89
−12	−5.98	——
−8	−3.90	——
−30	−12.40	−1.55
−50	−40.0	−10.21

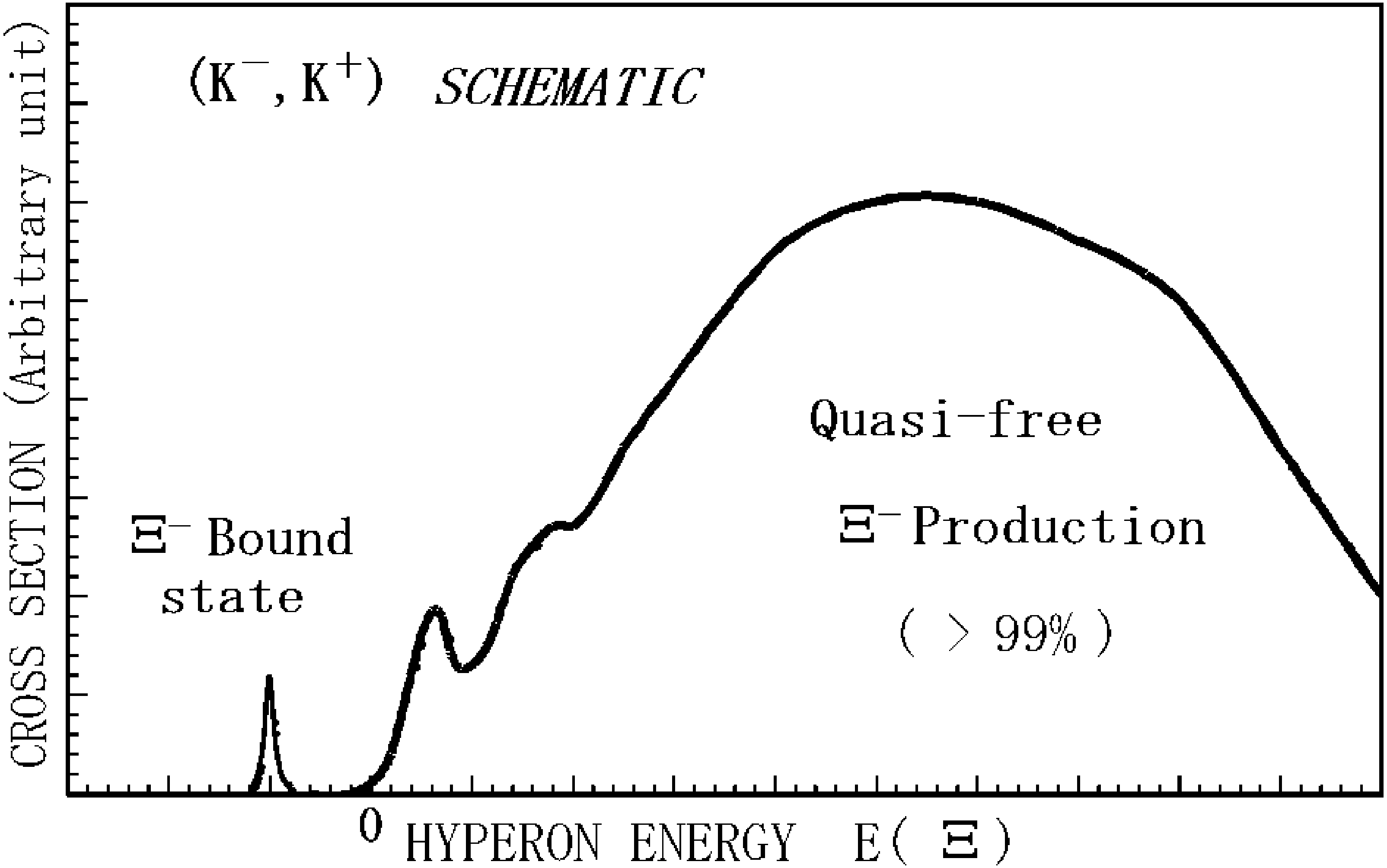
- 0.224 μb/sr
- 0.151
- 0.086
- 0.049

Table I. Effective proton numbers $Z_{\rm eff}$ (in units of $\times 10^{-3}$) as calculated in DWIA for the ^{12}C$(K^-, K^+)^{12}_{\Xi}$Be reaction ($p_{K^-}=1.6$ GeV/c, $\theta_{K^+}^{\rm Lab}=0°$) leading to the Ξ-bound states. The bars denote that the Ξ p-state is unbound. The single-particle wave functions of the $[j_N^{-1} j_\Xi]_J$ configuration are generated with the Woods-Saxon potentials described in the text. In the round brackets are the cross sections (in μb/sr) estimated from $Z_{\rm eff}^{\rm DW}$ by multiplying the empirical elementary cross section $\xi\ (d\sigma/d\Omega)_{K^-p\to\Xi^-K^+}=0.73\times 35=26\ \mu$b/sr.[4)]

Ξ^- well depth V_0^Ξ		−24 MeV	−20 MeV	−16 MeV	−12 MeV
proton→Ξ^-	J^π	$Z_{\rm eff}^{\rm DW}$ $(d\sigma/d\Omega)$	$Z_{\rm eff}^{\rm DW}$ $(d\sigma/d\Omega)$	$Z_{\rm eff}^{\rm DW}$ $(d\sigma/d\Omega)$	$Z_{\rm eff}^{\rm DW}$ $(d\sigma/d\Omega)$
		$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$
$0p_{3/2}\to 0s_{1/2}$	$1^-_{\rm gs}$	8.29 (0.215)	6.46 (0.168)	4.72 (0.123)	3.42 (0.081)
$\to 0p_{3/2}$	0^+	1.11 (0.029)	0.77 (0.020)	— (—)	— (—)
	2^+_1	6.29 (0.164)	3.96 (0.103)	— (—)	— (—)
$\to 0p_{1/2}$	2^+_2	5.83 (0.152)	3.58 (0.093)	— (—)	— (—)
$0s_{1/2}\to 0s_{1/2}$	0^+	1.14 (0.030)	0.79 (0.021)	0.51 (0.013)	0.29 (0.008)
$\to 0p_{3/2}$	1^-	1.62 (0.042)	0.91 (0.024)	— (—)	— (—)
$\to 0p_{1/2}$	1^-	0.74 (0.019)	0.41 (0.011)	— (—)	— (—)
Sum: bound states		25.02 (0.651)	16.88 (0.439)	5.23 (0.136)	3.42 (0.089)
[Sum/$Z_{\rm eff}^{\rm total}$]:		[0.83%]	[0.56%]	[0.17%]	[0.11%]

$Z_{\rm eff}^{\rm total}=2.99$ for DW [6.00 for PW].

Schematic strength function for a nuclear target



Bound state strength is less than 1% of the total XS, but it is absolutely important to discriminate Ξ-N interactions.

Interesting theoretical aspects:

Widths of X-states in nuclei:

D.J.Millener, C.B.Dover and A. Gal:

Prog. Theor. Phys. Sup.117, 304 (1994)

Y. Yamamoto, T. Motoba, T. Fukuda, M.Takahashi, K. Ikeda,

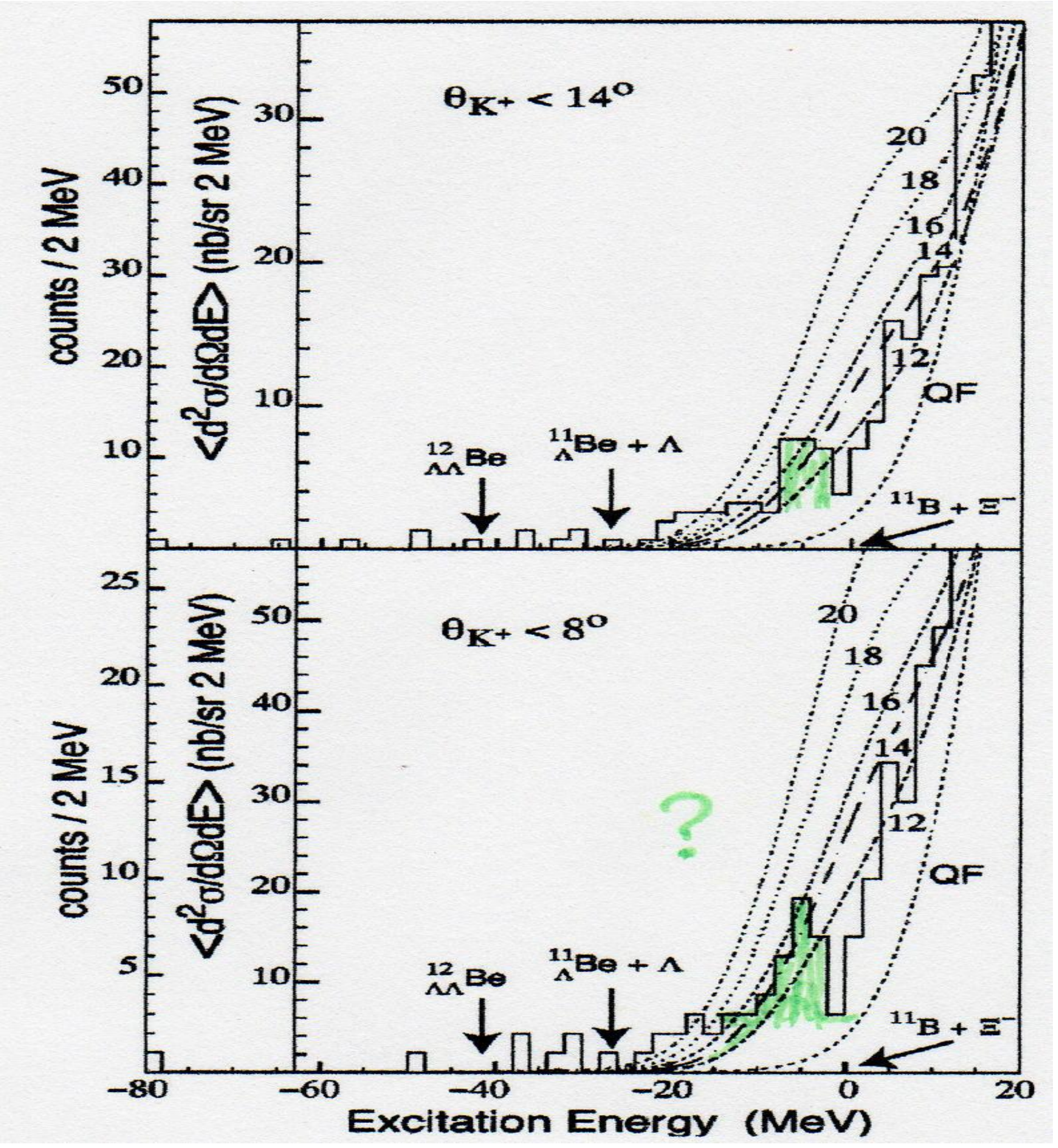
Prog. Theor. Phys. Sup.117, 281 (1994)

Two pioneering experiments done:

Two reports of (K-,K+) experiments on ^{12}C:

Exp-1: T. Fukuda et al., PRC 58, 1306 (1998)

Exp-2: P. Khaustov et al., PRC 61 (2000)



BNL-E885

P. Khaustov et al, PRC 61 (2000)

BS strengths observed,but **peaks not confirmed.**

Suggesting:

only WS-pot. depth:

U=12-14 MeV or less.

"shallow"

Ξ-N interactions are not taken up to this stage (except W-S pot.)

→**Use basic XN interactions**

Typical meson theoretical models:

Nijmegen model-D, Ehime (Ueda)

Nijmegen ESC04d, ESC08

(Gmatrix: 3-range Gaussian expressions, YNG)

Quark models,

EFT cal. , Lattice cal,

2. Many-body calculations

2.1 Energy levels and Ξ-N interactions

2.2 (K-,K+) spectra calculated with several Ξ-N interactions

- $\mathrm{H}_{\mathrm{total}} = H^{(\text{Cohen-Kurath})} + t_{\mathrm{Y}} + \sum \boldsymbol{V}^{(\mathrm{YNG})}$

$\boldsymbol{V}^{(\mathrm{YNG})}$: NHC-D, Ehime, ESC04d, ESC08

2. Many-body calculations

were performed by taking into account of

1) dynamical nuclear core excitations ($^{11}B+\Xi^-$),
2) NN and ΞN effective interactions on equal footing,
3) tail (radial) behavior of $\phi_\Xi(r)$ carefully.

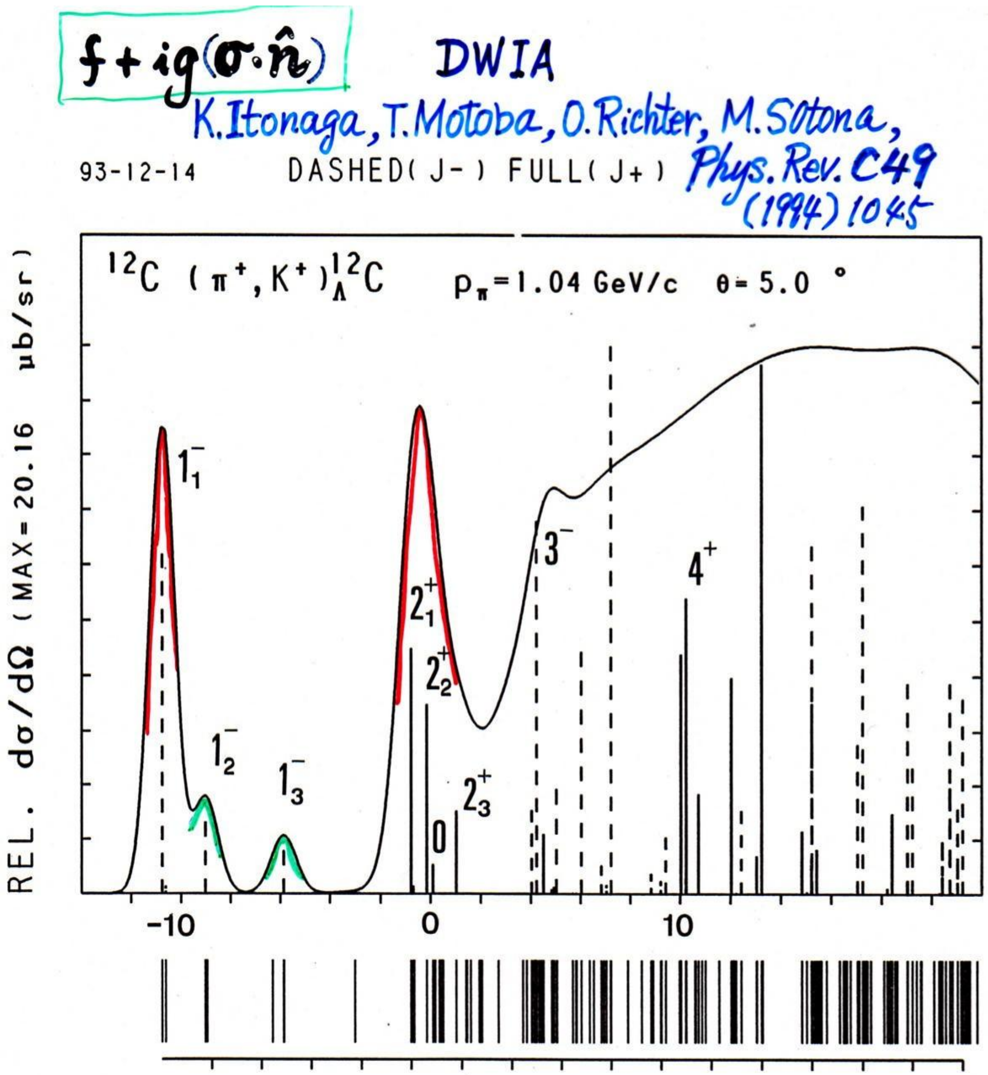
0s+1s+2s, 0p+1p+2p, ….

- $H_{total} = H^{(Cohen\text{-}Kurath)} + t_Y + \sum V^{(YNG)}$

$V^{(YNG)}$: NHC-D, Ehime, ESC04d, ESC08

This theory works quite nice as proved in (π+,K+).

CAL (Itonaga et al 1994) VS. EXP (Hotchi et al, 2001)



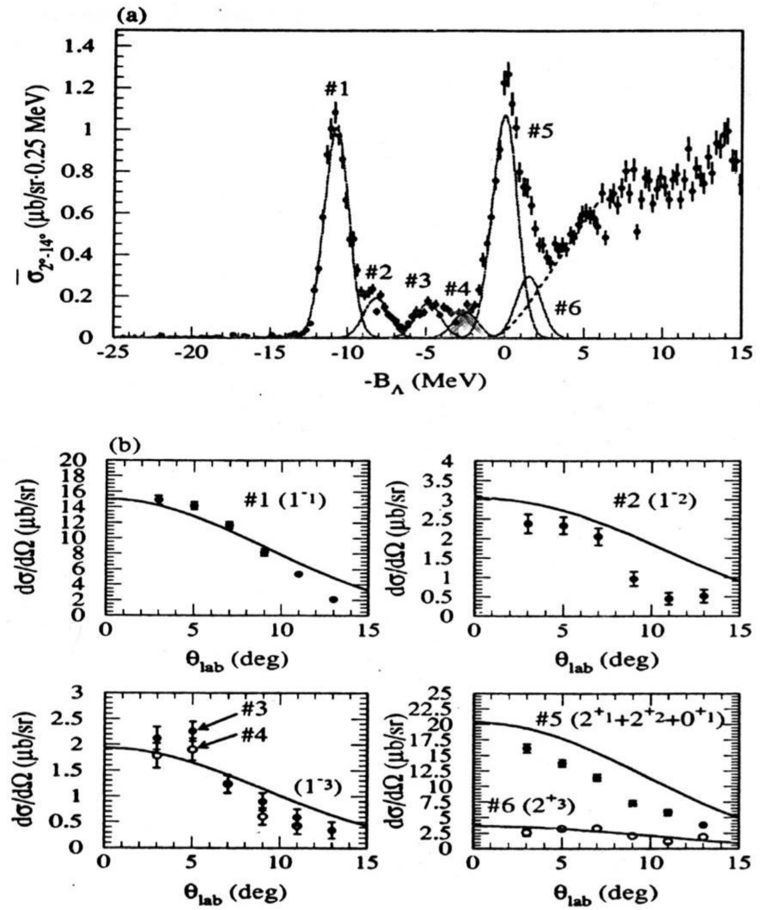
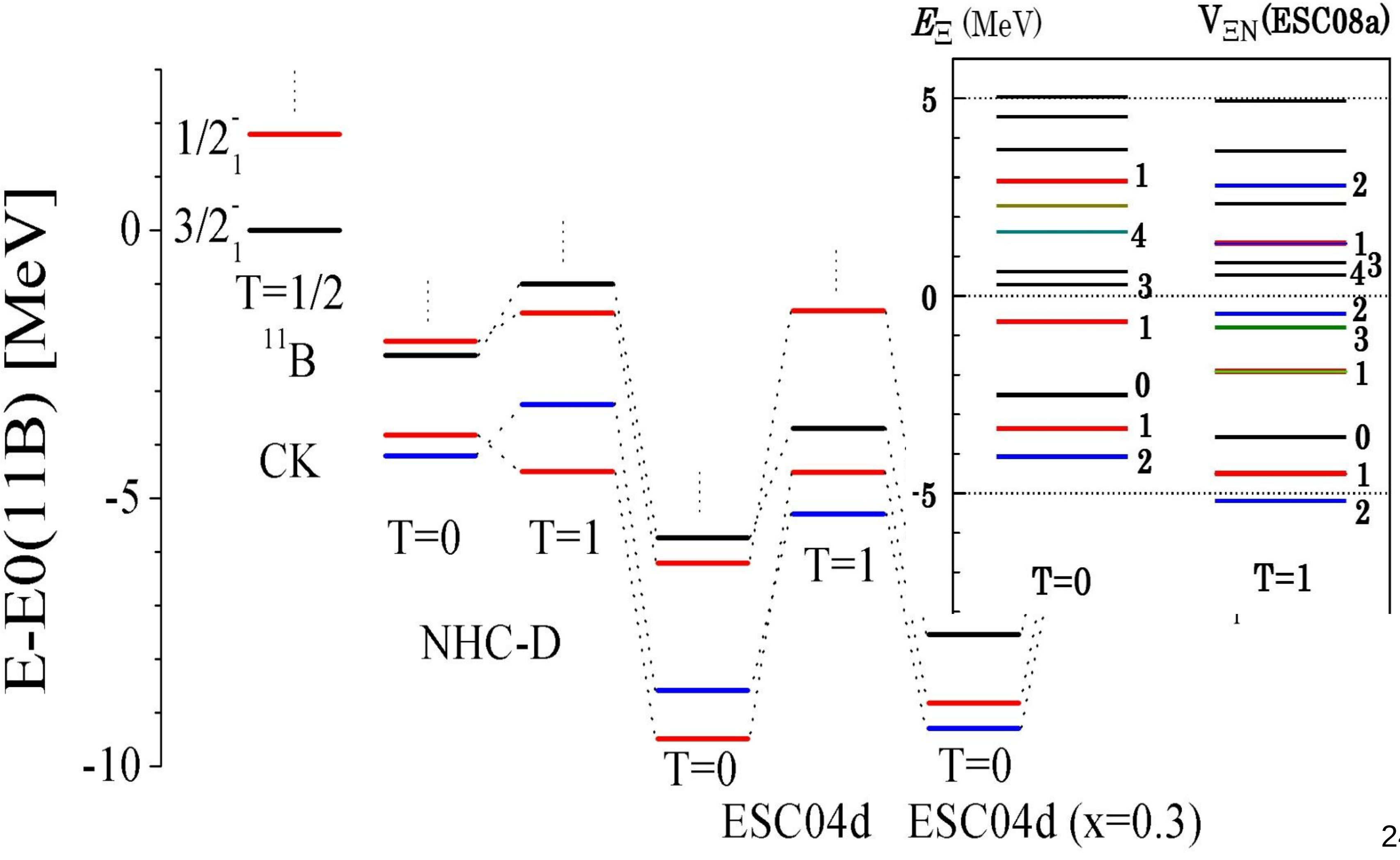


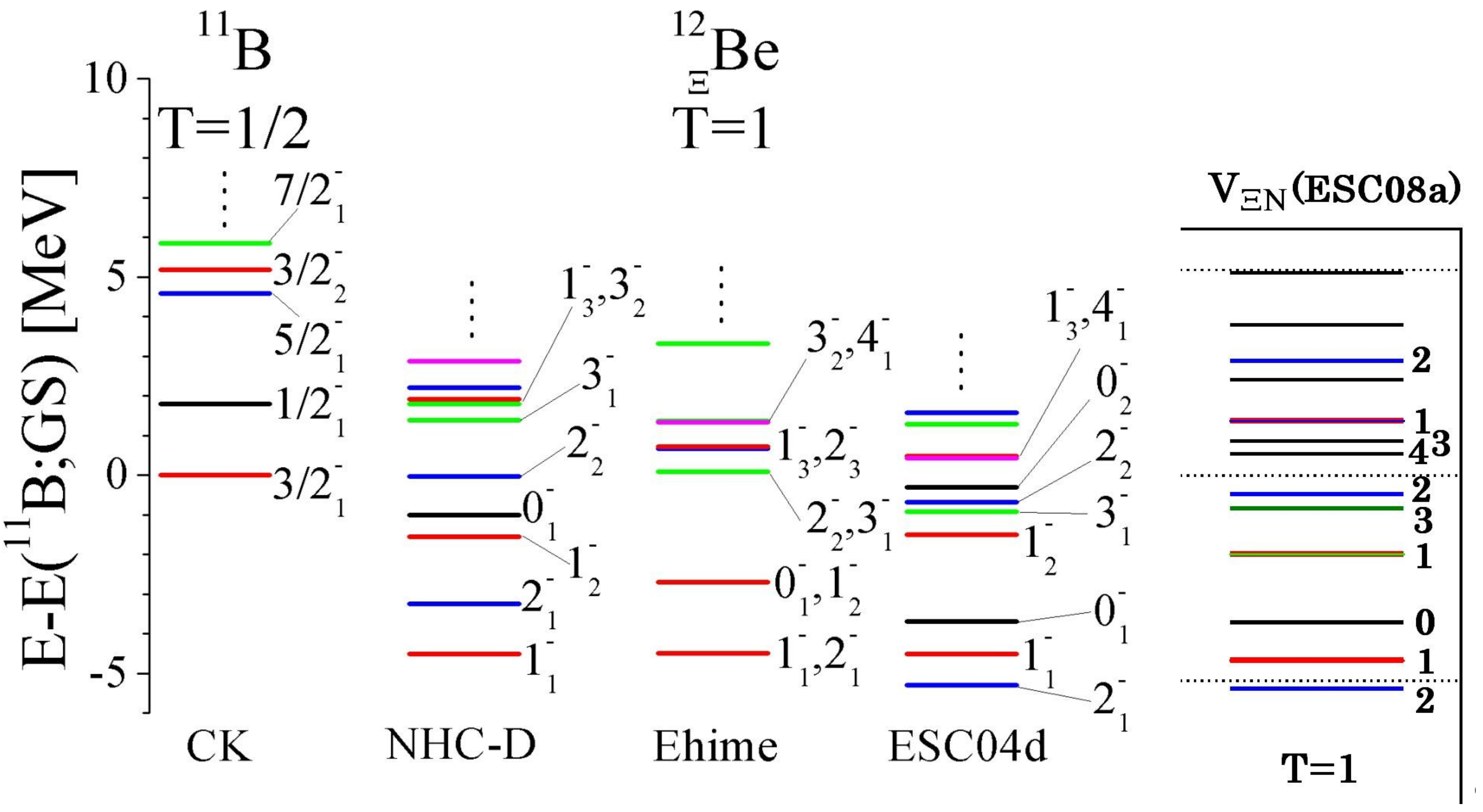
FIG. 8. (a) $^{12}_{\Lambda}C$ spectrum obtained with the thick carbon target. (b) Angular distributions of kaons leading to the observed peaks for the $^{12}C(\pi^+, K^+)$ reaction derived from the above high-statistics

ESC04d: deep T=0 states,
ND, ESC08: T=0 & 1 states coexist.

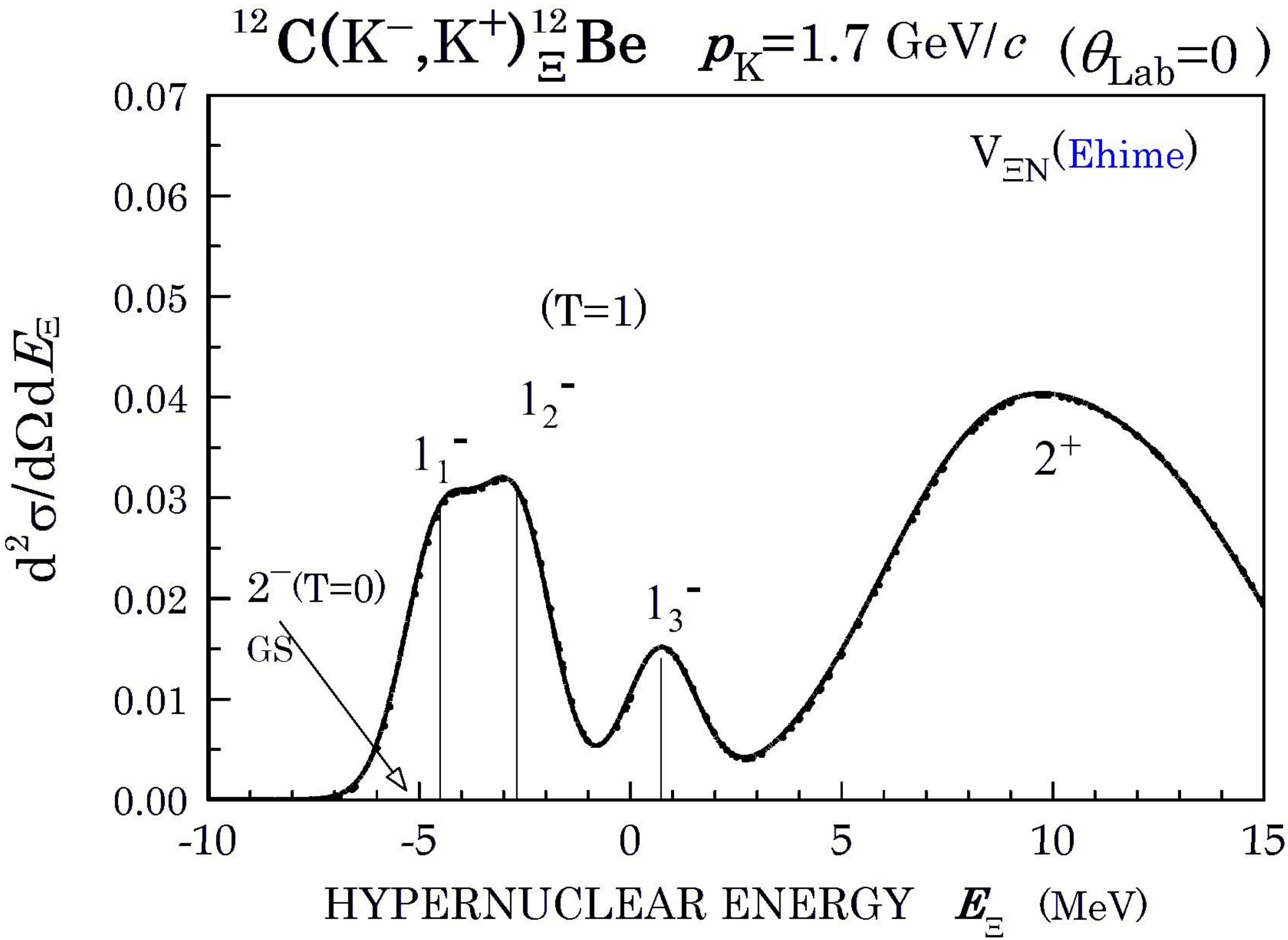


Reference position of J=1-(T=1) states.

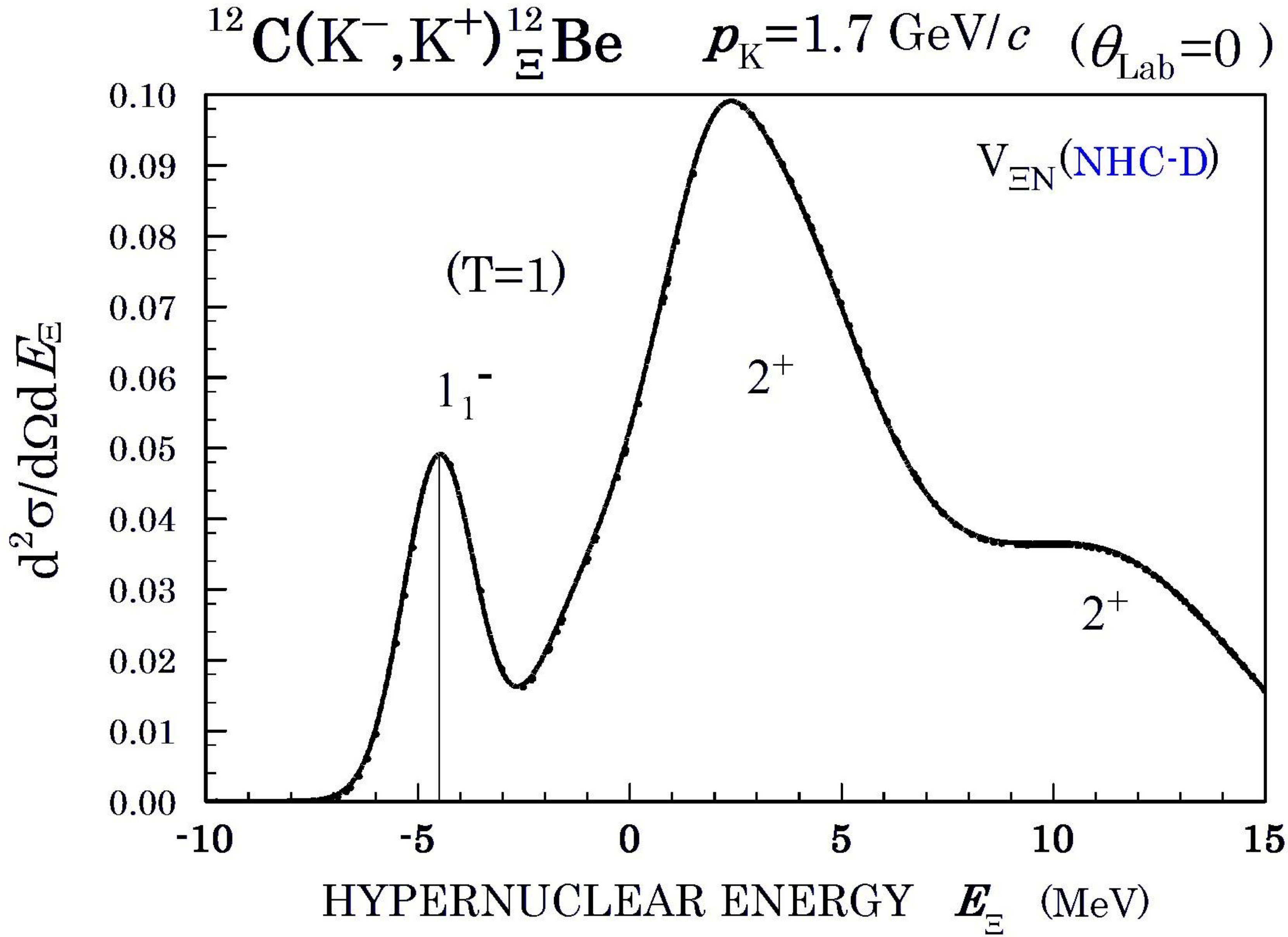
The relative positions of J_n show the different σ.σ interaction nature of $V_{\Xi N}$.



Spectrum (1): Ehime case.



Spectrum(2): ND case (p-state attraction)



Sensitive interaction dependence
Nijmegen NHC-D vs. ESC04d

- **Different partial-wave contributions**
  NHC-D (large p-state attraction) vs. ESC04d(s-state)
- **ESC04d (quite large spin- & isospin-dependence)**

Table 1: Ξ single particle energies U_Ξ and conversion widths Γ_Ξ at normal density calculated with ESC04d and NHC-D. S-state contributions in $(TSLJ)$ states and total P-state contributions are also given. All entries are in MeV.

	$^{11}S_0$	$^{13}S_1$	$^{31}S_0$	$^{33}S_1$	P	U_Ξ	Γ_Ξ
ESC04d($\alpha = 0$)	6.4	−19.6	6.4	−5.0	−6.9	−18.7	11.4
ESC04d($\alpha = .18$)	6.3	−18.4	7.2	−1.7	−5.6	−12.1	12.7
NHC-D	−2.6	0.7	−2.3	−0.4	−16.8	−21.4	1.1

Core-Ξ coupling: ESC04d vs. ND

The spatial symmetry of target GS does not persist due to large spin-dep., when a proton is replaced by Ξ.

Mixing probability in $^{12}{}_{\Xi}Be$

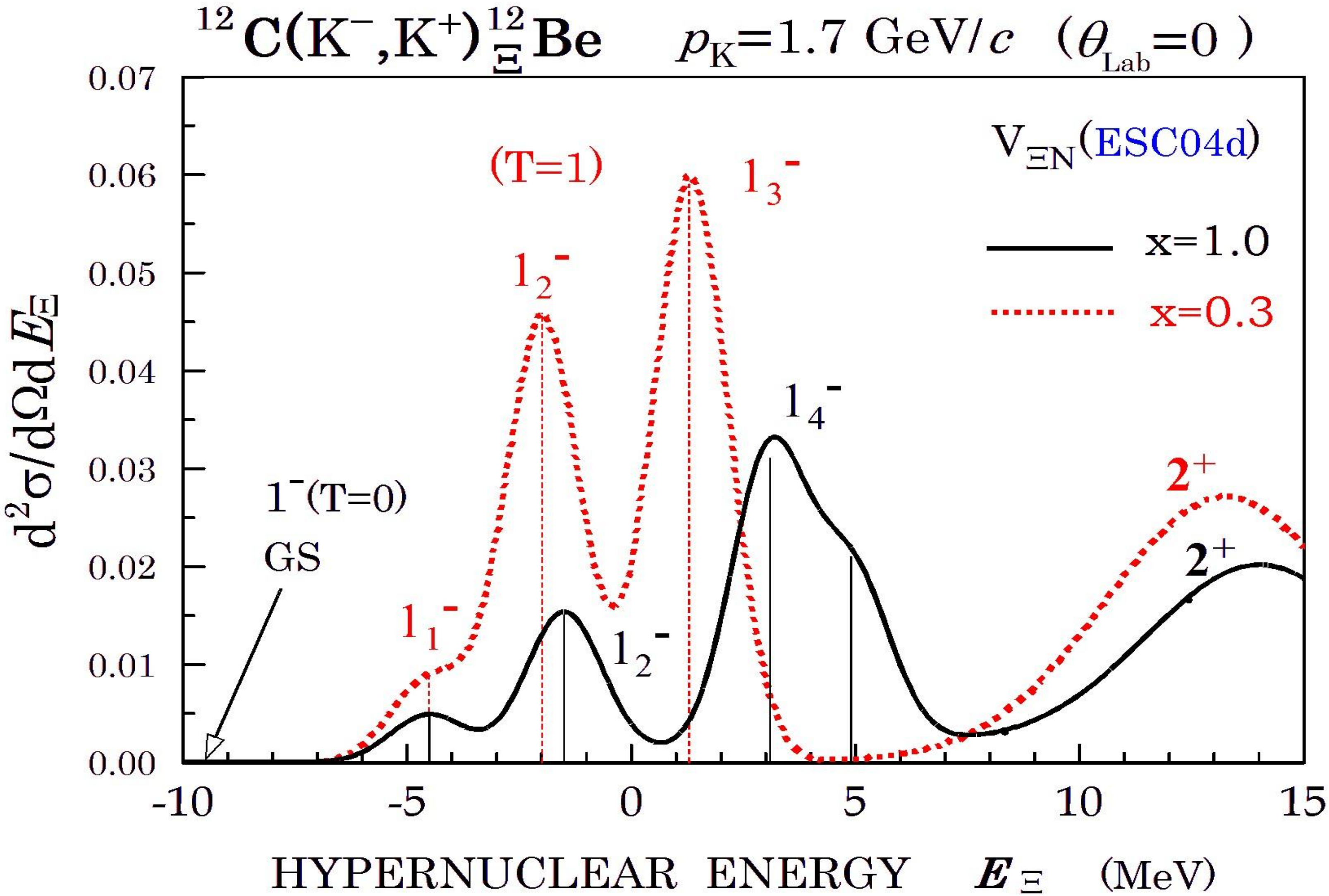
ESC04d		Jπ of the core (^{11}B)		
T=1 Jπ	BE(Ξ)	3/2- 1st (GS)	1/2- 1st (1.8MeV)	3/2- 2nd (5.2MeV)
1st 1-	4.5MeV	58.5%	39.0%	1%
2nd 1-	1.5MeV	13.3%	34.0%	48.7%

NHC-D		Jπ of the core (^{11}B)		
T=1 Jπ	BE(Ξ)	3/2- 1st (GS)	1/2- 1st (1.8MeV)	3/2- 2nd (5.2MeV)
1st 1-	4.5MeV	85.5%	12.3%	2%
2nd 1-	3.0MeV	13.1%	85.8%	1%

- Probability for $0s_{1/2}(\Xi) \otimes J^{\pi}(\mathrm{Core})$
- ESC04d induce large mixing.
- It probably reflects on the spectra of (K^{+},K^{-}) reaction.

Spectrum (3): ESC04d case

(large S- & T-dependence)



Ξ-N interactions used in Shell Model

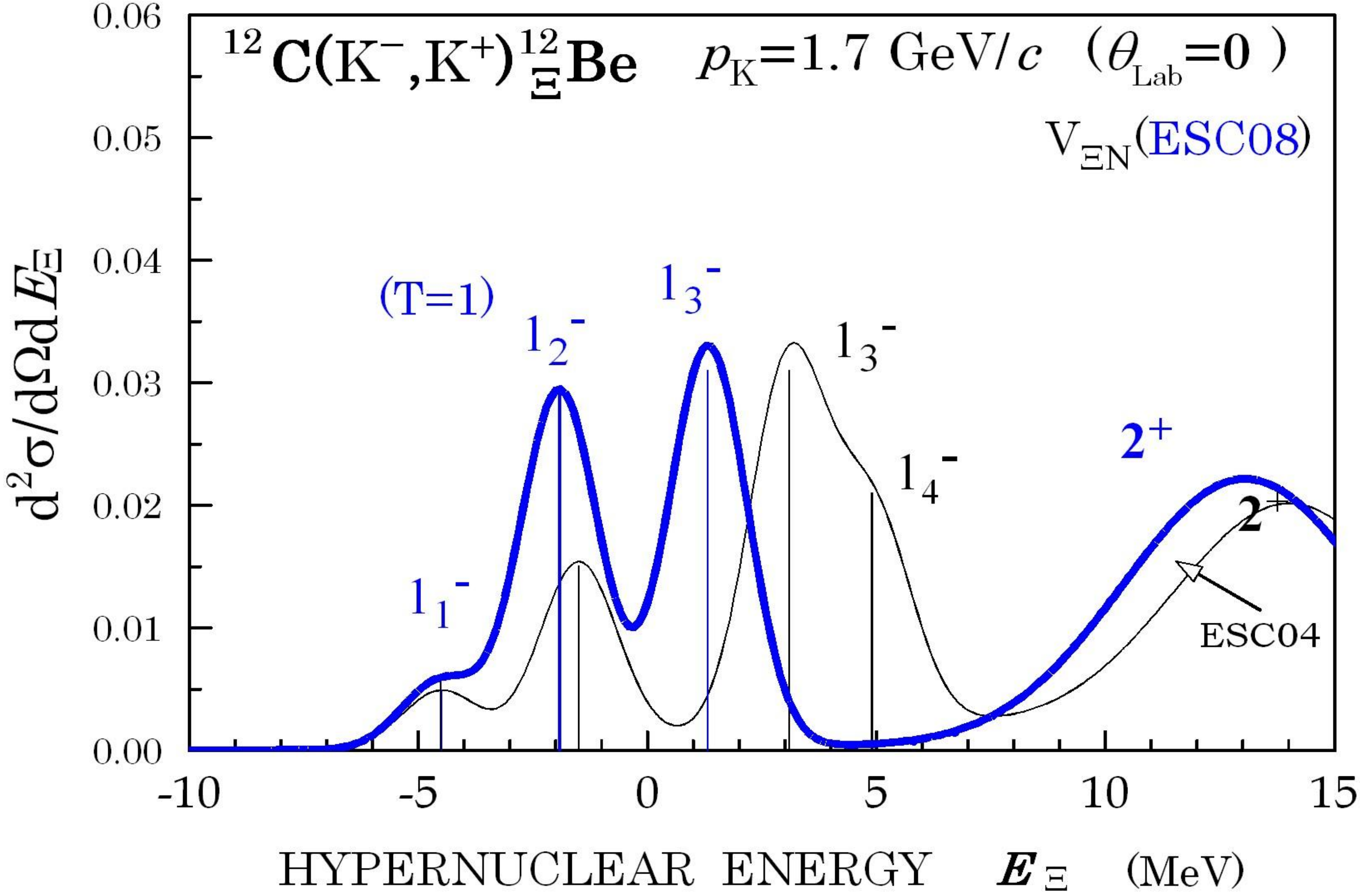
Comparison in the form of $V = -V_0 + \Delta(\sigma.\sigma)$

		V_0	Δ	Δ/V_0
NΞ(ESC04d)	T=0	4.98	-15.81	-3.18
	T=1	0.30	-2.96	-9.88
NΞ(NHC-D)	T=0	2.14	4.75	2.23
	T=1	1.55	0.79	0.51
NΛ(NSC97f)	T=1/2	1.05	0.04	0.04

$\sigma.\sigma$ strengths are quite different for ESC and ND, so further trials and improvements are required.

ESC04 modified to be ESC08 !

(continuum bump should be larger)



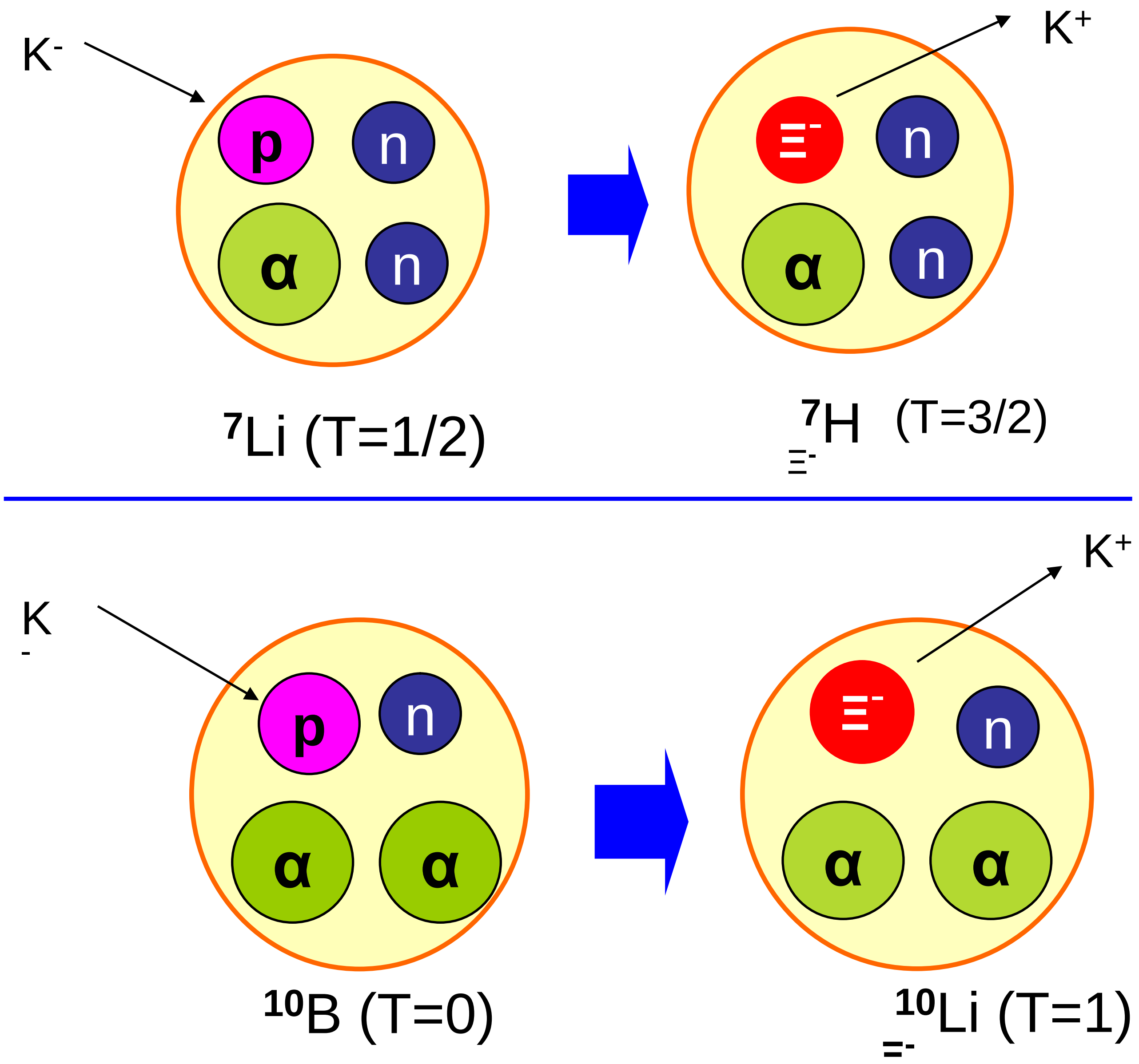
Trying to use the most recent Ξ-N interaction from Nijmegen (ESC08)

Table 1: Partial wave contributions to $U_\Xi(\rho_0)$

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	U_Ξ	Γ_Ξ
ESC08	0	4.0	−2.7	0.2	−2.2	0.6	−1.1		
	1	7.0	−19.5	−0.3	0.1	−3.6	−0.7	−18.0	6.0
ESC04d	0	6.4	−19.6	1.1	1.2	−1.3	−2.0		
	1	6.4	−5.0	−1.0	−0.6	−1.4	−2.8	−18.7	11.4

- Let us wait for the J-PARC Day-1 Exp. by Nagae et al, which will provide us a very important restriction on V(Ξ-N).

As the second best candidates to extract information about the spin-, isospin-independent term V_0, we propose to perform…

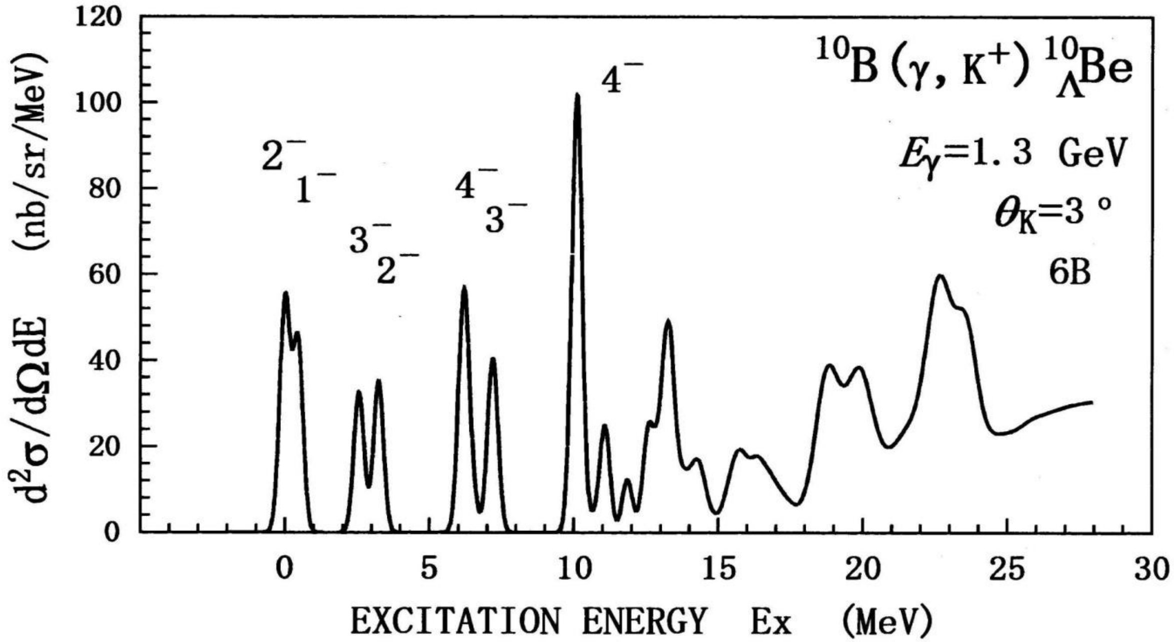


Why they are suited for investigating V_0?

^{10}B target:

Prediction

Motoba-Sotona-Itonaga, P.T.P. Suppl.117 (1994)

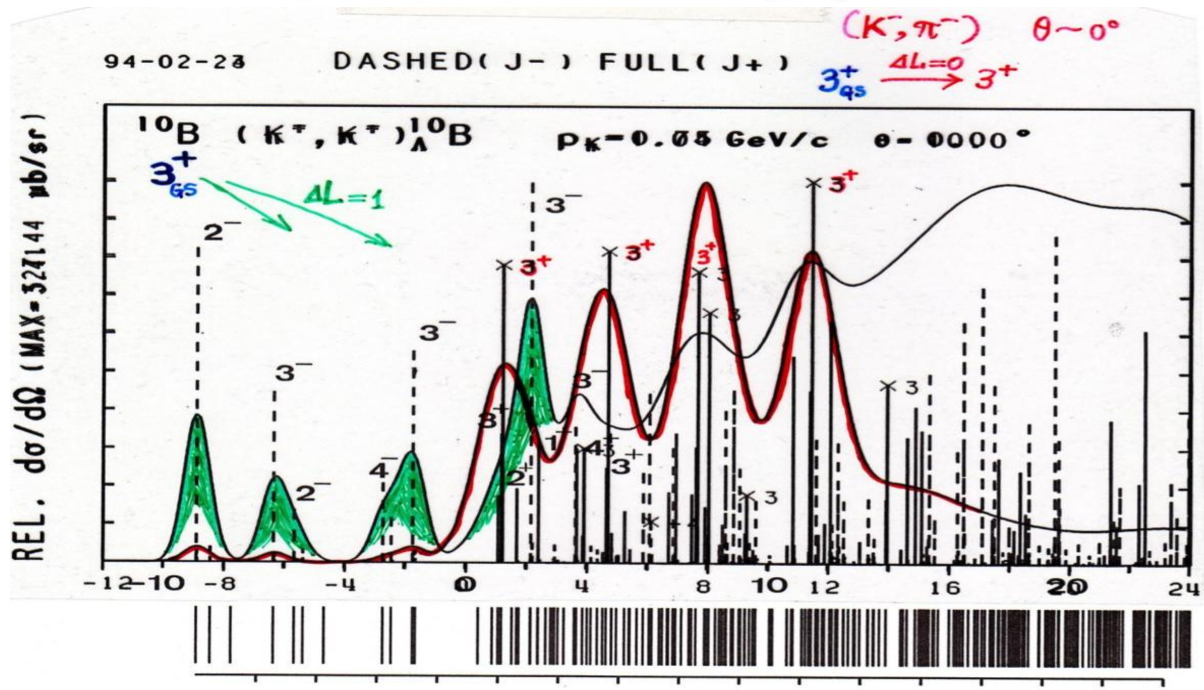


compared with

Theoretical results for

(K-,π-)

(π+,K+)



Summary

1. Shell-Model many-body WF have been applied to $^{12}C(K^-,K^+)_{\Xi}{}^{12}B$ in DWIA. + ^{10}B

2. Low-lying energy levels and the reaction strength functions are quite sensitive to the choice of Ξ-N interactions.

3. Therefore the coming experiments will surely provide us with good opportunity of discriminating the existing Ξ-N interactions.

4. There are still improvement in the theoretical estimates of XS spectrum.