Extraction of hadron interactions from Lattice QCD

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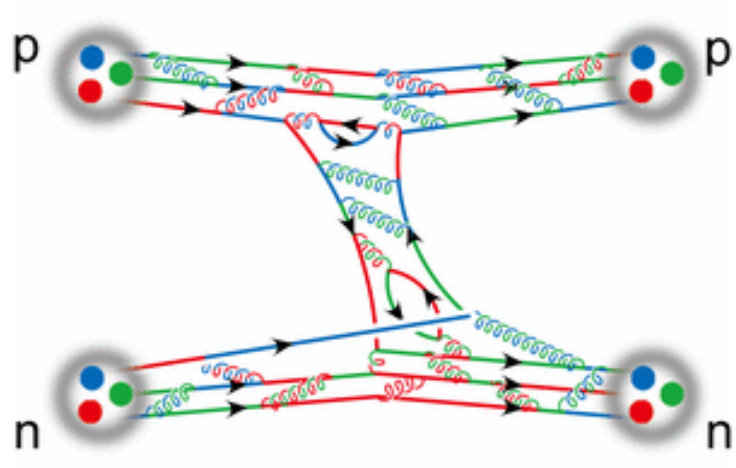
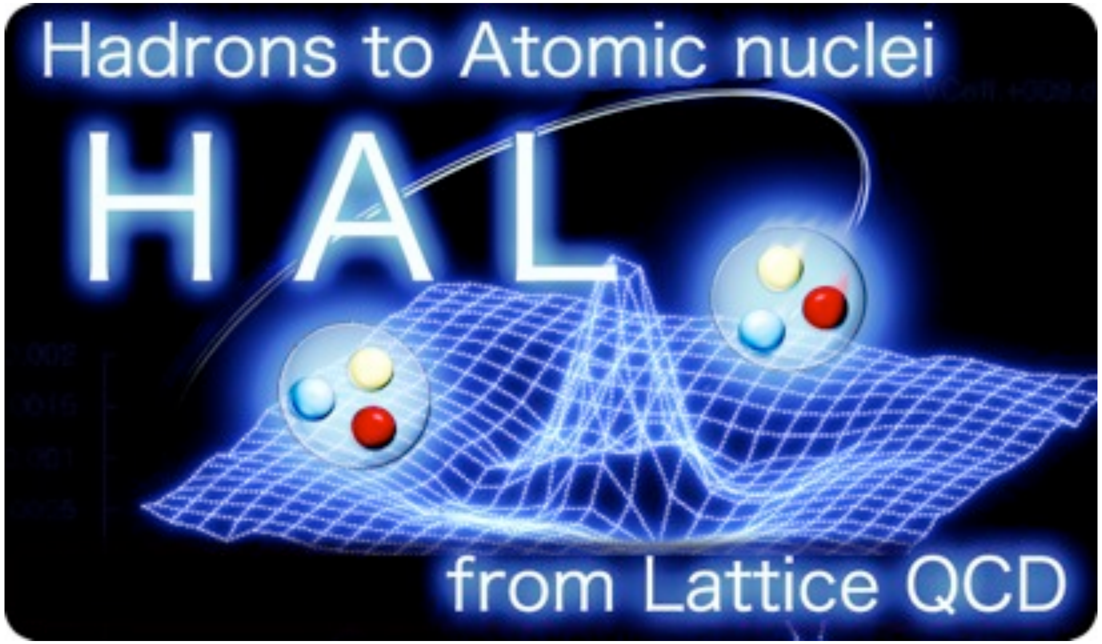


Lattice QCD confronts experiments
- Japanese-German Seminar 2010 -
4 - 6 November 2010, Mishima, Japan

HAL QCD Collaboration



S. Aoki (Tsukuba)
T. Doi (Tsukuba)
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N. Ishii (Tokyo)
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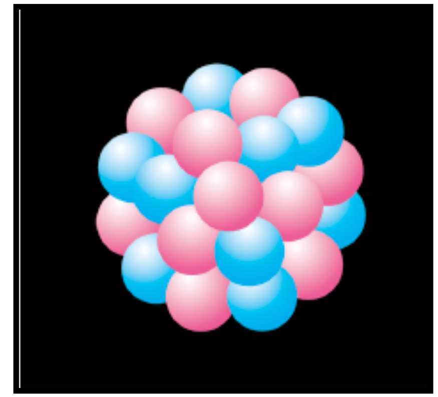
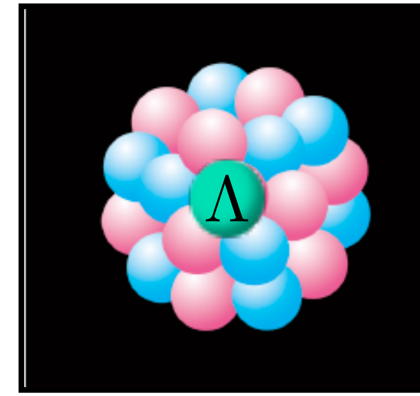
H. Nemura (Tohoku)
K. Sasaki (Tsukuba)

1. Motivation

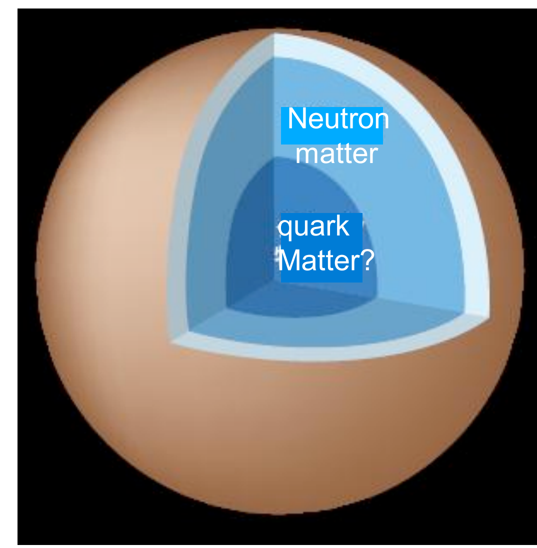
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei



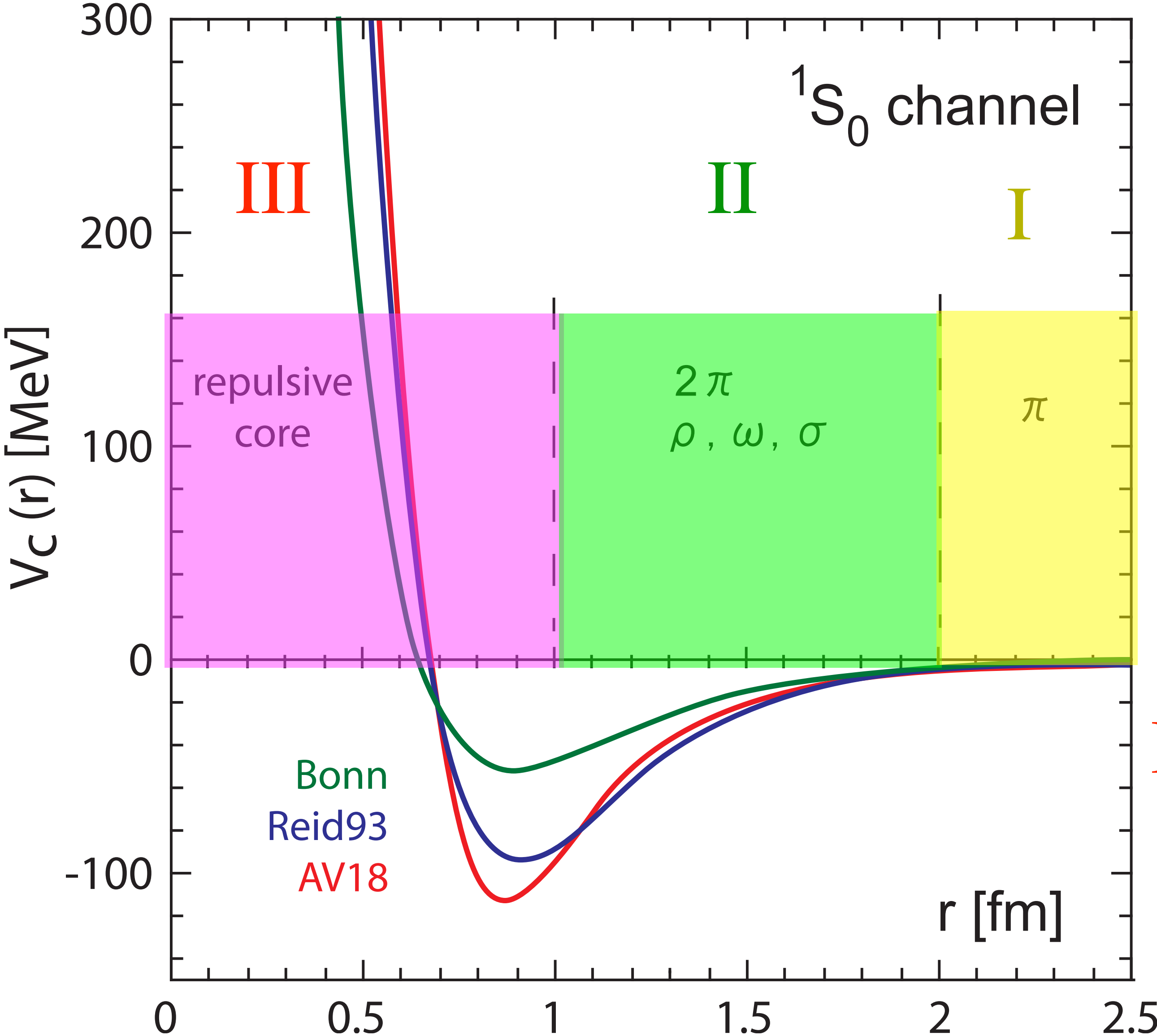


- Structure of neutron star



- Ignition of Type II SuperNova

Phenomenological NN potential
(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

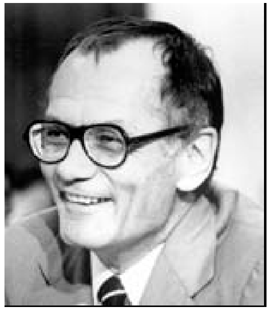
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

Jastrow(1951)



Plan of my talk

1. Motivation
2. Strategy in (lattice) QCD to extract “potential”
3. Inelastic scattering: octet baryon interactions
    1. Baryon-Baryon interactions in an SU(3) symmetric world
    2. Proposal for S=-2 inelastic scattering
    3. H-dibaryon
4. New method for hadron interactions in lattice QCD
5. Summary

2. Strategy in (lattice) QCD to extract “potential”

Challenge to Nambu’s statement

“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation.”

Y. Nambu, “Quarks: Frontiers in Elementary Partcile Physics”, World Scientific (1985)

Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives

$$S = e^{2i\delta}$$

- Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | 6q, E\rangle$$

6 quark QCD eigen-state with energy E

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

Asymptotic behavior

$$r = |\mathbf{r}| \to \infty$$

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \qquad E = \frac{k^2}{2\mu_N} = \frac{k^2}{m_N}$$

partial wave

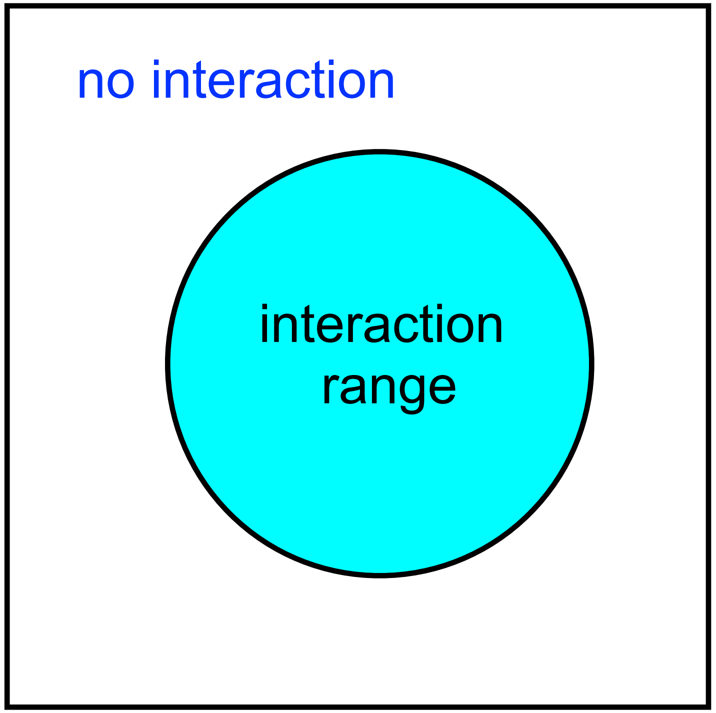
⇒ $\delta_l(k)$ is the scattering phase shift

We define the potential as

$$[\epsilon_k - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.



Velocity expansion

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

Okubo-Marshak (1958)

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$

We calculate observables: phase shift, binding energy etc.
using this approximated potential.

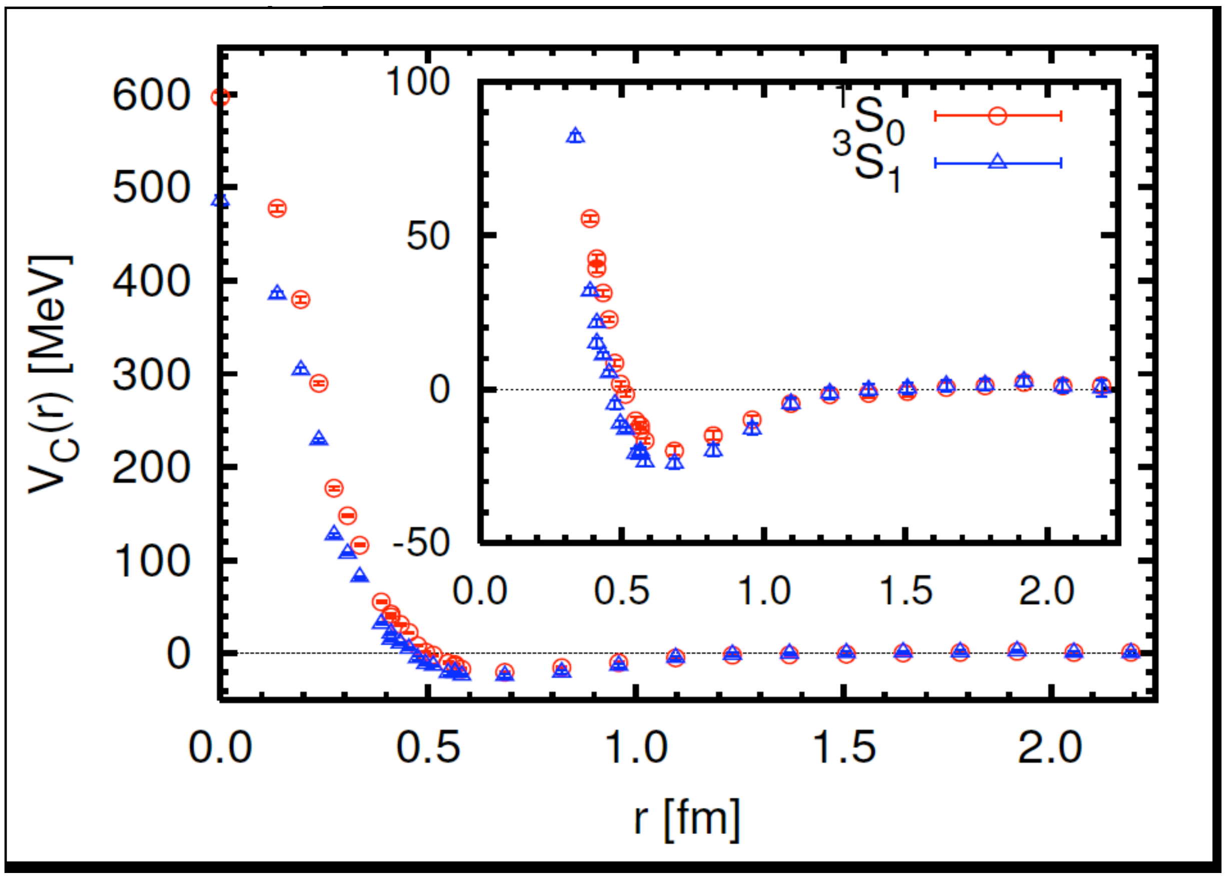
(quenched) potentials

LO (effective) central Potential

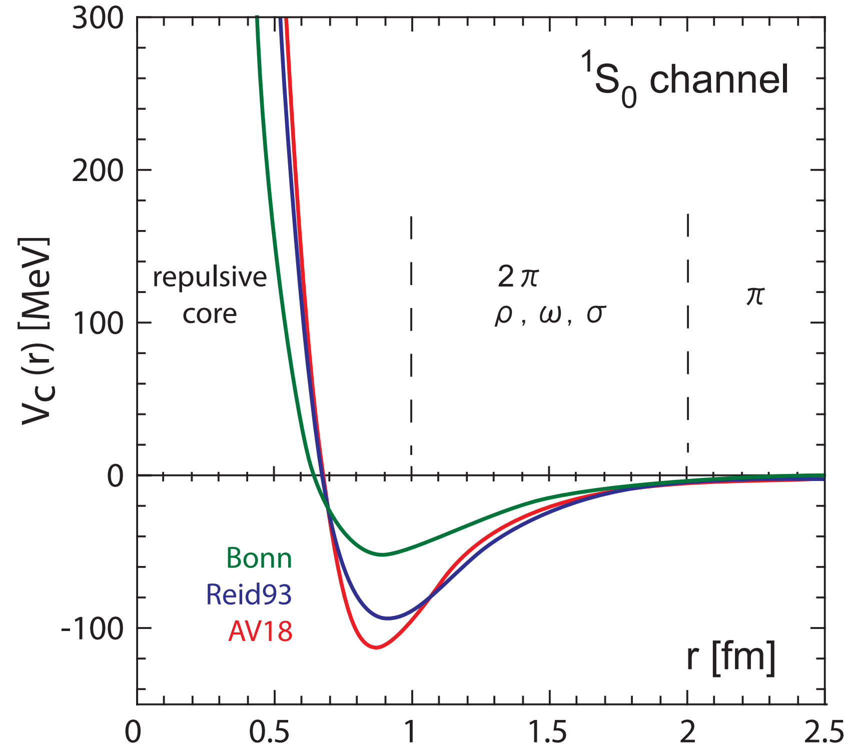
$$V(r;{}^1S_0) = V_0^{(I=1)}(r) + V_\sigma^{(I=1)}(r)$$

$$V(r;{}^3S_1) = V_0^{(I=0)}(r) - 3V_\sigma^{(I=0)}(r)$$

$E \simeq 0$ $\quad m_\pi \simeq 0.53\ \mathrm{GeV}$



a=0.137 fm L=4.4fm



Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

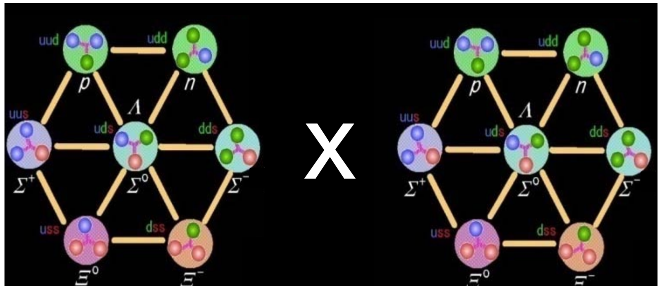
This paper has been selected as one of 21 papers in Nature Research Highlights 2007

3. Inelastic scattering: octet baryon interactions

3-1. Baryon-Baryon interactions in an SU(3) symmetric world

$$m_u = m_d = m_s$$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric　　　Anti-symmetric

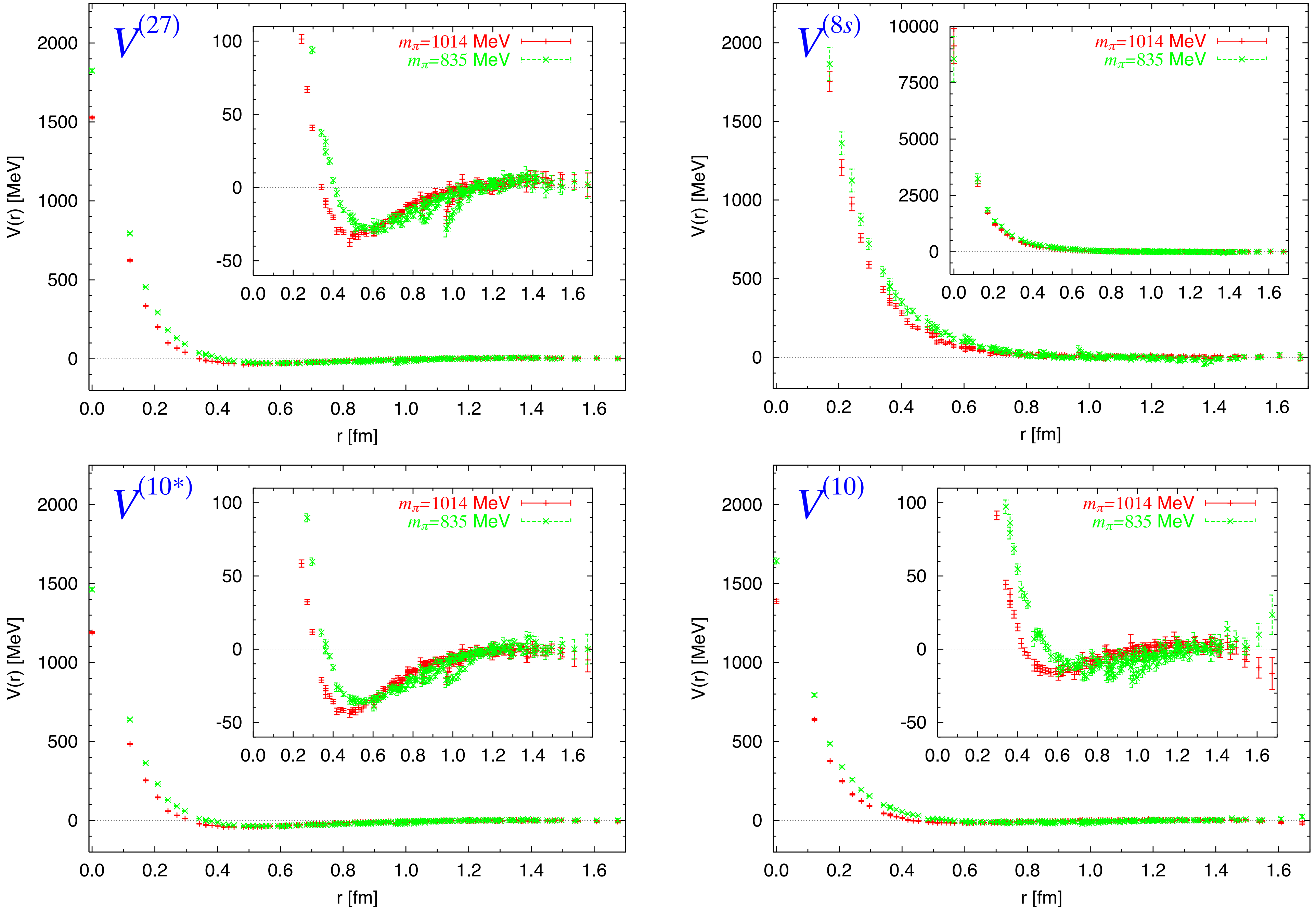
6 independent potential in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

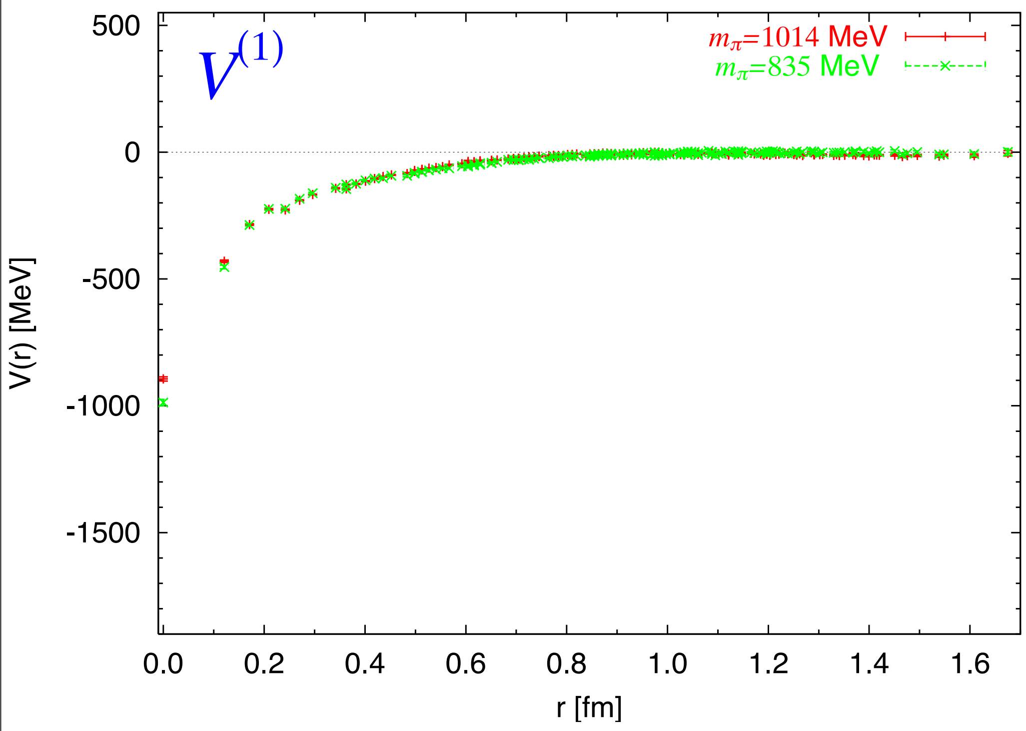
Potentials

Inoue for HAL QCD Collaboration

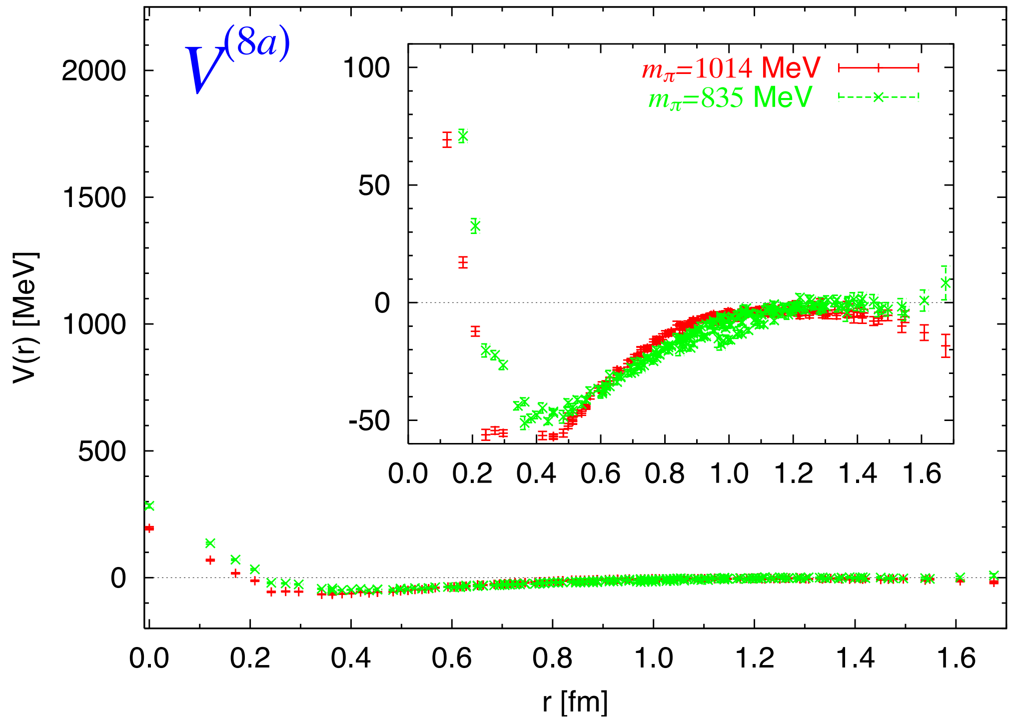


27, 10*: same as before, NN channel

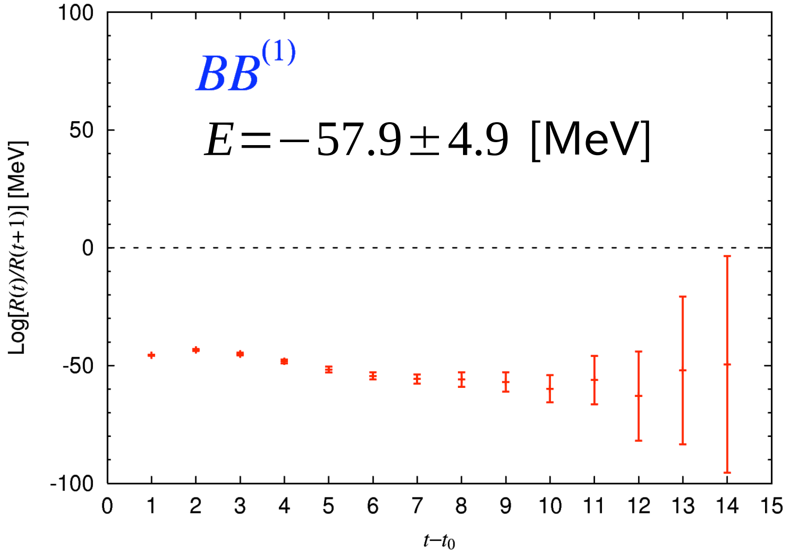
8s, 10: strong repulsive core



1: no repulsive core, attractive core !
No quark mass dependence



8a: week repulsive core,
deep attractive pocket

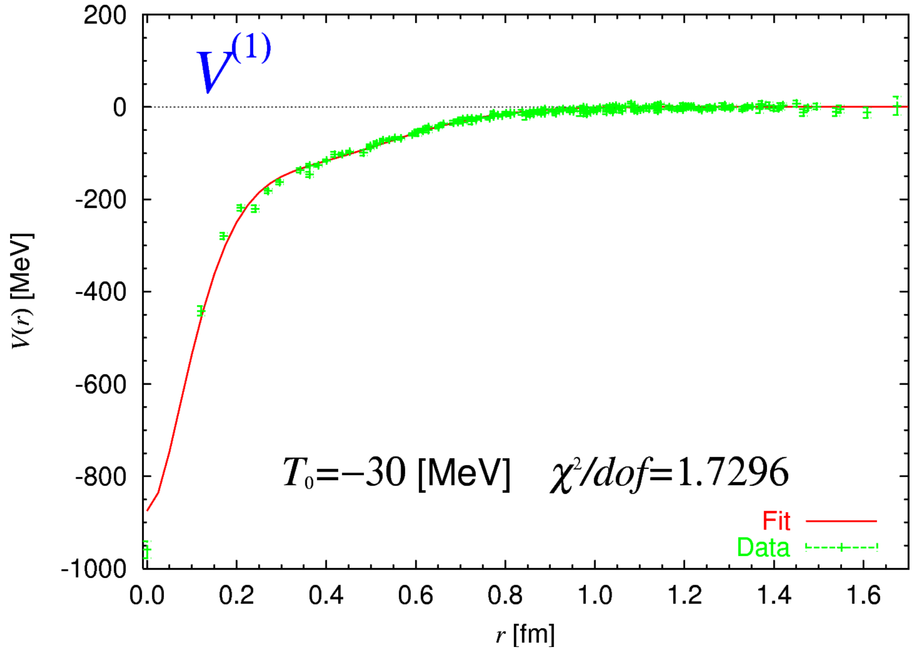


Bound state in 1(singlet) channel ?
H-dibaryon ?

However, it is difficult to determine E precisely, due to contaminations from excited states.

Singlet potential with a certain value of E

Schroedinger eq. predicts a bound state at E < -30 MeV



E [MeV]	E_0 [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]
E = −30	−0.018	24.7
E = −35	−0.72	4.1
E = −40	−2.49	2.3

Finite size effect is very large on this volume.
(consistent with previous results.)

$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

larger volume calculations are in progress.

3-2. Proposal for S=-2 In-elastic scattering

$$m_N = 939 \text{ MeV}, m_\Lambda = 1116 \text{ MeV}, m_\Sigma = 1193 \text{ MeV}, m_\Xi = 1318 \text{ MeV}$$

S=-2 System(I=0)

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

The eigen-state of QCD in the finite box is a mixture of them:

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

HAL's proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$
$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$
$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the "potential" from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \underbrace{V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x})}_{\text{diagonal}} + \underbrace{V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})}_{\text{off-diagonal}}$$

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}} + \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = \underbrace{V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x})}_{\text{off-diagonal}} + \underbrace{V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})}_{\text{diagonal}}$$

μ: reduced mass

$$\begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

$$E_\alpha = \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}, \ \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}$$

$X \neq Y \qquad X, Y = \Lambda\Lambda \text{ or } \Xi N$

$$\alpha = 1, 2$$

Using the potentials:
$$\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$$

we solve the coupled channel Schroedinger equation in the infinite volume with an appropriate boundary condition.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several "in"-states.

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

Lattice is a tool to extract the interaction kernel ("T-matrix" or "potential").

Preliminary results from HAL QCD Collaboration

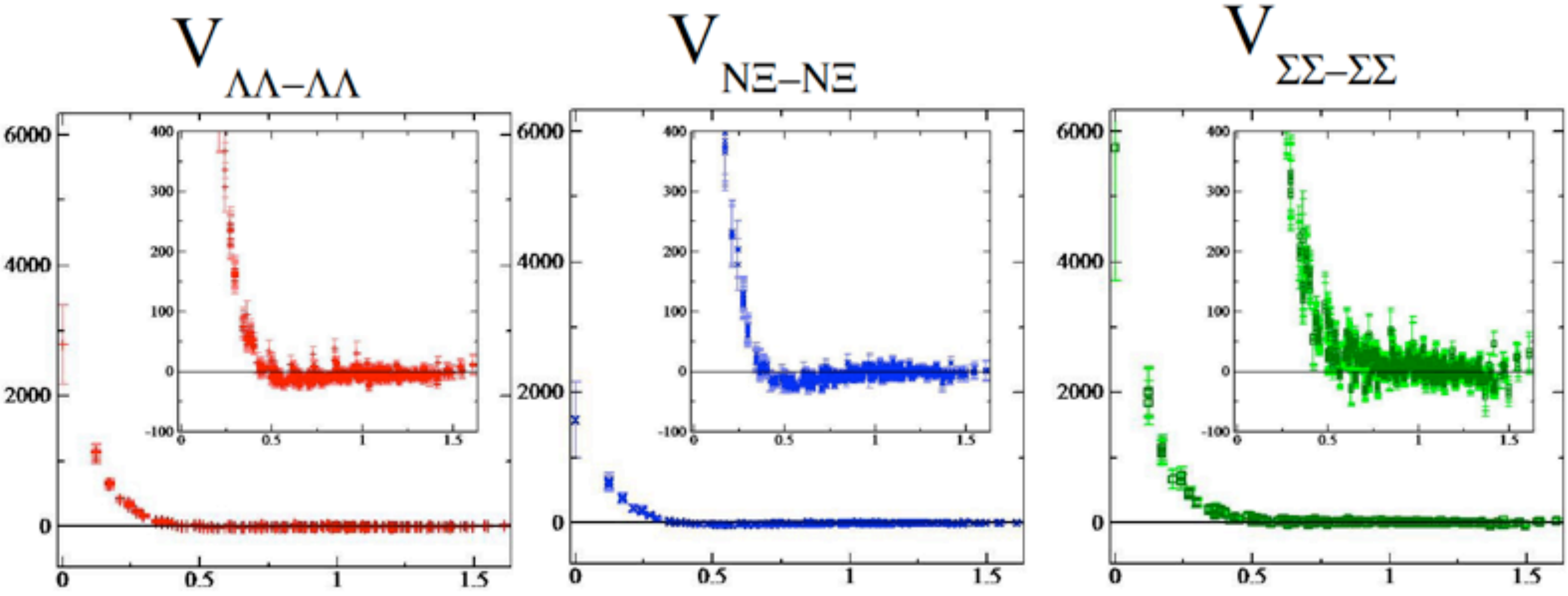
Sasaki for HAL QCD Collaboration

2+1 flavor full QCD

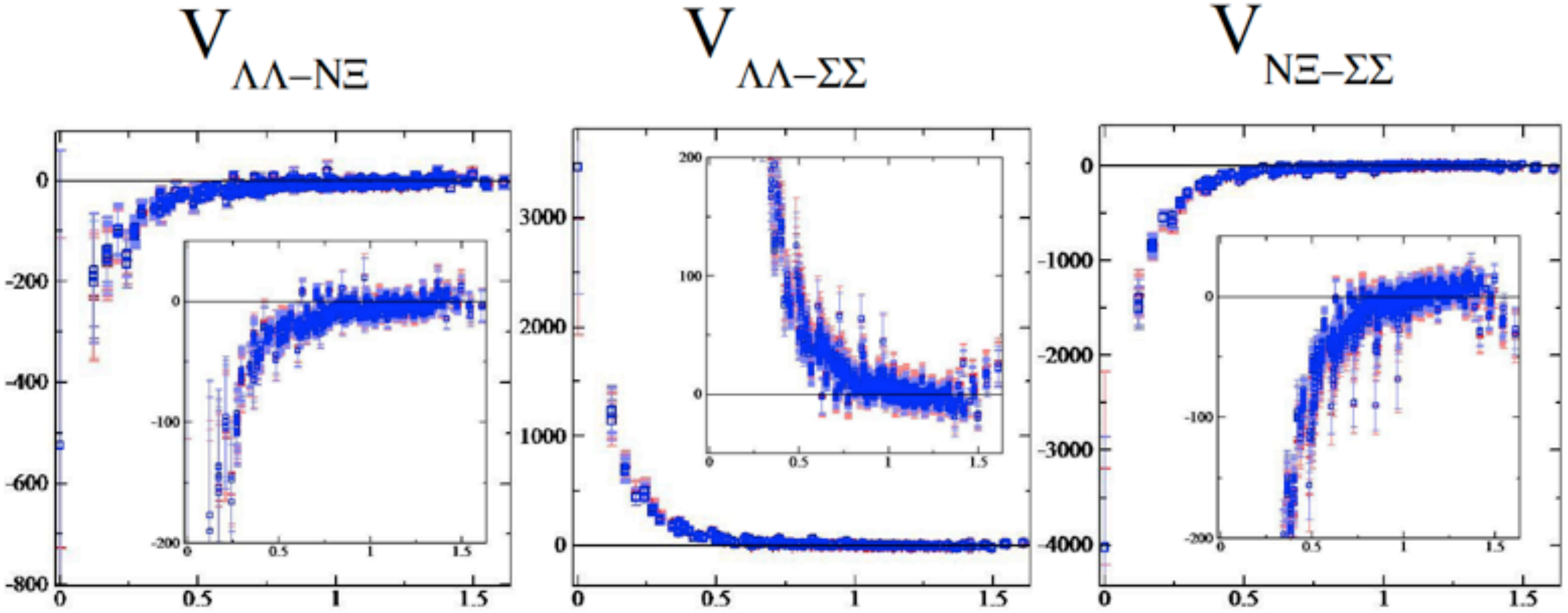
a=0.1 fm, L=2.9 fm

$m_\pi \simeq 870\ \text{MeV}$

Diagonal part of potential matrix



Non-diagonal part of potential matrix



$$V_{A-B} \simeq V_{B-A}$$

Hermiticity ! (non-trivial check)

3-3. H-dibaryon

1. S=-2 singlet state may become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance may appear above ΛΛ but below ΞN threshold.

3. Trial demonstration:

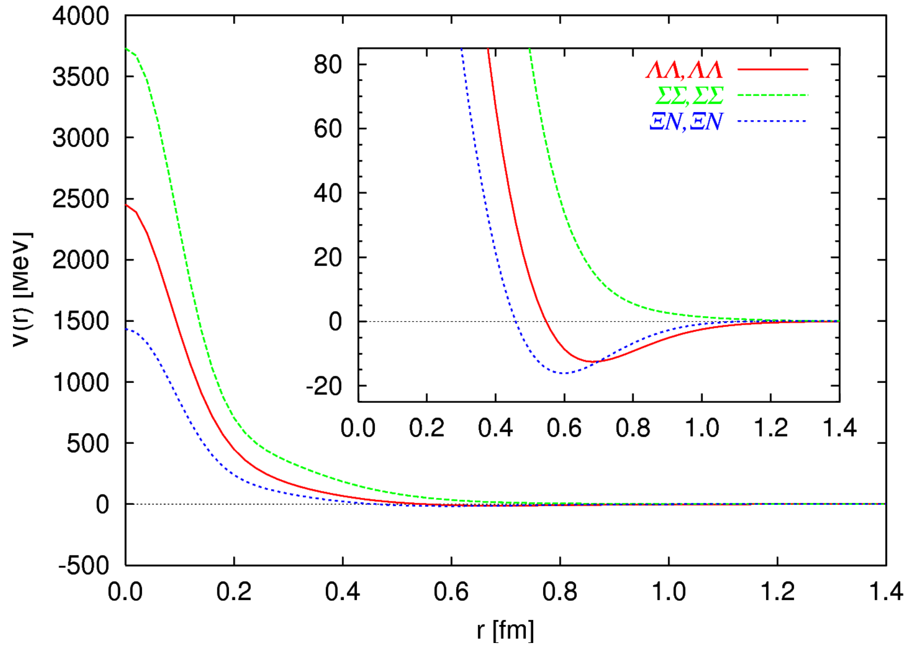
   3.1. Use potential in SU(3) limit

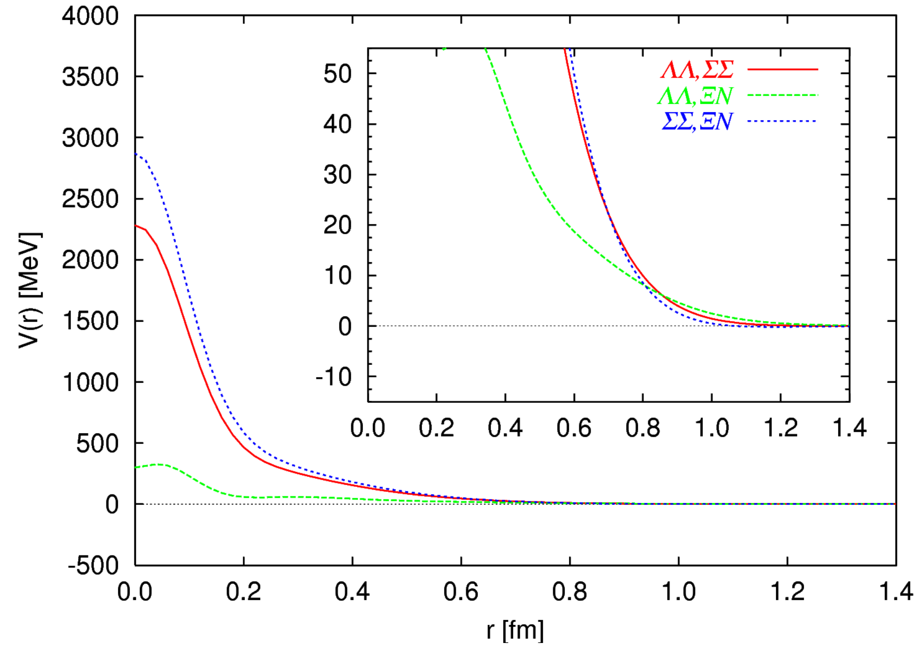
   3.2. Introduce only mass difference from 2+1 simulation

Inoue for HAL QCD Collaboration

Potentials in particle basis in SU(3) limit

$$\begin{pmatrix} \Lambda\Lambda \\ \Sigma\Sigma \\ \Xi N \end{pmatrix} = U \begin{pmatrix} |27\rangle \\ |8\rangle \\ |1\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$

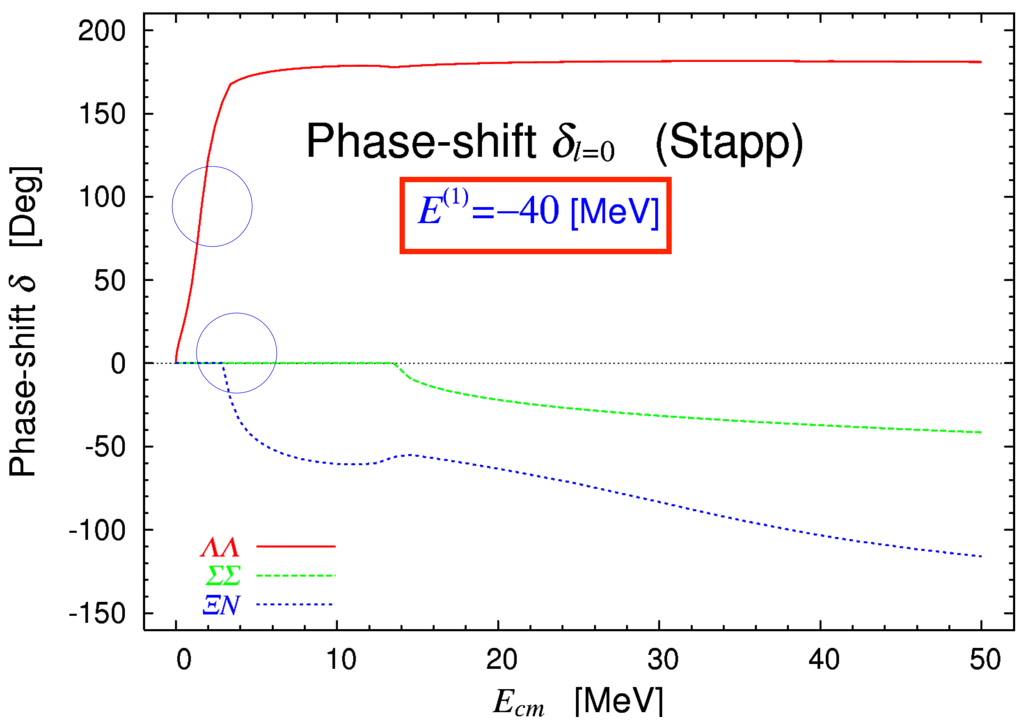


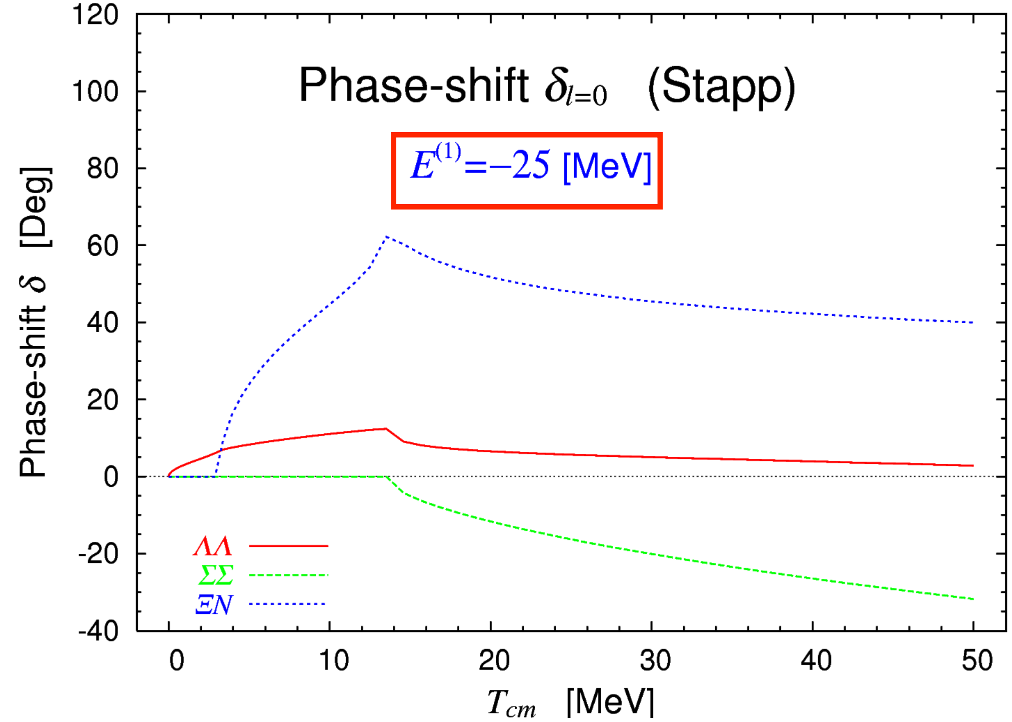


where $T_0^{(1)} = -25$, $T_0^{(8)} = 25$, $T_0^{(27)} = -5$ [MeV] are used

$S = -2, I = 0, {}^1S_0$ scattering

"2+1 flavor"





"2+1 flavor" resonance no resonance

SU(3) limit bound state no bound state

4. New method for hadron interactions in lattice QCD

Inelastic scattering II: particle production

$$E \ge E_{th} = 2m_N + m_\pi$$

elastic scattering $NN \leftarrow NN$

NBS wave function

$$\varphi_E(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p}, -\mathbf{p} \leftarrow \mathbf{k}, -\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon}$$
$$+ \mathcal{I}(\mathbf{r})$$

inelastic contribution $NN\pi \leftarrow NN$ $\quad \propto e^{i\mathbf{q}\cdot\mathbf{r}}$ $\quad |\mathbf{q}| = O(E - E_{th})$

Consider additional NBS wave function

$$\varphi_{E,\pi}(\mathbf{r}, \mathbf{y}) = \langle 0 | N(\mathbf{r} + \mathbf{x}, 0) \pi(\mathbf{y} + \mathbf{x}, 0) N(\mathbf{x}, 0) | 6q, E \rangle$$

Note that

$$|6q, E\rangle = c_1 |NN, E\rangle_{\text{in}} + c_2 |NN\pi, E\rangle_{\text{in}} + \cdots$$

Coupled channel equations

$$
\begin{aligned}
(E-H_0)\varphi_E(\mathbf{x}) &= \int d^3y\, U_{11}(\mathbf{x};\mathbf{y})\varphi_E(\mathbf{y}) + \int d^3y d^3z\, U_{12}(\mathbf{x};\mathbf{y},\mathbf{z})\varphi_{E,\pi}(\mathbf{y},\mathbf{z}) \\
(E-H_0)\varphi_{E,\pi}(\mathbf{x},\mathbf{y}) &= \int d^3z\, U_{21}(\mathbf{x},\mathbf{y};\mathbf{z})\varphi_E(\mathbf{z}) + \int d^3z d^3w\, U_{22}(\mathbf{x},\mathbf{y};\mathbf{z},\mathbf{w})\varphi_{E,\pi}(\mathbf{z},\mathbf{w})
\end{aligned}
$$

Velocity expansion at LO, two values of E

$i = 1, 2$

$$
\begin{aligned}
(E_i-H_0)\varphi_{E_i}(\mathbf{x}) &= V_{11}(\mathbf{x})\varphi_{E_i}(\mathbf{x}) + V_{12}(\mathbf{x},\mathbf{x})\varphi_{E_i,\pi}(\mathbf{x},\mathbf{x}) \\
(E_i-H_0)\varphi_{E_i,\pi}(\mathbf{x},\mathbf{y}) &= V_{21}(\mathbf{x},\mathbf{y})\varphi_{E_i}(\mathbf{x}) + V_{22}(\mathbf{x},\mathbf{y})\varphi_{E_i,\pi}(\mathbf{x},\mathbf{y})
\end{aligned}
$$

$V_{11}(\mathbf{x}) : NN \leftarrow NN$ $\qquad$ $V_{12}(\mathbf{x},\mathbf{x}) : NN \leftarrow NN\pi$

$V_{21}(\mathbf{x},\mathbf{y}) : NN\pi \leftarrow NN$ $\qquad$ $V_{22}(\mathbf{x},\mathbf{y}) : NN\pi \leftarrow NN\pi$

Solve Schroedinger equation with these potentials and a specific B.C.

General prescription

- Consider a QCD eiegnstate with given quantum numbers Q and energy E.
- Take all possible combinations with Q of stable particles whose threshold is below or near E.

$$\text{ex. } Q = 6q: \quad NN, NN\pi, NN\pi\pi, NNK^+K^-, NN\bar{N}N, \cdots$$

- Calculate NBS wave functions for all combinations.
- Extract coupled-channel potentials in a finite volume.
- Solve Schroedinger equation with these potentials in the infinite volume with a suitable B.C. to obtain physical observables.

In practice, of course, final states more than 2 particles are very difficult to deal with.

5. Summary

Summary

- Potentials from NBS wave function are useful tools to extract hadron interactions in lattice QCD. Finite size effect is smaller and quark mass dependence is milder than the phase shift.
  - Combined with Schroedinger equation in the infinite box. Rotational symmetry is recovered.

- Inelastic scattering can also be analysed in terms of coupled channel “potentials”.

  - ΛΛ scattering, H-dibaryon as a resonance

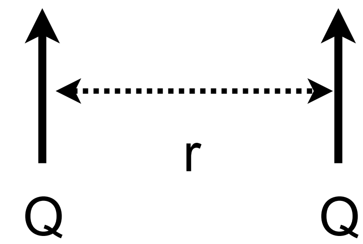
- unstabel particle as a resonace
  - ρ meson, Δ, Roper etc.
  - exotic: penta-quark, X, Y etc.
- 3-Baryon forces : NNN (Doi) , BBB-> Neutron star
- Weak decay ?

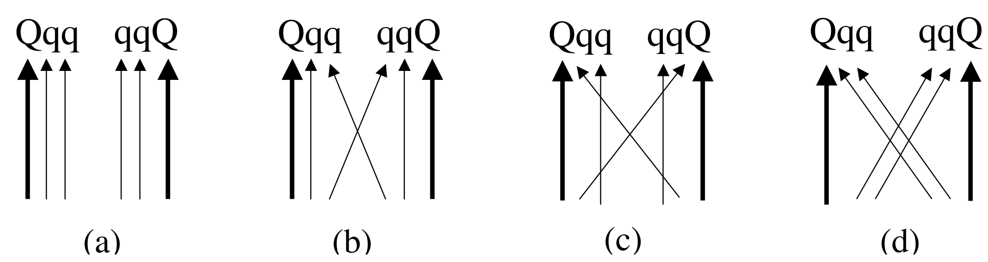
Definition of "Potential" in (lattice) QCD ?

Previous attempt

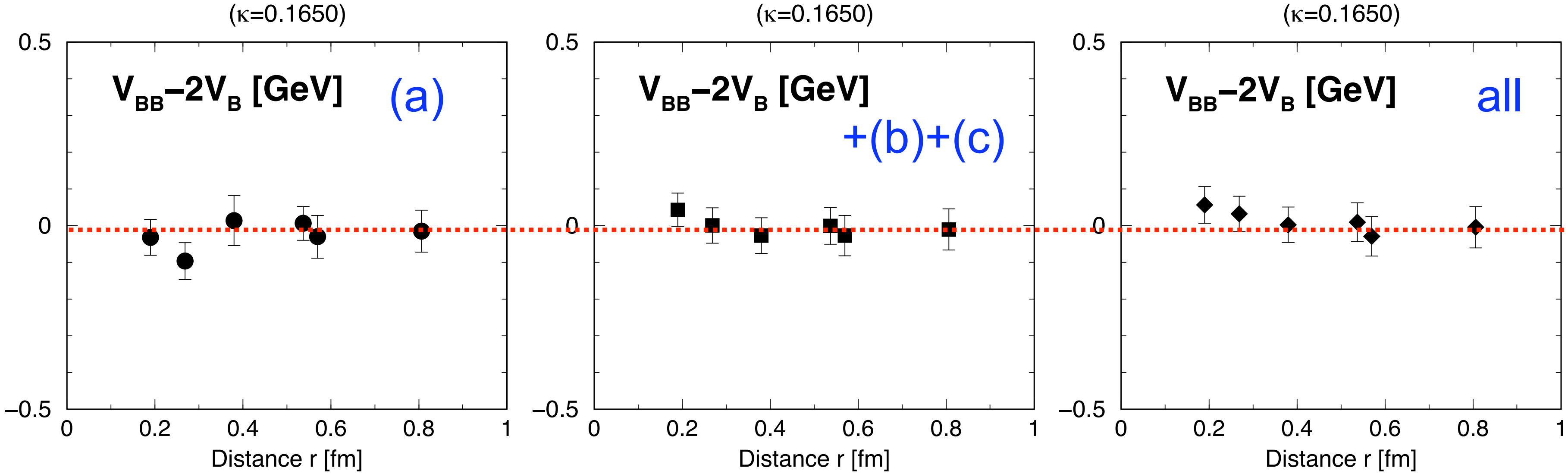
Takahashi-Doi-Suganuma, AIP Conf.Proc. 842,249(2006)

calculate energy of Qqq +Qqq as a function of r between 2Q.
Q:static quark, q: light quark





Quenched result



Almost no dependence on r !

cf. Recent successful result in the strong coupling limit
(deForcrand-Fromm, PRL104(2010)112005)

Frequently Asked Questions

[Q1] Operator dependence of the potential

[Q2] Energy dependence of the potential

[A1] choice of operator = scheme, cf. running coupling

(N(x), U(x,y)) is a combination to define ovservables

QM: (Φ, U) $\rightarrow$ observables

QFT: (asymptotic field, vertices) $\rightarrow$ observables

EFT: (choice of field, vertices) $\rightarrow$ observables

- local operator = convenient choice for reduction formula

[A2] U(x,y) is E-independent by construction

- non-locality can be determined order by order in velocity expansion (cf. ChPT)

Non-local, E-independent $\Longleftrightarrow$ **Local, E-dependent**

$$\left(E + \frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y}) \qquad V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E + \frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$$

Validity of the velocity expansion of U

Leading Order $$V_C(r) = \frac{(E - H_0)\varphi_E(\mathbf{x})}{\varphi_E(\mathbf{x})}$$ Local potential approximation

E-dependent ⟷ Non-locality

From E-dependence, one may determine higher order terms:

$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

Numerical check in quenched QCD

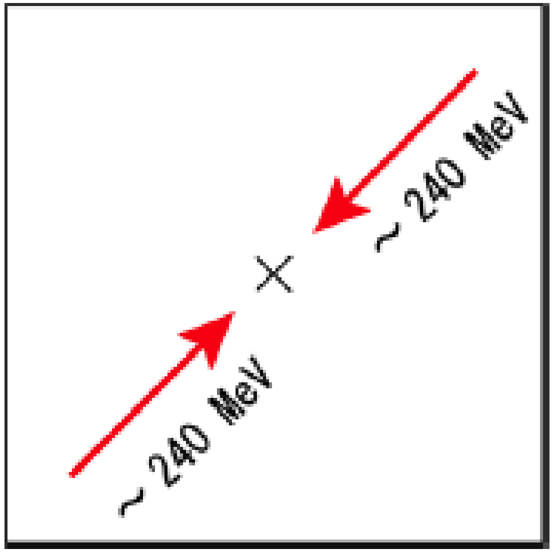
$m_\pi \simeq 0.53$ GeV

a=0.137fm

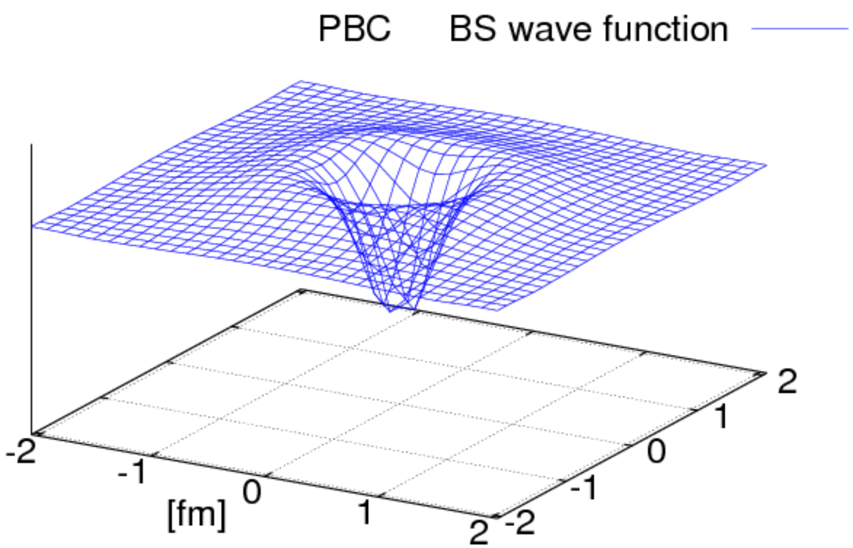
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

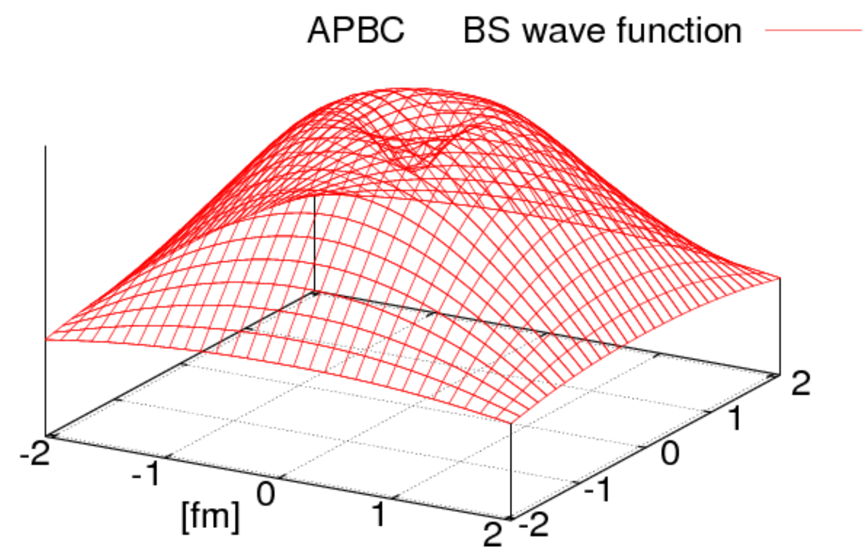
PoS Lattice2009 (2009)126.

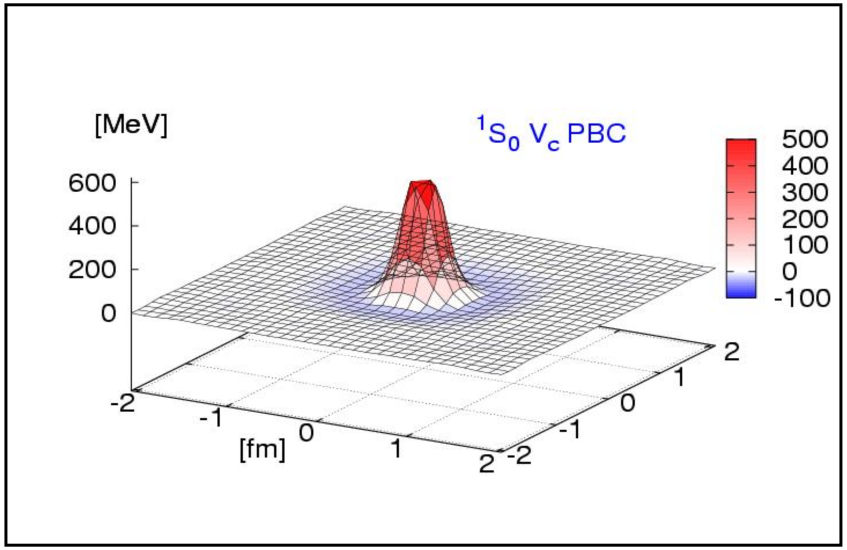
Anti-Periodic B.C.

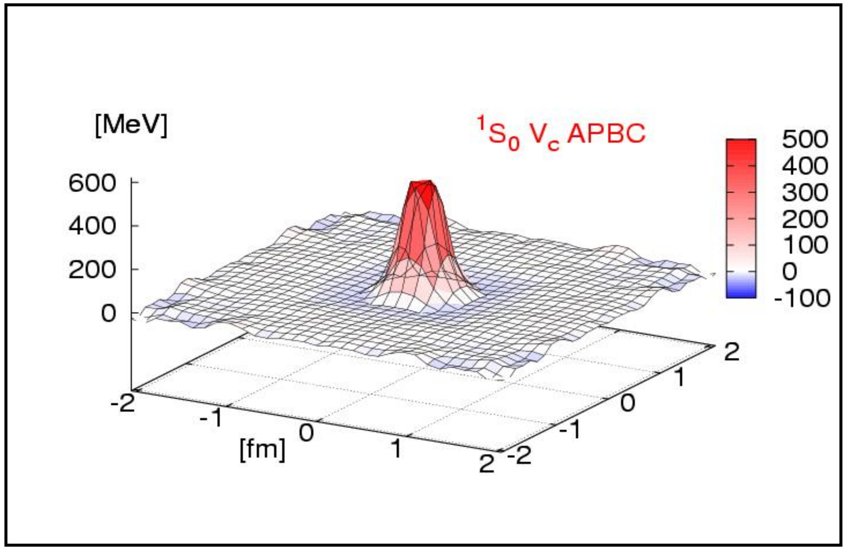


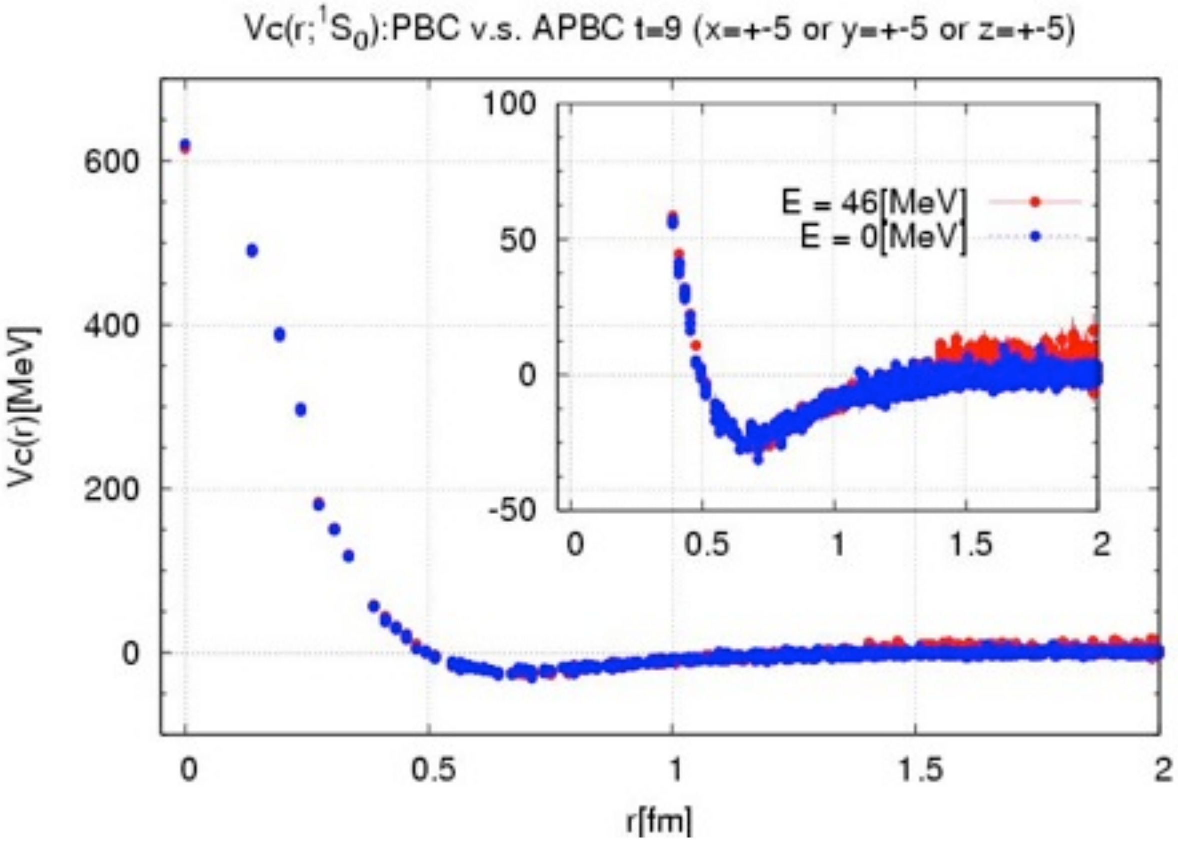
● PBC (E～0 MeV)　　● APBC (E～46 MeV)









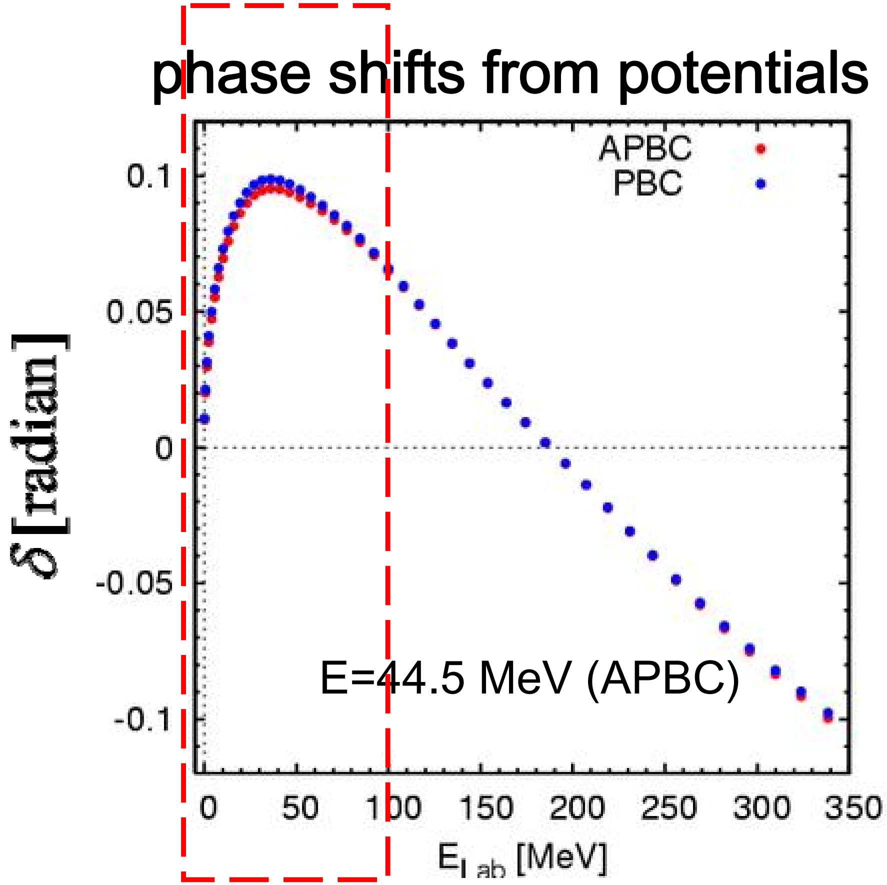


Quenched QCD

$m_\pi \simeq 0.53$ GeV

a=0.137fm

phase shifts from potentials

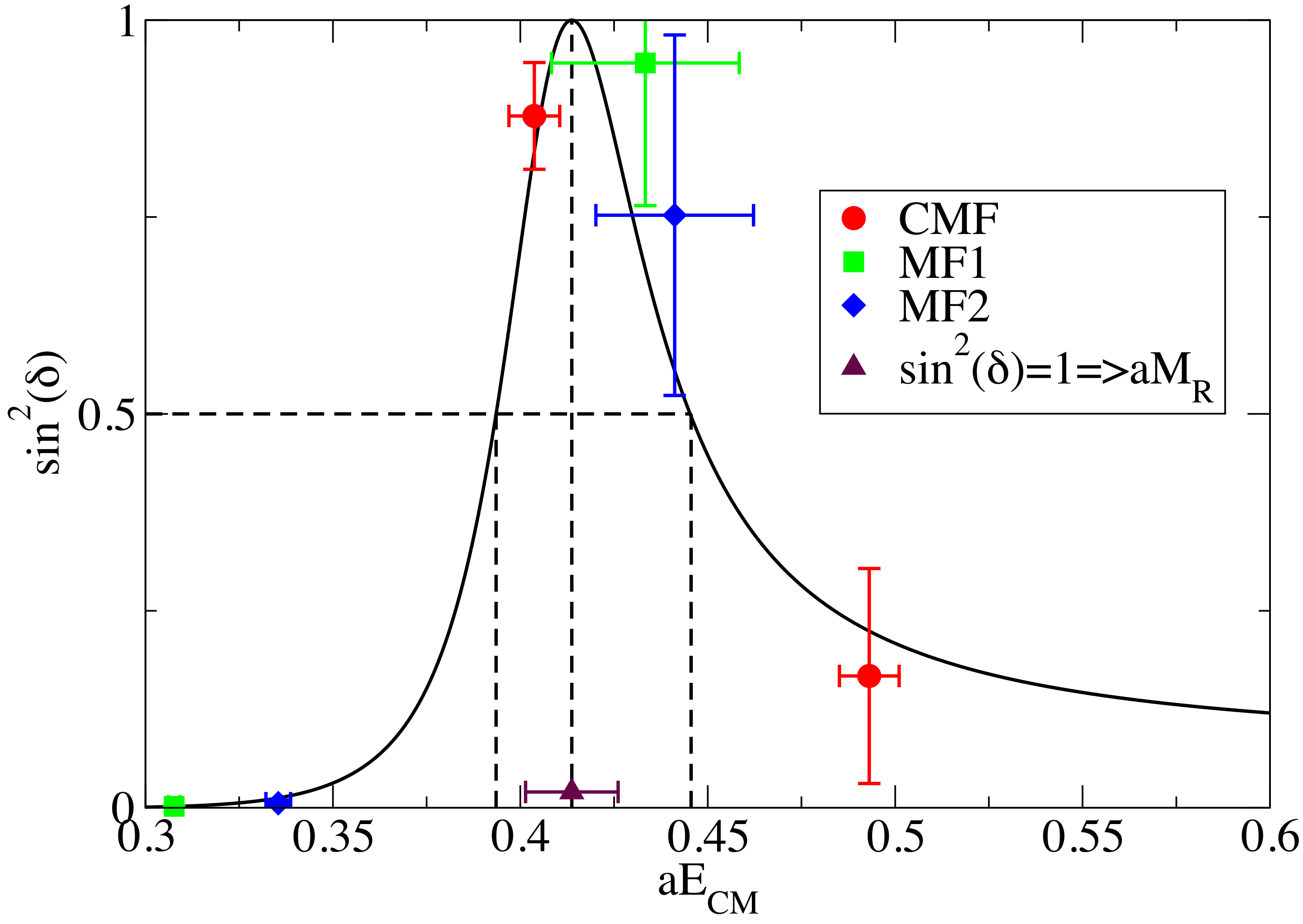


E-dependence of the local potential turns out to be very small at low energy in our choice of wave function.

$\pi^+\pi^-$ scattering (ρ meson width)

Finite volume method

ETMC: Feng-Jansen-Renner, PLB684(2010)



$$\varphi_E(\mathbf{x}) = \langle 0|\pi(\mathbf{x},0)\pi(\mathbf{0},0)|\rho, E\rangle \Rightarrow V(\mathbf{x}) \Rightarrow \sin^2\delta(s)?$$