# Light meson physics with dynamical overlap simulations

Jun Noaki for the JLQCD and TWQCD collaborations

We summarize the project of the light meson spectrum with $N_f$= 2+1 overlap fermions by the JLQCD and TWQCD collaborations. We study the finite size effect by comparing the analytical correction with the data on a larger volume lattice. With the degenerate quark masses $m_{ud} = m_s$ , we study the convergence property of ChPT. We also update the results of the chiral extrapolation to obtain physical quantities

## Introduction

Dynamical overlap fermions [1] conserve chiral symmetry on the lattice and enable us to study the continuum ChPT for any $N_f$. In particular, the convergence property of ChPT at $m_s \sim$ 500 MeV is phenomenologically important. As an extension of our previous study for the $N_f$ = 2 case [2], we generate $N_f$= 2+1 gauge configurations [3]. In this study, chiral extrapolation with NNLO ChPT formulae is necessary. We also update the chiral extrapolation with the increased number of data points.

## Data points

- Our gauge configurations are summarized as

| volume | $m_{ud}$ | $m_s$ | $Q$ | trajs | |
|---|---|---|---|---|---|
| $16^3$x48 | 0.015 – 0.080 (5pts) | 0.080 | 0 | 5,000 | |
| | 0.015 – 0.100 (5pts) | 0.100 | 0 | 5,000 | |
| | 0.025 | 0.025 | 0 | 1,200 | additional degenerate mass |
| | 0.035 | 0.035 | 0 | 1,250 | for the convergence study |
| | 0.015 | 0.080 | 1 | 900 | non-trivial topology |
| $24^3$x48 | 0.015 | 0.080 | 0 | 2,500 | large volume to check finite size |
| | 0.025 | 0.080 | 0 | 2,500 | corrections |

- We determine the lattice scale $a^{-1}$ = 1.759(8)(5) GeV from the Ω-baryon mass as in Figure 1.
- Our pion mass covers 290 MeV < $m_\pi$ < 780 MeV.
- Low-lying modes are computed & stored in disk. Used to improve the correlator (Low-Mode-Averaging).
- Quark mass is renormalized non-perturbatively through RI/MOM scheme [4].

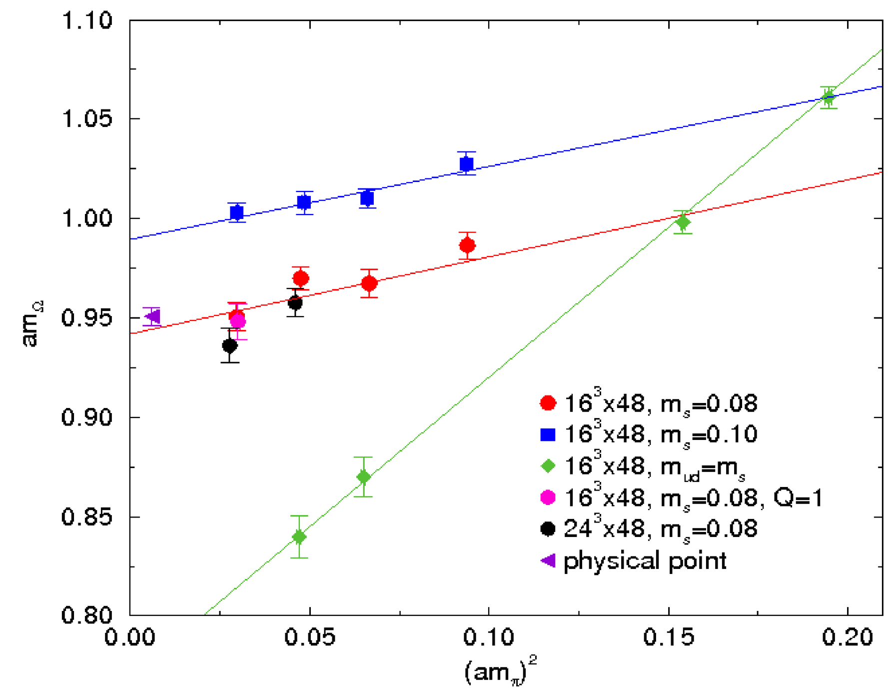

Figure 1: Ω-baryon mass as a function of pion mass in the lattice unit

## Finite size effect (FSE)

We correct the data by a combination of the formulae for the two sources of FSE.

Conventional FSE: Caused by the pion wrapping around the spatial directions [5].

Fixed topology effect: Deviation from the θ-vacuum [6,7]. Topological susceptibility $\chi_t$ needed for the correction is calculated in [8].

Figure 2 shows how much FSE corrections the original data receive on the different volumes ( $m_\pi L$ = 2.75 and 4.01). The smaller volume receive significant correction.

The remaining difference between the fully corrected values might be explained by higher order effects of the fixed-topology FSE. In this case, the correlators may have non-exponential functional [7]. We take this difference into account in the systematic error of the final result.

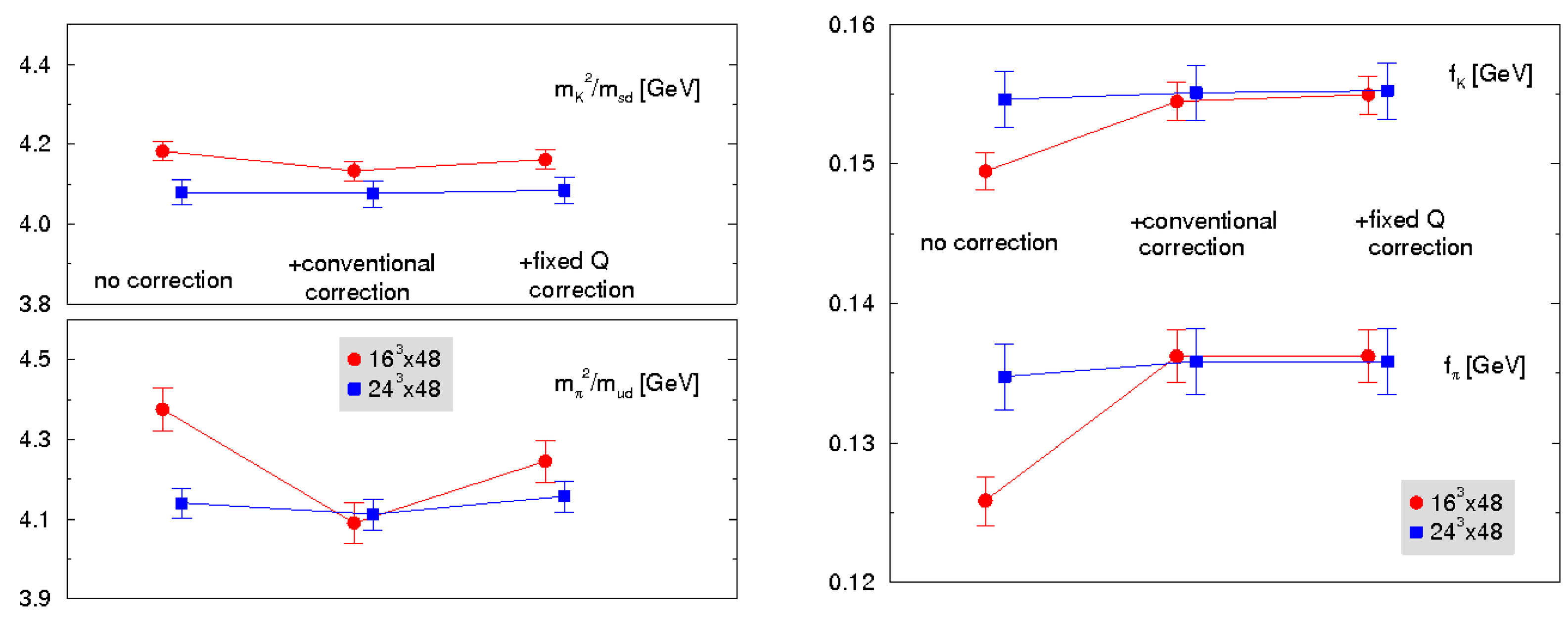

Figure 2: Transition of the data (the lightest quark mass) by FSEs

## Convergence of SU(3) ChPT

The discussion on the convergence can be made simpler by considering ChPT in the SU(3) limit with the degenerate quark masses. Using eight such data points, we carry out the chiral extrapolation. The deviations from the tree level values are plotted in Figure 3. The convergence ratio around 500 MeV is summarized as follows.

| | $m_\pi^2/m_{ud}$ (NLO) | $m_\pi^2/m_{ud}$ (NNLO) | $f_\pi$ (NLO) | $f_\pi$ (NNLO) |
|---|---|---|---|---|
| $N_f$ = 2+1 | –56(71)% | +95(268)% | +41(29)% | +23.7(5.6)% |
| $N_f$ = 2 | –4.5(2.1)% | +1.91(63)% | +29.6(5.7)% | +16.0(1.0)% |

Also, results from $N_f$ = 2 case is listed in the table for comparison. While the large error does not allow solid conclusion for $m_\pi^2/m_{ud}$ , we see, for $f_\pi$ , decreasing ratio and similar magnitude of convergence to the $N_f$ = 2 case.

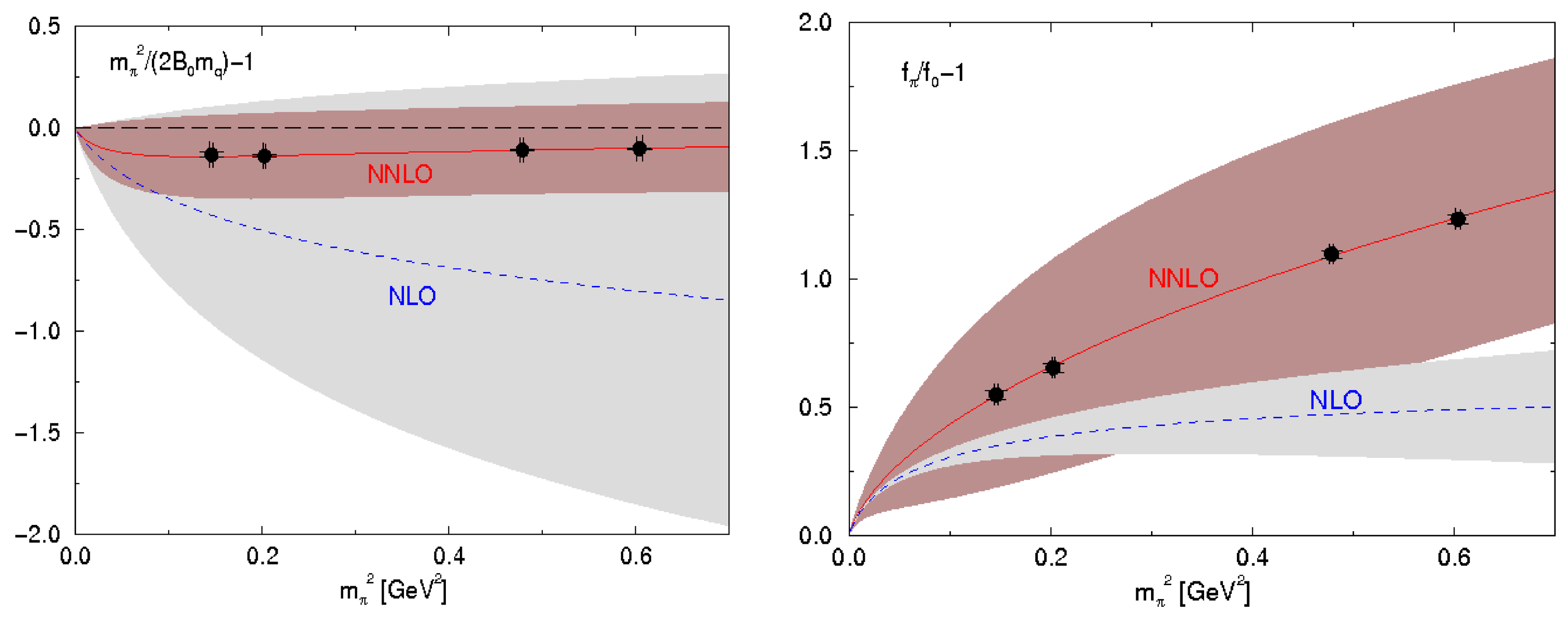

Figure 3: Results of the simultaneous fit to the NNLO ChPT at the SU(3) limit. Dashed curves indicate the truncation to NLO.

## Chiral extrapolation at NNLO

With increased data points explained above, we update the chiral extrapolation of the light meson observables using the NNLO ChPT formulae. We use expansion parameters $\xi_\pi = 2m_\pi^2/(4\pi f_\pi)^2$, $\xi_K = 2m_K^2/(4\pi f_K)^2$ for a stable fit. See [9] for more detail about the chiral fit. Figure 4 shows the fit curves obtained from the correlated simultaneous fit with $\chi^2$/dof = 2.6.

Because of the degenerate mass point, we obtain more stable fit results for SU(3) LECs than before. The pre-final results are

$$f_0 = 74.0(6.6) \text{ MeV}, \quad \Sigma_0^{1/3} = 177(12) \text{ MeV},$$

$$L_4^r(m_\rho) = 8.2(3.4) \times 10^{-4}, \; L_5^r(m_\rho) = -8.0(6.7) \times 10^{-4},$$

$$L_6^r(m_\rho) = 3.5(2.5) \times 10^{-4}, \; L_8^r(m_\rho) = -3.2(3.0) \times 10^{-4}.$$

Result of $f_0$ is substantially smaller than the phenomenological estimate $f_0$ = 124 MeV. However, as seen in Figure 5, the $N_f$ dependence of our data can be described by ChPT. Therefore, it is inevitable $f_0 < f$ =110 MeV (the $N_f$ = 2 value we obtain). Also, there is a large difference between $\Sigma_0^{1/3}$ the SU(2) chiral condensate $\Sigma^{1/3}$ = 230 MeV.

Results of the physical quantities are

$$f_\pi = 118.5(3.6) \text{ MeV}, \quad f_K = 145.8(2.7) \text{ MeV}, \quad f_K / f_\pi = 1.230(19),$$

$$m_{ud} = 4.028(57) \text{ MeV}, \quad m_s = 113.4(1.2) \text{ MeV}, \quad m_s / m_{ud} = 28.15(23).$$

There are also systematic errors to be considered.

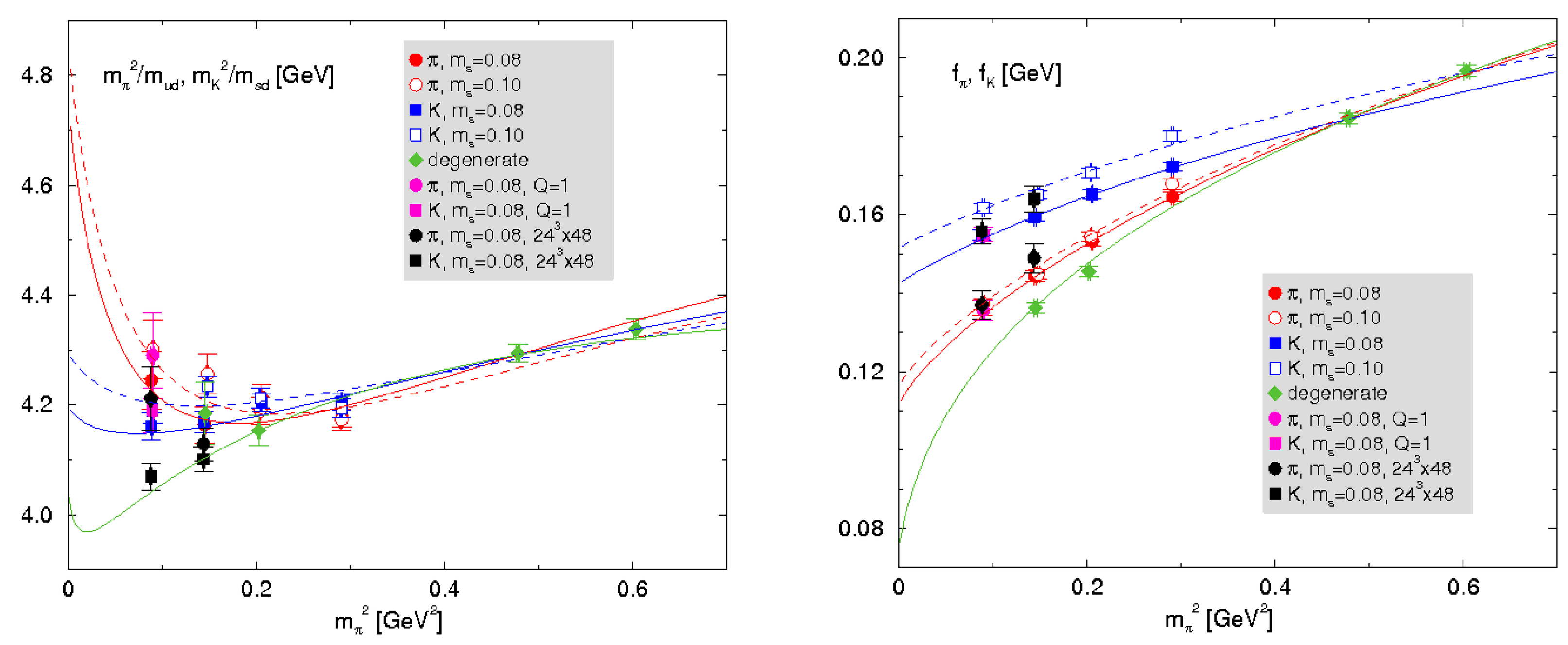

Figure 4: Chiral extrapolation with the NNLO ChPT using all available data points.

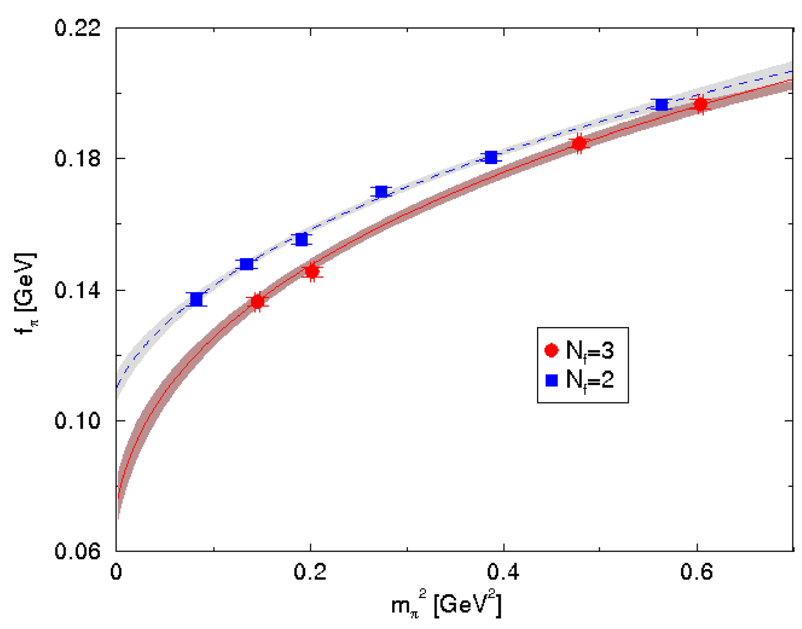

Figure 5: $f_\pi$ for $N_f$ = 2 and 3.

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