Dirac operator eigenvalues in QCD at zero and finite temperature

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[JLQCD collaboration]

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Spontaneous symmetry breaking in QCD

- **Bose-Einstein condensation of quark anti-quark pairs**
  - **"effective mass" of quarks (chiral symmetry broken)**
  - **light pions (⇒ chiral effective theory)**

- **Their origin or mechanism?**
  - **Unknown until QCD is solved.**
  - **Plausible "story"**
    1. **Topological excitations of gauge field (instanton)**
    2. **Associated quark (near-)zero modes**
    3. **Dominate in the vacuum (classical solution)**

$U_A(1)$ anomaly

give chiral condensate

Dirac eigenvalue spectrum

- **Banks-Casher relation**
  - **Dirac eigenvalue spectrum**
  - **chiral condensate**

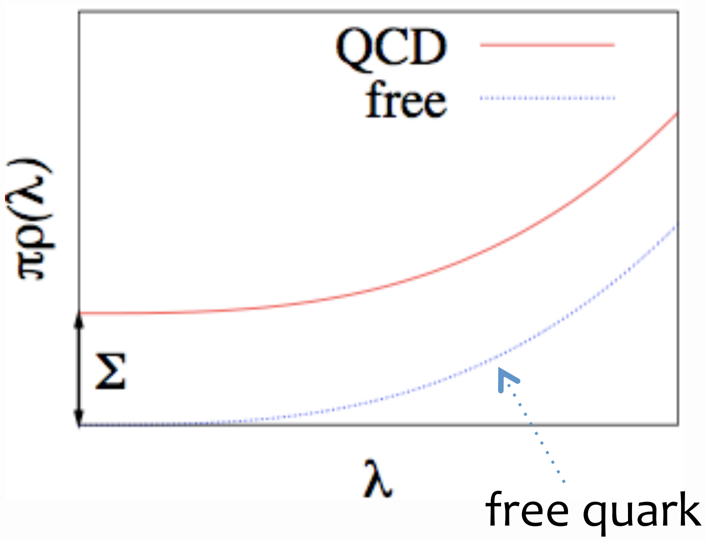
$$\rho(\lambda) = \frac{1}{V}\sum_k \langle \delta(\lambda - \lambda_k) \rangle,$$

$$\langle \bar{q}q \rangle = \frac{1}{V}\sum_k \left\langle \frac{1}{m + i\lambda_k} \right\rangle$$

Banks, Casher (1980)

$$\lim_{m\to 0}\lim_{V\to\infty} \rho(\lambda = 0) = \frac{\Sigma}{\pi}$$

accumulation of near-zero modes =
"quark-antiquark condensate"



Lattice study?

- Dynamical lattice QCD calculation
  - Eigenvalue distribution of the overlap-Dirac operator (JLQCD, 2007, 2009)

$$\boxed{D = \frac{\rho}{a}\left[1 + \gamma_5 \,\mathrm{sgn}(aH_W)\right],\ aH_W = \gamma_5 (aD_W - \rho)}$$

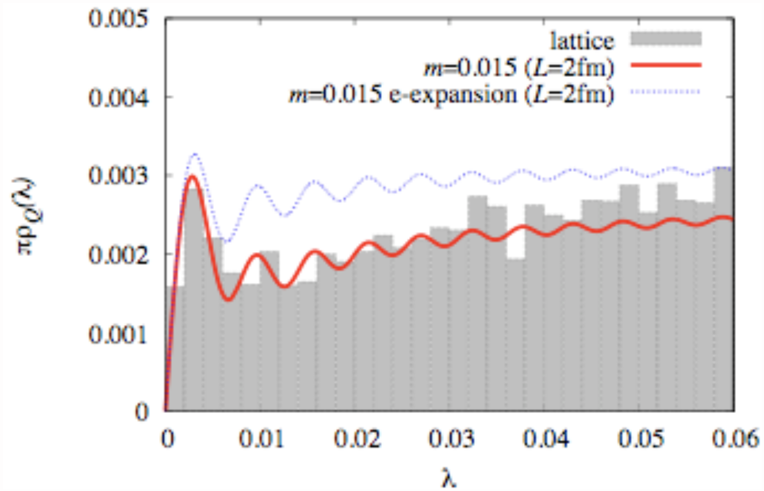
    - sign function approximated by Zolotarev after projecting out lowmodes of H_W. Precise to the level of $10^{-(7\text{-}8)}$.
  - Exact chiral symmetry plays the key role.
    - allows for the study of lowest eigenvalues.
    - ε-regime simulation is possible.

Role of near-zero mode

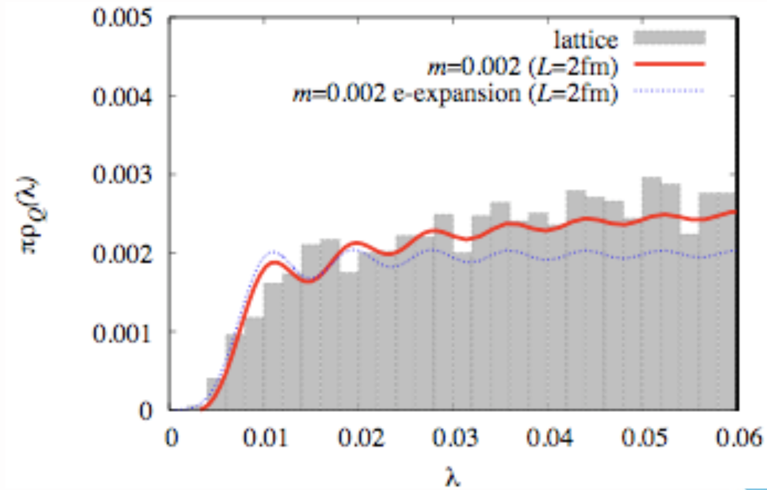
- ε-regime
  - pion zero-mode gives the main contribution.
  - quark mass becomes negligible in the fermion determinant.

$$\prod_k \left(\lambda_k^2 + m^2\right)$$

p-regime



ε-regime

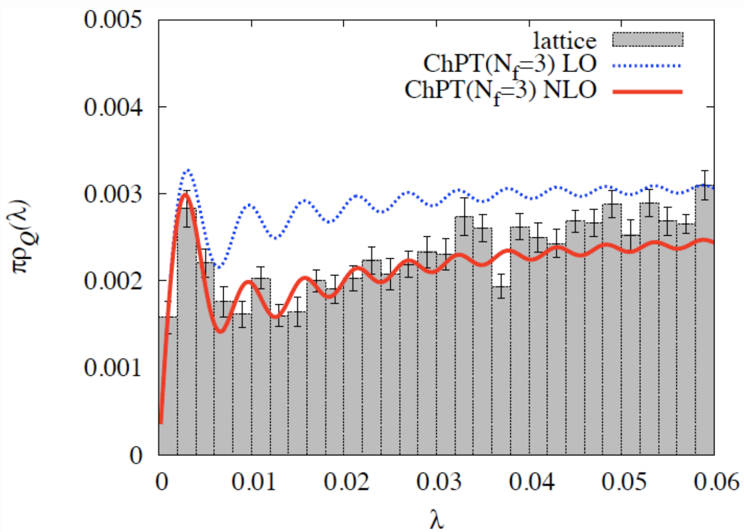


Full NLO analysis

- **Damgaard-Fukaya (2009): NLO ChPT valid in p and ε.**

$$\rho_Q(\lambda) = \Sigma_{\rm eff}\ \hat{\rho}^{\epsilon}_Q(\lambda\Sigma_{\rm eff}V, \{m_{sea}\Sigma V\}) + \rho^p(\lambda),$$

pion zero-mode integral pion loop



Lattice results:

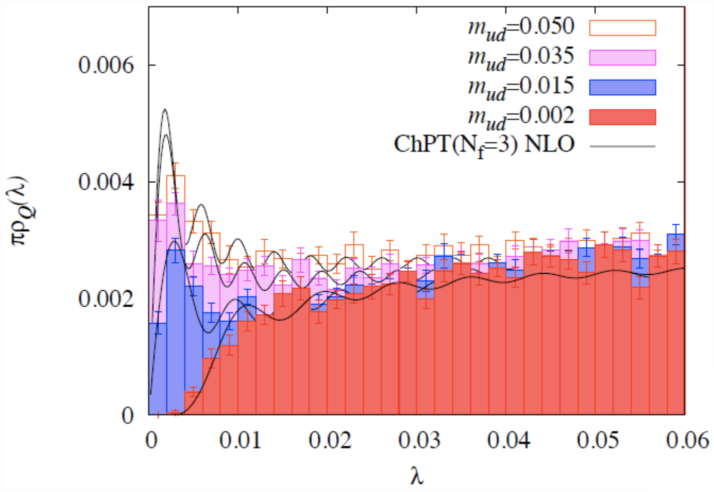
JLQCD, PRL101, 122002 (2010)

and a full paper (in preparation)

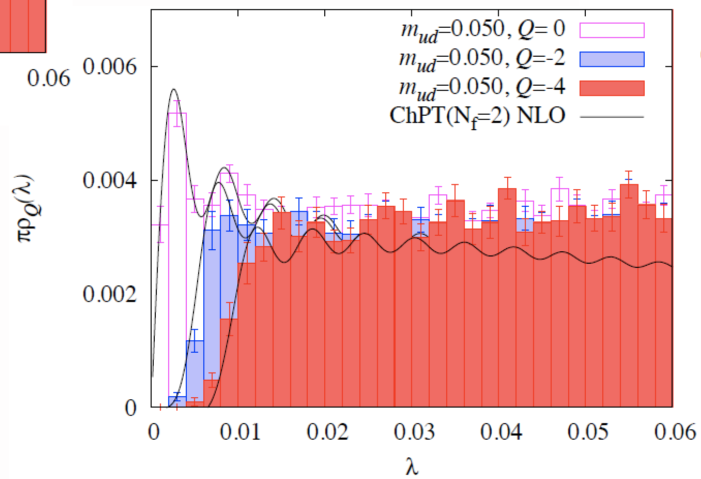
- 2 and 2+1 flavors
- p and e regimes (various quark masses)
- 16^3x48 and 24^3x48
- various topological charges

Scaling tests

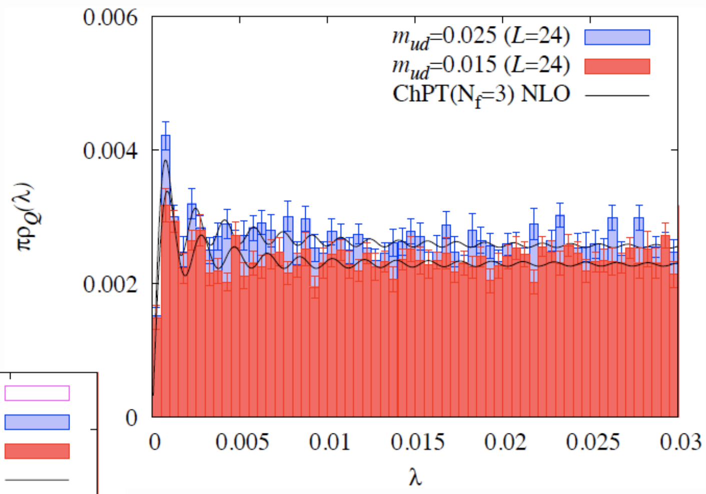
- **Quark mass m**



- **Topological charge Q**



- **Volume V**

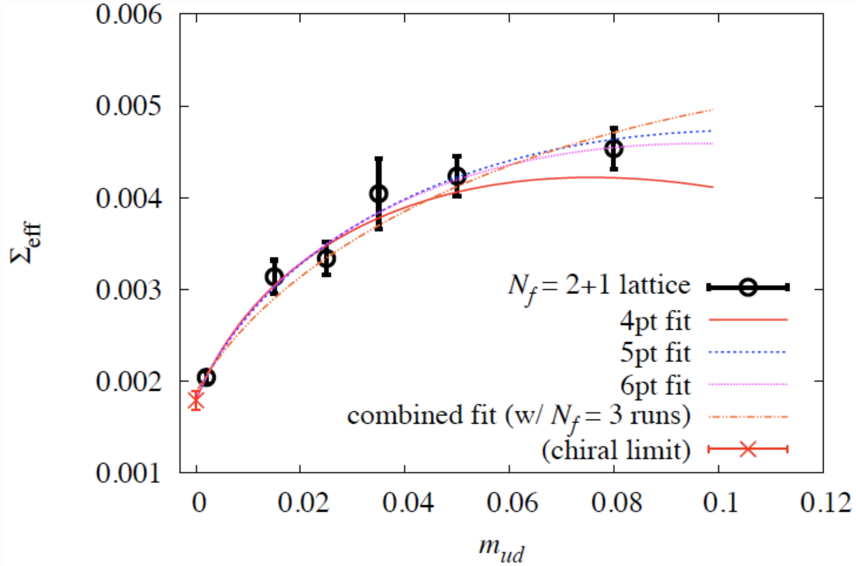


Chiral condensate at T=0

- Chiral extrapolation
  - Very stable because of the ε-regime point.
  - NLO ChPT

$$\Sigma(m_{ud},m_s)=\Sigma(0,m_s)\times\left[1-\frac{3M_\pi^2}{32\pi^2F^2}\ln\frac{M_\pi^2}{\mu^2}+\frac{32L_6M_\pi^2}{F^2}\right]$$

  - In the real analysis, finite V effects are included.



$$\Sigma^{\overline{MS}}(2\,\mathrm{GeV})=[242(04)(^{+19}_{-18})\ \mathrm{MeV}]^3$$

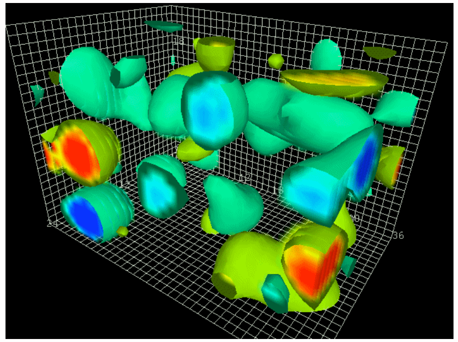
Near-zero modes are ...

- **Nearly chiral (L or R) and localized**
  - **Contributes to topological susceptibility**

$$\chi_t = \frac{\langle Q^2 \rangle}{V} = \int d^4x \langle q(x) q(0) \rangle$$

  - **Can be extracted from topological charge density correlator**

Derek Leinweber



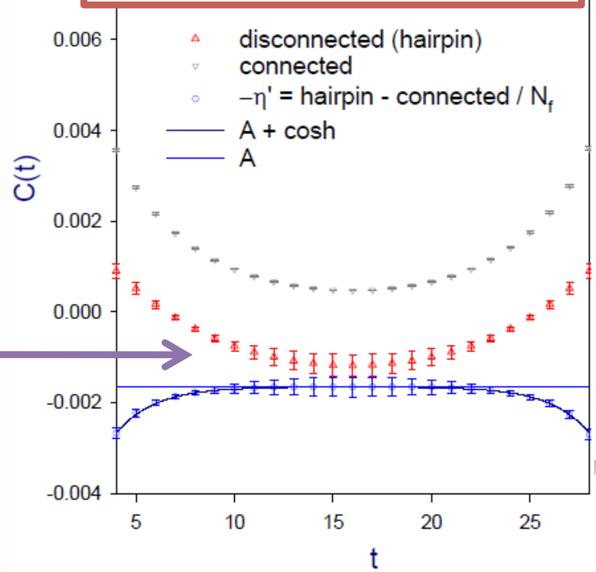
Aoki et al, PRD76, 054508 (2007)

$$\lim_{x\to\infty} \langle mP(x) mP(0) \rangle_Q = -\frac{1}{V}\left(\chi_t - \frac{Q^2}{V} + \ldots \right) + O(e^{-m_{\eta'} x})$$

even at fixed Q

Near-zero modes dominates in the disconnected diagrams

JLQCD, PLB665, 294 (2008)



Topological susceptibility at T=0

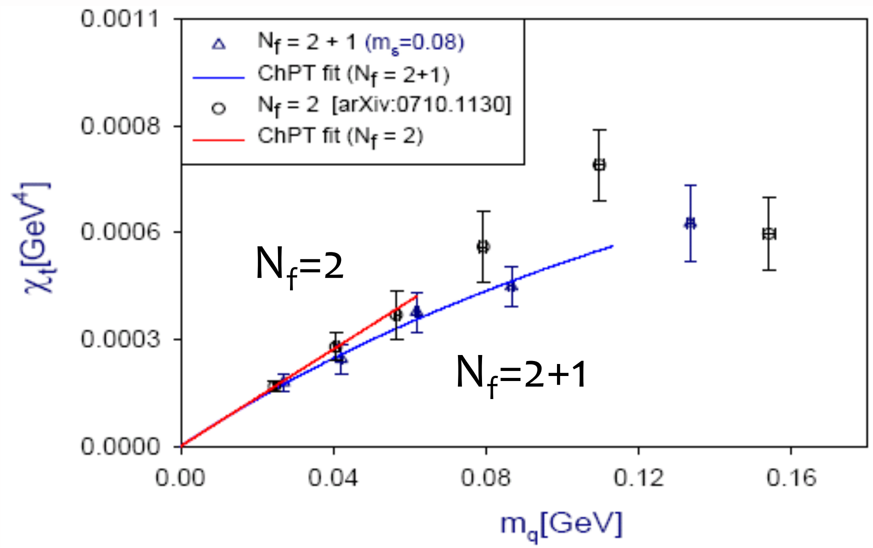
- **Chiral extrapolation**

Crewther (1977), Leutwyler-Smilga (1992)

$$\chi_t = m\Sigma / N_f, \quad \text{or}$$

$$\chi_t = \frac{\Sigma}{m_u^{-1} + m_d^{-1} + m_s^{-1}}$$

… indicates that fixed Q doesn't freeze local topology change.



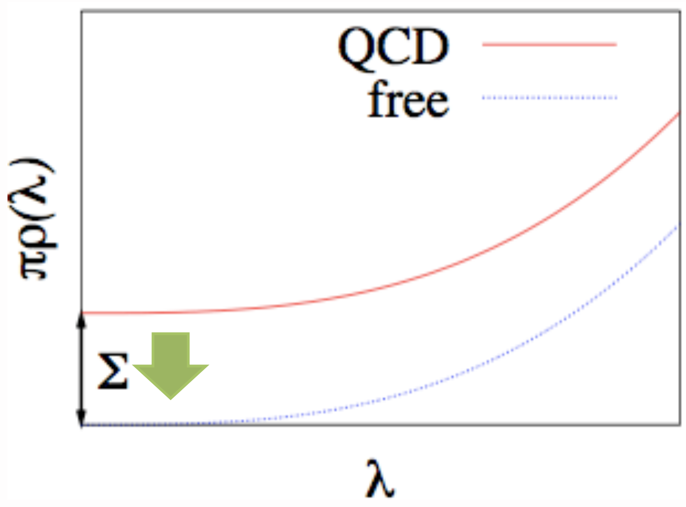
$$\Sigma^{\overline{MS}}(2\,\mathrm{GeV}) = [247(03)(XX)\ \mathrm{MeV}]^3$$

What happens at finite T

- Restoration of chiral symmetry above T_c

$$\Sigma \equiv -\langle \bar{q}q \rangle = 0$$

- , which means…
  - Near-zero modes vanish.
  - Topological susceptibility vanishes.
  - Everything coming from the disconnected quark-loop vanishes?



Restoration of $U_A(1)$?

- **Flavor-singlet axial rotation**
  - **axial anomaly = broken by quantization, at any T**

$$\partial_\mu j_{\mu 5} = \frac{N_f}{16\pi^2} G^a_{\alpha\beta}\tilde{G}^a_{\alpha\beta}, \quad j_{\mu 5} = \sum_f \bar{q}^f \gamma_\mu \gamma_5 q^f$$

  - **How does it show up in hadron spectra?**

	non-singlet	singlet
scalar	δ	σ
pseudo-scalar	π	η’

$U_A(1)$

- **Degenerate when $U_A(1)$ restores**

Near-zero mode dominates.
Others $m^2 \to 0$

$$\chi_\pi - \chi_\delta = \int d^4x \langle j_\pi(x) j_\pi(0) - j_\delta(x) j_\delta(0) \rangle = \int d\lambda \rho(\lambda) \frac{4m^2}{\left(\lambda^2 + m^2\right)^2}$$

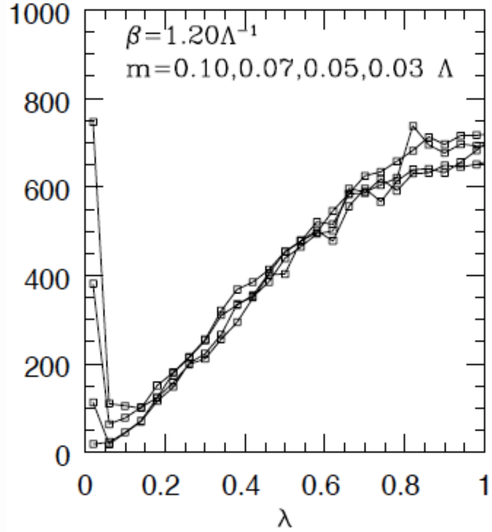
Restoration of $U_A(1)$?

- **Paradox ?** Shuryak (1994), Cohen (1996).
  - **$U_A(1)$ is violated by quantization, no restoration**
  - **On the other hand,**
    - **non-singlet** $$\Sigma = \int d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = 0$$
    - **singlet** $$\chi_\pi - \chi_\delta = \int d\lambda \rho(\lambda) \frac{4m^2}{\left(\lambda^2 + m^2\right)^2} = 0$$

always occurs together?

- **In order that $U_A(1)$ remains broken,**

$$\rho(\lambda) \sim m^2 \delta(\lambda)$$



see also, Lee, Hatsuda (1996)

Schafer (1996)
Instanton Liquid Model

Finite T lattices

- Quenched QCD (N_f=0)
  - 16^3x6, 24^3x6
  - Iwasaki + Q-fixed

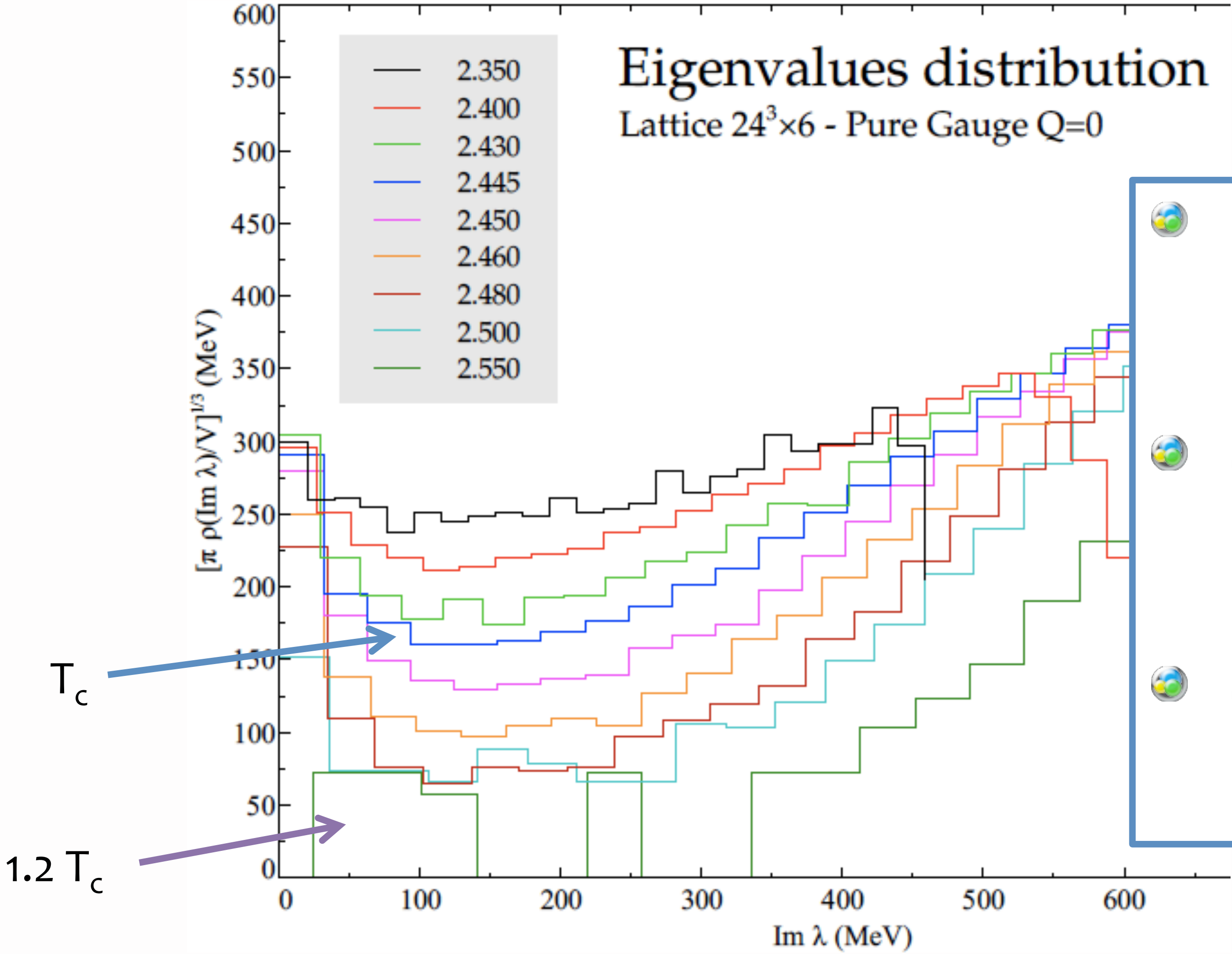
β	a (fm)	T (MeV)	T/T_c
2.35	0.13	250	0.86
…			
2.445	0.114	288	1
…			
2.55	0.094	350	1.20

- N_f=2 QCD
  - 16^3x8
  - Iwasaki + Q-fixed + overlap
  - am=0.050, 0.025

β	a (fm)	T (MeV)	T/T_c
2.18	0.144	172	0.90
2.20	0.139	177	0.93
2.30	0.118	209	1.09
2.40	0.101	243	1.27
2.45	0.094	262	1.38

Tc = 190 MeV (fixed)

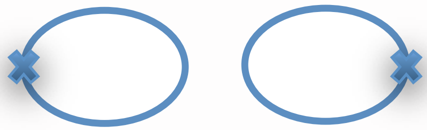
Quenched QCD: Dirac spectrum



- Peak near the origin
  - Shoulder suppressed at larger T
- ~ Tc
  - Nothing significantly different
- ~ 1.2 Tc
  - Peak disappears

Topological susceptibility

- **At a fixed Q**
  - **Long range correlation of q(x)**

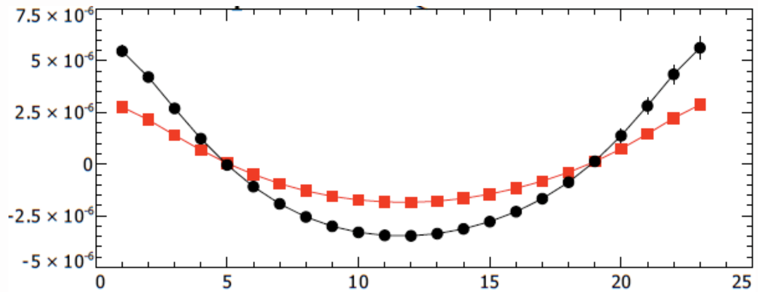


$$\lim_{x\to\infty}\left\langle q(x)q(0)\right\rangle_Q = \lim_{x\to\infty}\left\langle mP(x)mP(0)\right\rangle_Q = -\frac{1}{V}\left(\chi_t - \frac{Q^2}{V} + \frac{c_4}{2\chi_t V}\cdots\right) + O(e^{-m_{\eta'}x})$$

  - **Quenched pathology: not η', but** pion **double-pole**

$$f_P \frac{1}{p^2+m_\pi^2} m_0^2 \frac{1}{p^2+m_\pi^2} f_P + \text{const}$$

$$\sim \frac{f_P^2 m_0^2}{4m_\pi^3}(1+m_\pi x)e^{-m_\pi x} + \text{const}$$



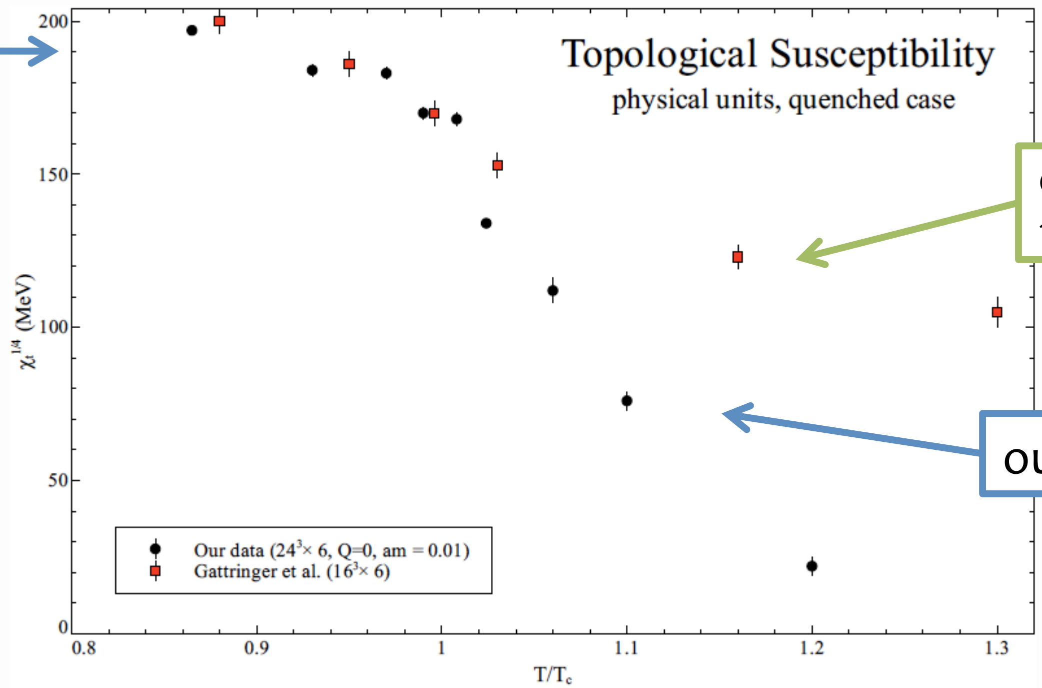
Approx the disconnected diagram by only the near-zero modes:

β=2.40. *a*m=0.025, 0.01

Topological susceptibility

- **In quenched QCD**

T=0: ~191(5) MeV
Del Debbio et al. (2008)

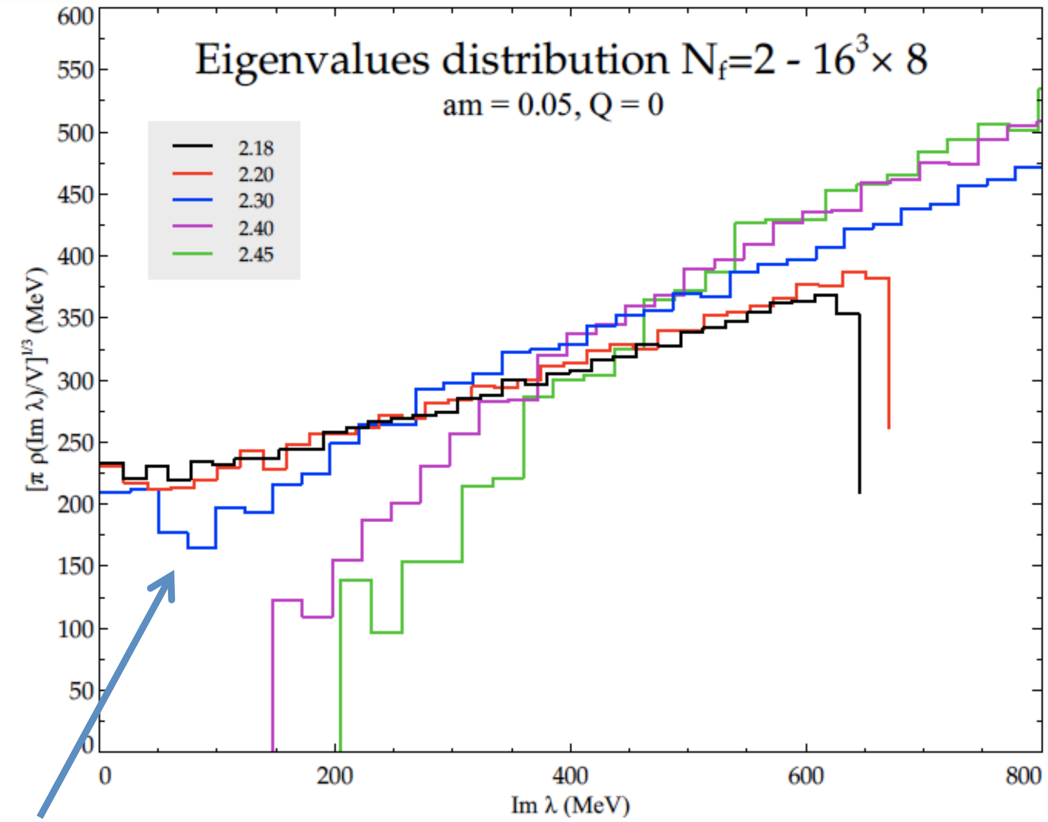


Gattringer et al. (2002)
16^3x6

ours, 24^3x6

Difference need to be understood, $c_4/\chi_t V$?

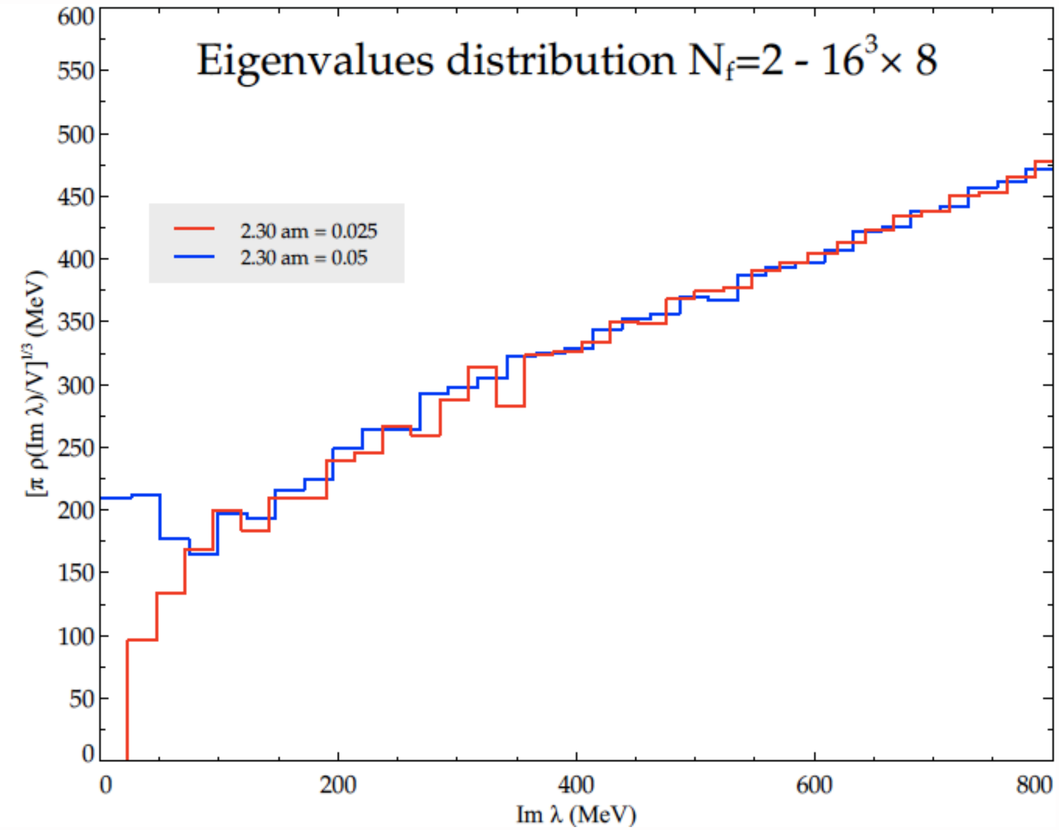
N_f=2 QCD: Dirac spectrum



T ~ 1.1 T_c

- *Q* fixed (=0)
- *am* fixed (~ 60-100 MeV)
- T ~ 1.1 Tc
  - Peak as in quenched QCD
- T~1.3Tc
  - Peak vanishes

N_f=2 QCD: Dirac spectrum



- *am* dependence
  - at T ~ 1.1 T_c
  - peak vanishes as *am* decreases
  - small gap near zero

N_f=2 QCD

Possible story (or speculation)

- **$T < T_c$**
  - $\rho(0) \neq 0$: **Spontaneous chiral symmetry breaking**
- **$T_c < T < 1.2(?) \times T_c$**
  - **chiral symmetry restores:** $\rho(\sim 0) = 0$ (infinite volume)
  - **A spike near zero. Gets lower as quark mass decreases. Will vanish at some point, where $U_A(1)$ restores too.**
- **$T > 1.2(?) \times T_c$**
  - **$U_A(1)$ restoration**

$$\rho(\lambda) \sim m^2 \delta(\lambda)$$

Summary

- Eigenvalue spectrum of the Dirac operator
  - source of a lot of info, e.g. spontaneous symmetry breaking
  - Well understood and tested at T=0
- Finite temperature QCD with exact chiral symmetry
  - now getting realistic (quenched and dynamical)
- $U_A(1)$ restoration
  - at T~1.2 T_c in quenched QCD
  - depends on m in Nf=2 QCD. Will approach T_c in the chiral limit?

Detailed analysis of correlators on-going.