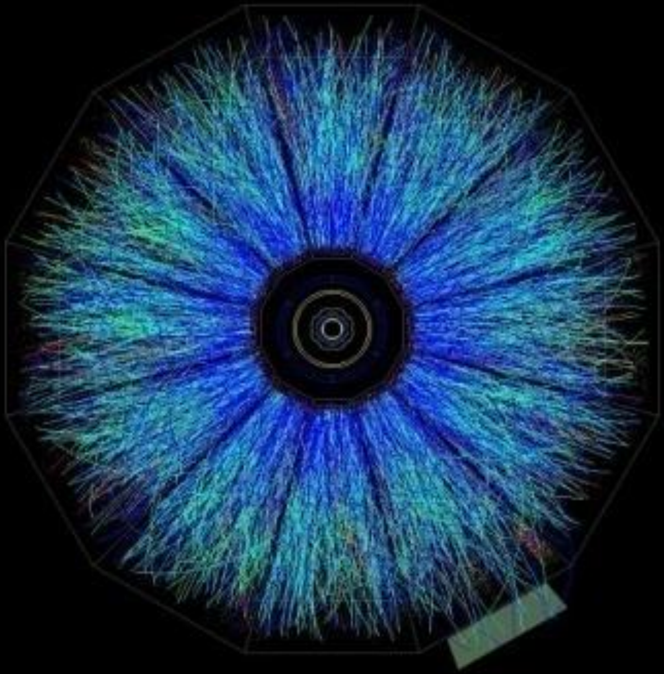
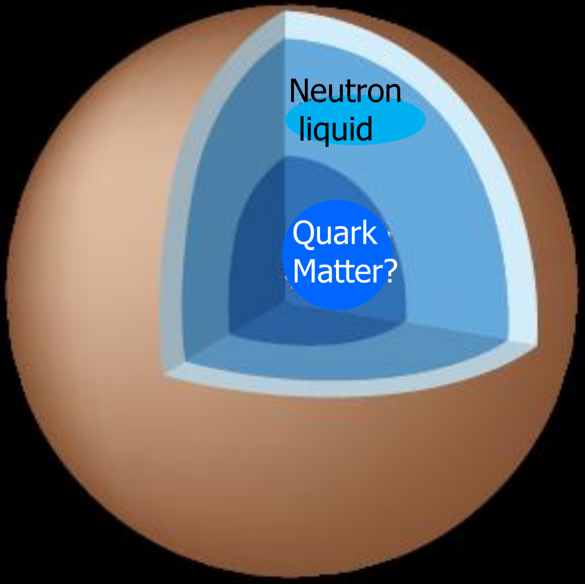
Many-body Problems in QCD

T. Hatsuda (University of Tokyo)



Hot matter： $T \sim 10^{12}$K



Dense matter：$\rho \sim 10^{12}$ kg/cm^3

Lattice QCD confronts experiments
Japanese-German Seminar 2010 Nov. 6 (2010) at Mishima

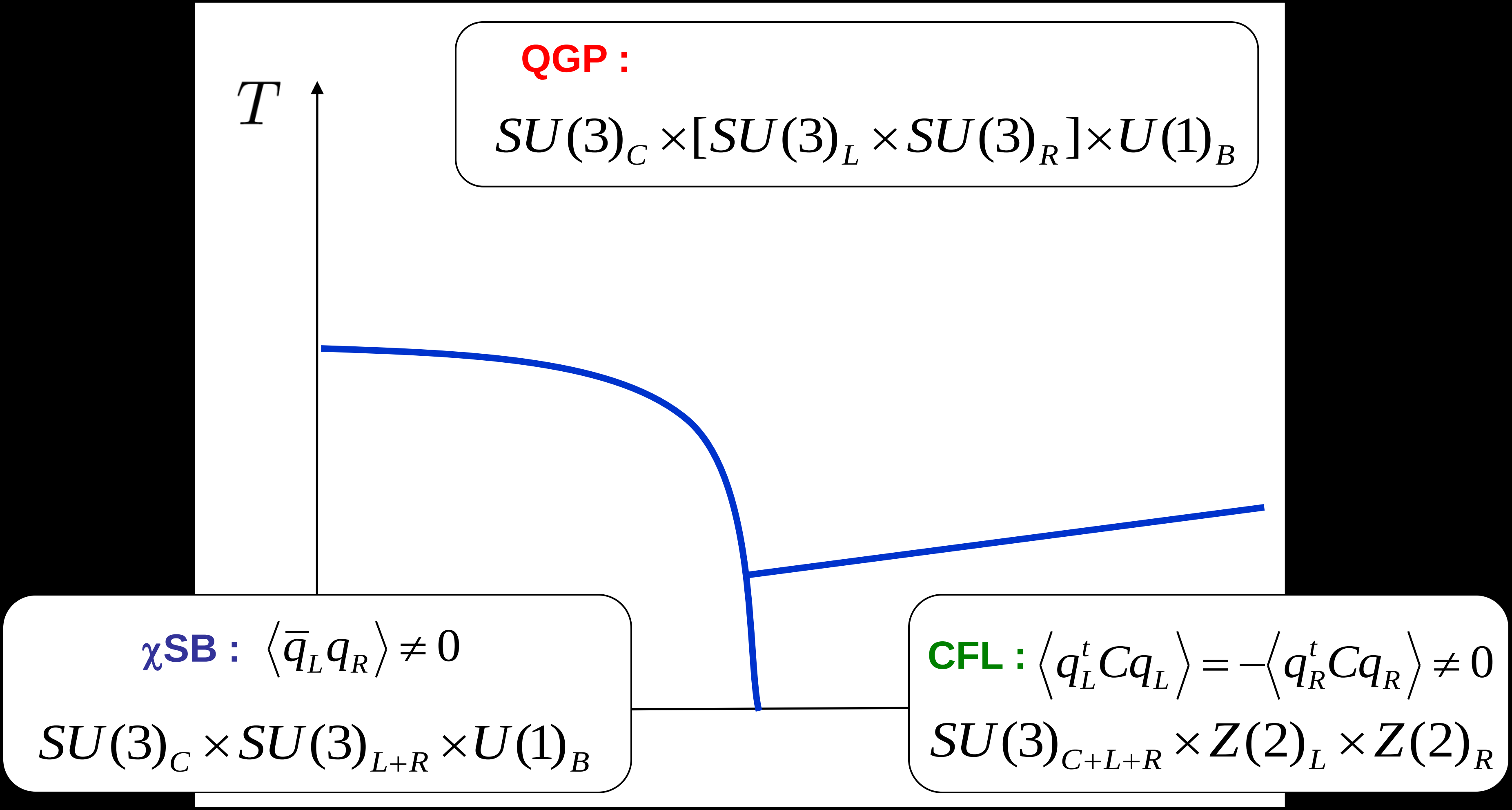
contents

Recent review:

"The Phase Diagram of Dense QCD"
K. Fukushima and T. Hatsuda, Rep. Prog. Phys. 73 (2010)
arXiv: 1005.4814 [hep-ph]

Symmetry realization in hot/dense QCD *(for $m_{u,d,s}$=0 case)*

Cabibbo & Parisi, PLB 59 (1975)
Collins & Perry, PRL 34 (1975)



QGP :

$$SU(3)_C \times [SU(3)_L \times SU(3)_R] \times U(1)_B$$

χSB : $\langle \bar{q}_L q_R \rangle \neq 0$

$$SU(3)_C \times SU(3)_{L+R} \times U(1)_B$$

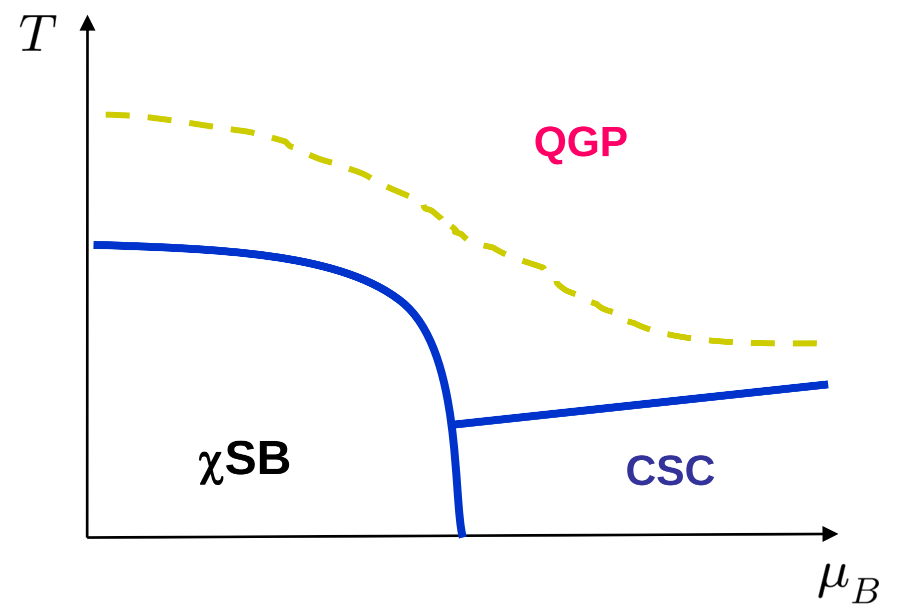
CFL : $\langle q_L^t C q_L \rangle = -\langle q_R^t C q_R \rangle \neq 0$

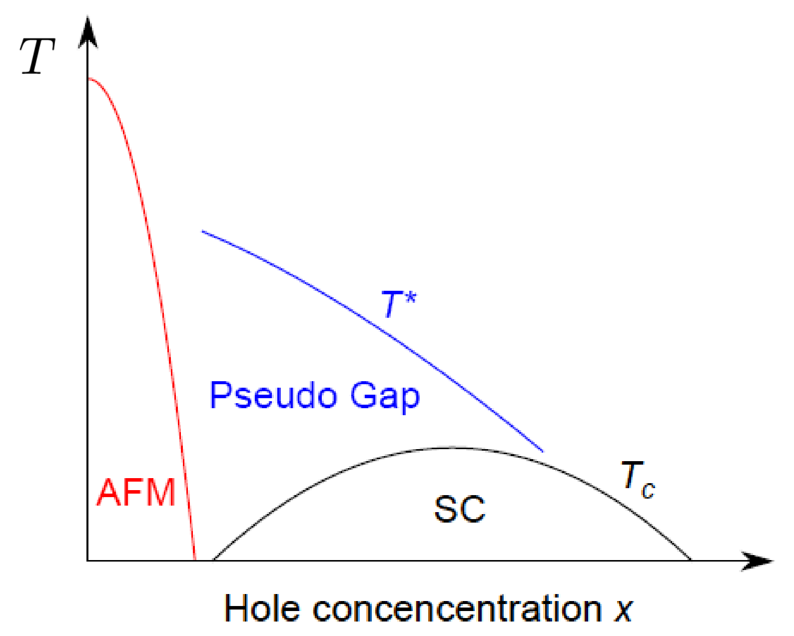
$$SU(3)_{C+L+R} \times Z(2)_L \times Z(2)_R$$

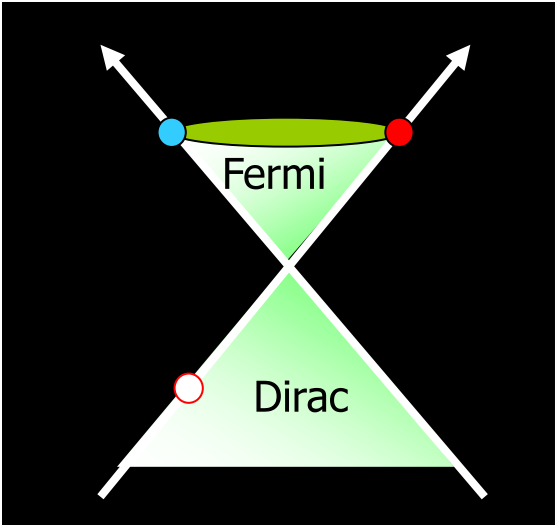
Nambu, PRL 4 (1960)

Alford, Rajagopal & Wilczek, NPB 537 (1999)

QCD vs. condensed matters



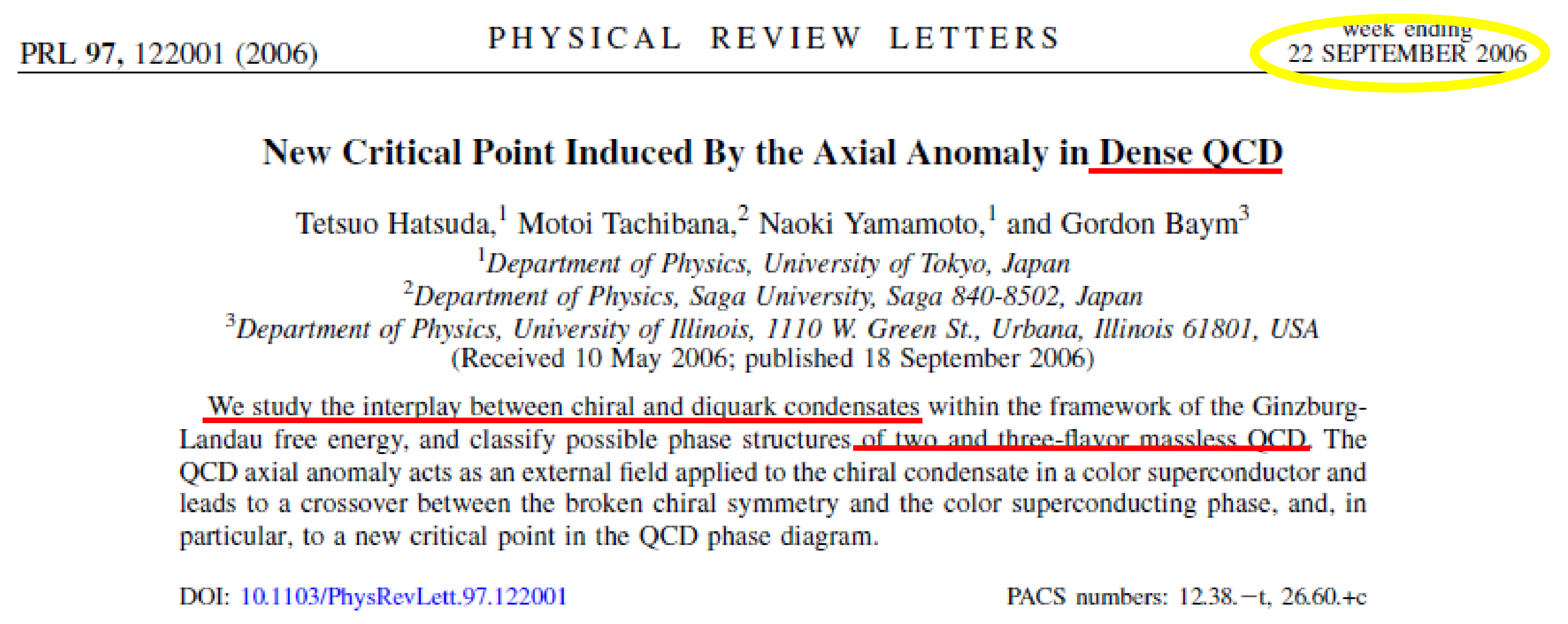




Common features

1. Competition between different orders
2. Strong coupling

- Kitazawa, Nemoto, Kunihiro, PTP ('02)
- Abuki, Itakura & Hatsuda, PRD ('02)
- Chen, Stajic, Tan & Levin, Phys. Rep. ('05)



PRL 97, 122001 (2006) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2006

New Critical Point Induced By the Axial Anomaly in Dense QCD

Tetsuo Hatsuda,[1] Motoi Tachibana,[2] Naoki Yamamoto,[1] and Gordon Baym[3]
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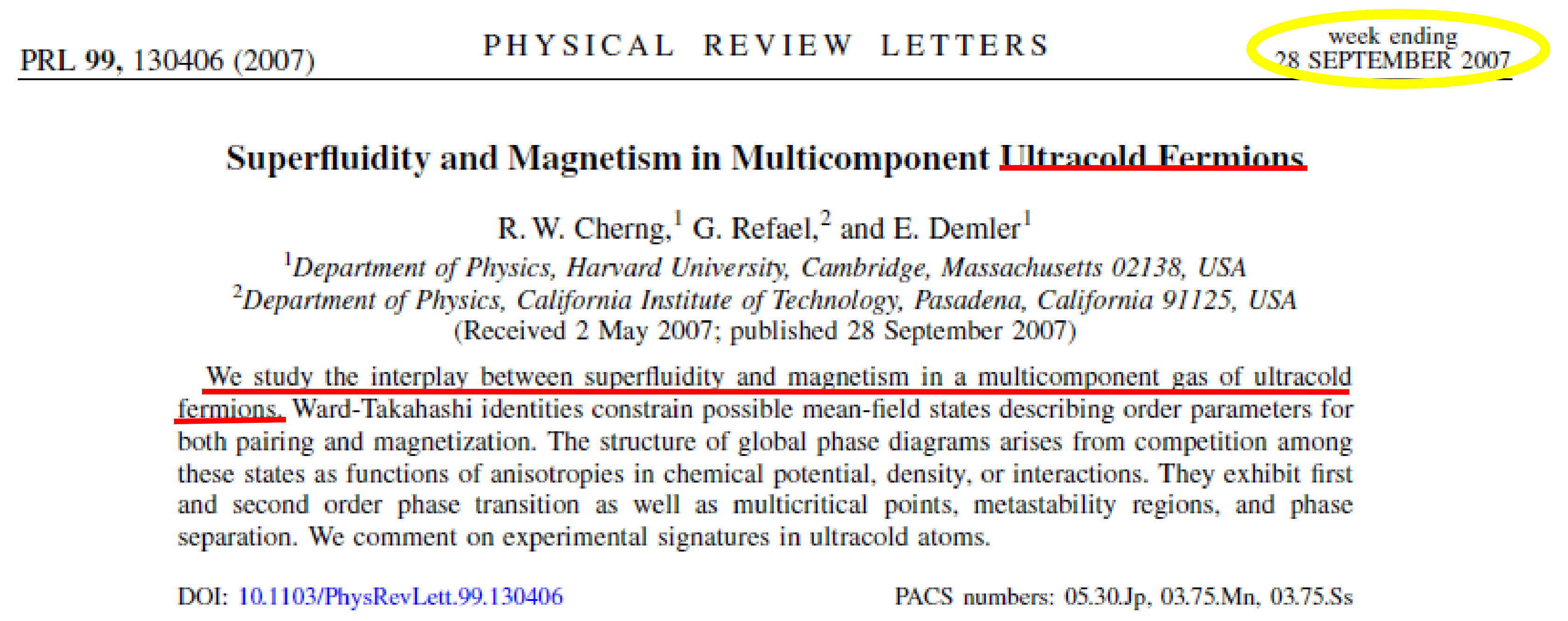
[2]*Department of Physics, Saga University, Saga 840-8502, Japan*
[3]*Department of Physics, University of Illinois, 1110 W. Green St., Urbana, Illinois 61801, USA*

(Received 10 May 2006; published 18 September 2006)

We study the interplay between chiral and diquark condensates within the framework of the Ginzburg-Landau free energy, and classify possible phase structures of two and three-flavor massless QCD. The QCD axial anomaly acts as an external field applied to the chiral condensate in a color superconductor and leads to a crossover between the broken chiral symmetry and the color superconducting phase, and, in particular, to a new critical point in the QCD phase diagram.

DOI: 10.1103/PhysRevLett.97.122001 PACS numbers: 12.38.−t, 26.60.+c



PRL 99, 130406 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Superfluidity and Magnetism in Multicomponent Ultracold Fermions

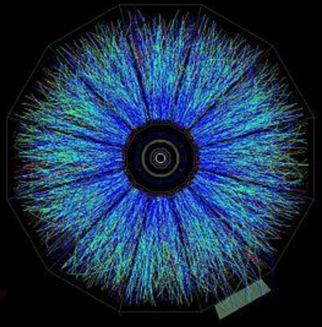
R. W. Cherng,[1] G. Refael,[2] and E. Demler[1]
[1]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*

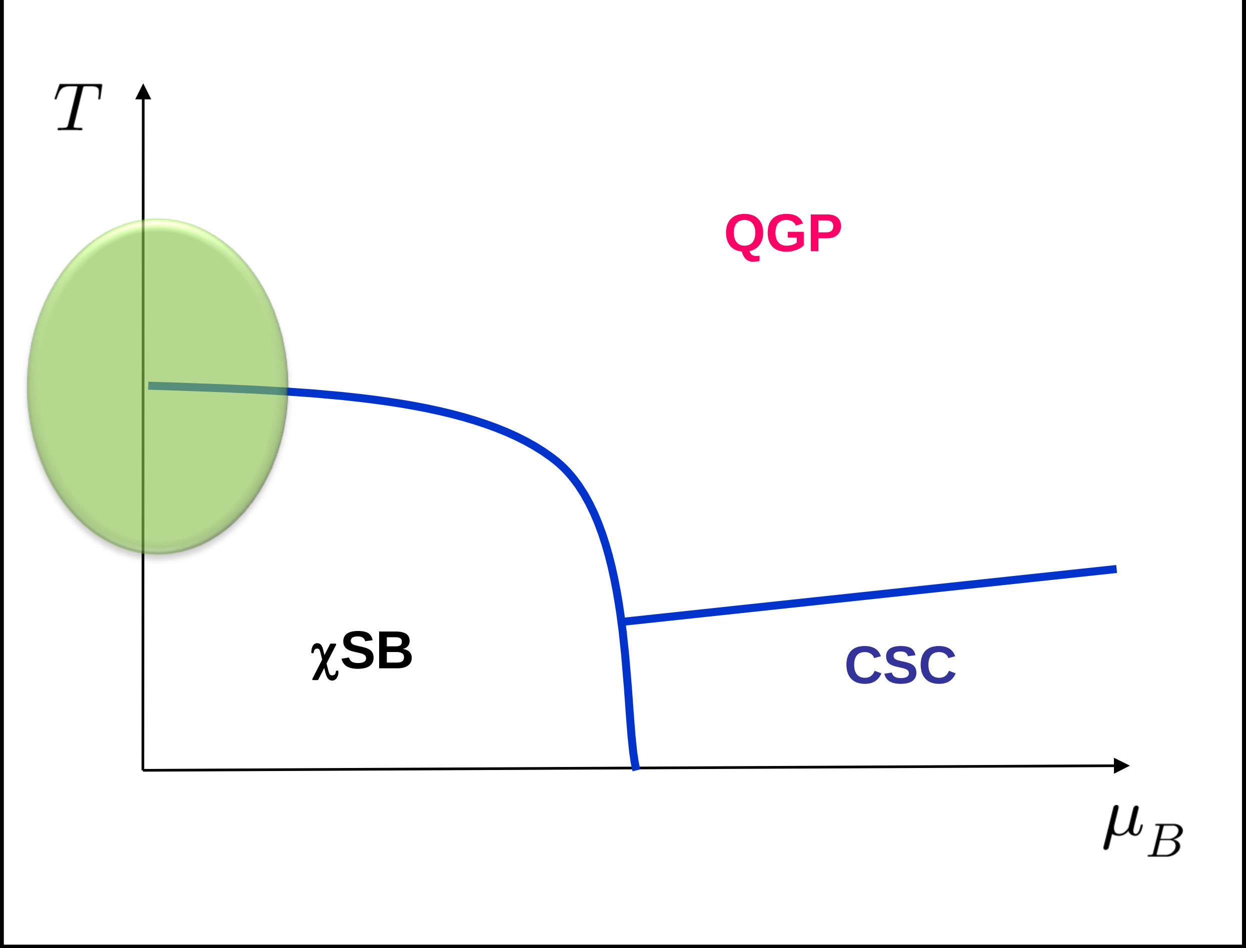
(Received 2 May 2007; published 28 September 2007)

We study the interplay between superfluidity and magnetism in a multicomponent gas of ultracold fermions. Ward-Takahashi identities constrain possible mean-field states describing order parameters for both pairing and magnetization. The structure of global phase diagrams arises from competition among these states as functions of anisotropies in chemical potential, density, or interactions. They exhibit first and second order phase transition as well as multicritical points, metastability regions, and phase separation. We comment on experimental signatures in ultracold atoms.

DOI: 10.1103/PhysRevLett.99.130406 PACS numbers: 05.30.Jp, 03.75.Mn, 03.75.Ss

Transition at Finite T





Ginzburg-Landau-Wilson analysis in hot QCD

Pisarski & Wilczek ('84)

Symmetry: $SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times \cancel{U(1)}_A$

$Z(2N_f)_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}_{\rm R}{}^j q_{\rm L}^i$

Chiral transformation: $\Phi \to e^{-2i\alpha_A}\; V_L\; \Phi\; V_R^\dagger$

$$\left.\begin{aligned}\mathcal{L}_{\rm eff} = {} & \frac{1}{2}\;\mathrm{tr}\;\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\;\mathrm{tr}\;\Phi^\dagger\Phi \\ & + \frac{b_1}{4!}\left(\mathrm{tr}\;\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\;\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2\end{aligned}\right\}\; SU(N_f)_L x SU(N_f)_R x U(1)_A$$

$$\left.-\frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)\right\}\; SU(N_f)_L x SU(N_f)_R$$

$$\left.-\frac{1}{2}\;\mathrm{tr}\;h(\Phi + \Phi^\dagger).\right\}\; \text{quark mass term}$$

Axial anomaly

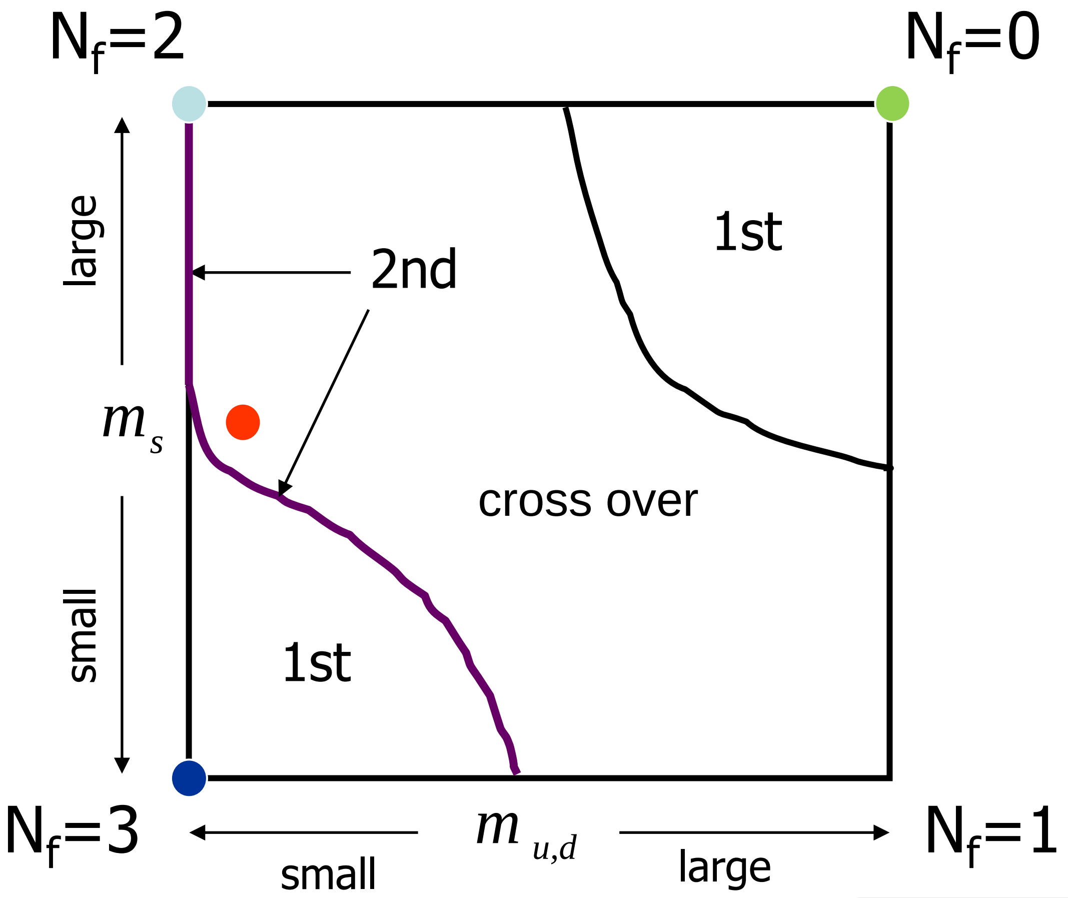
Order of the thermal QCD transition (μ=0)

Svetitsky & Yaffe, NPB210 ('82)

Pisarski and Wilczek, PRD29 ('84)

$$\mathcal{V} = \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4 - h\phi$$

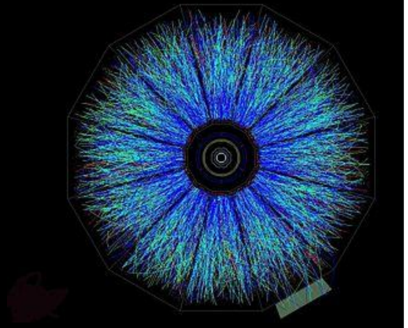
$$\mathcal{V} = \frac{a}{2}L^2 - \frac{c}{3}L^3 + \frac{b}{4}L^4 - hL$$

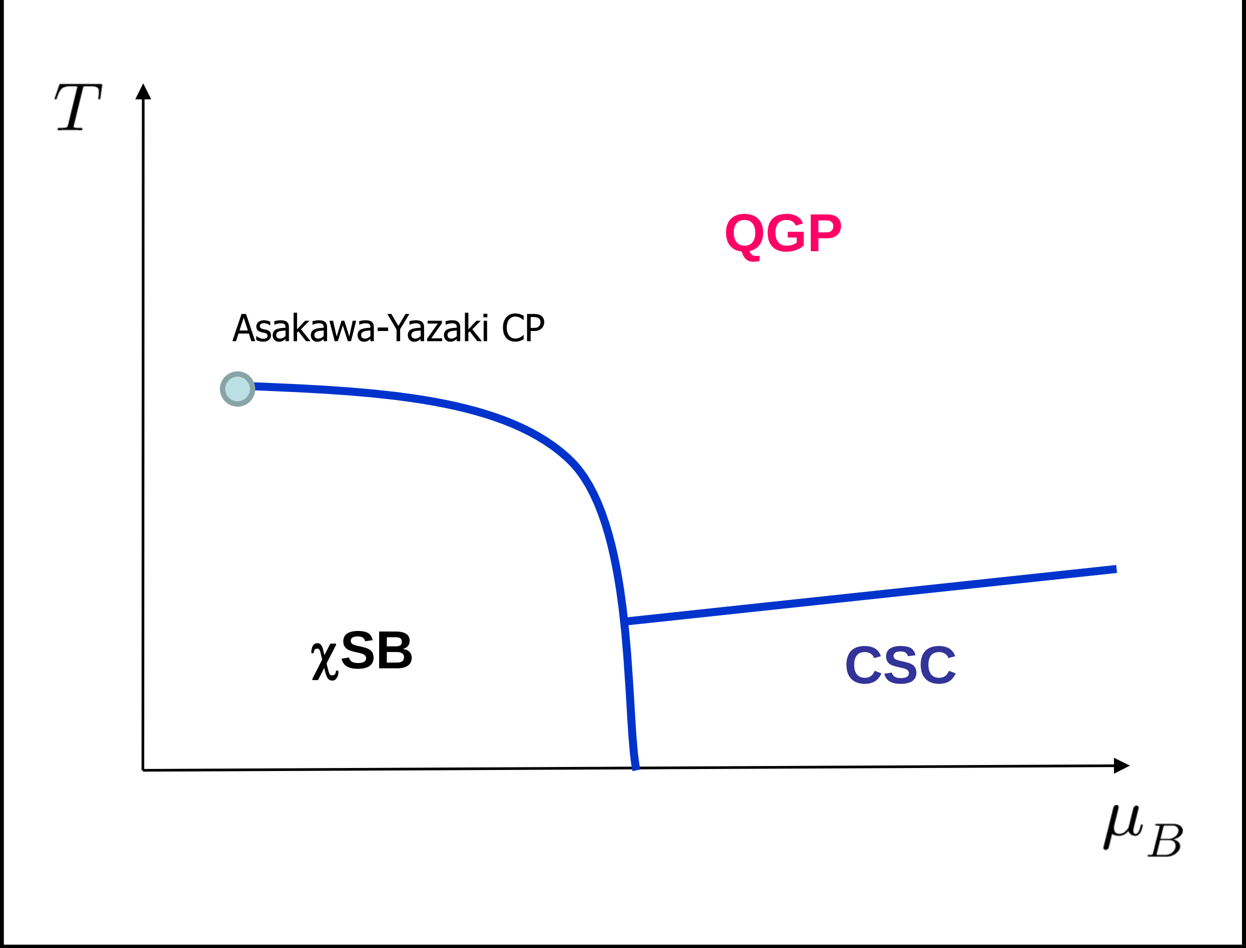


$$\mathcal{V} = \frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4 - h\sigma$$

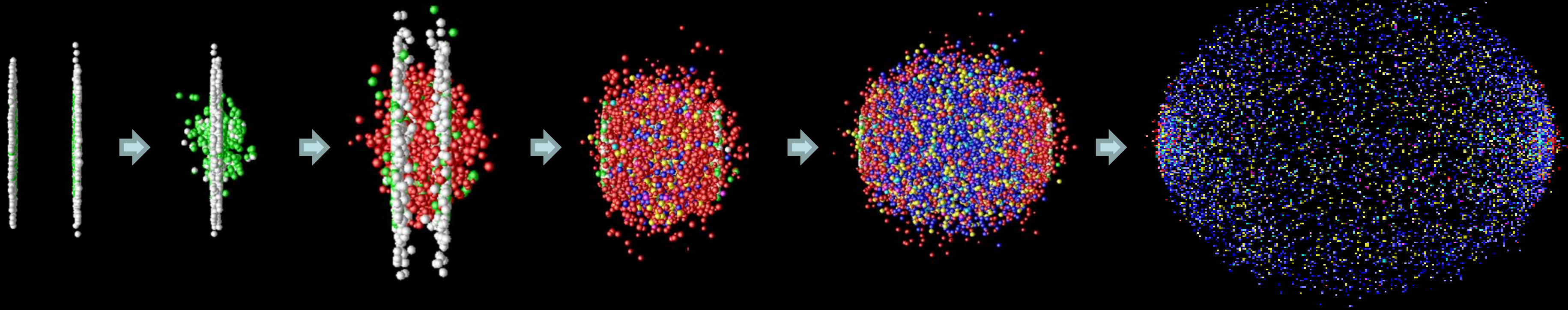
$$\mathcal{V} = -A\chi + \frac{a}{2}\chi^2 + \frac{b}{4}\chi^4$$

Transition at Finite T





Relativistic Heavy Ion Collisions



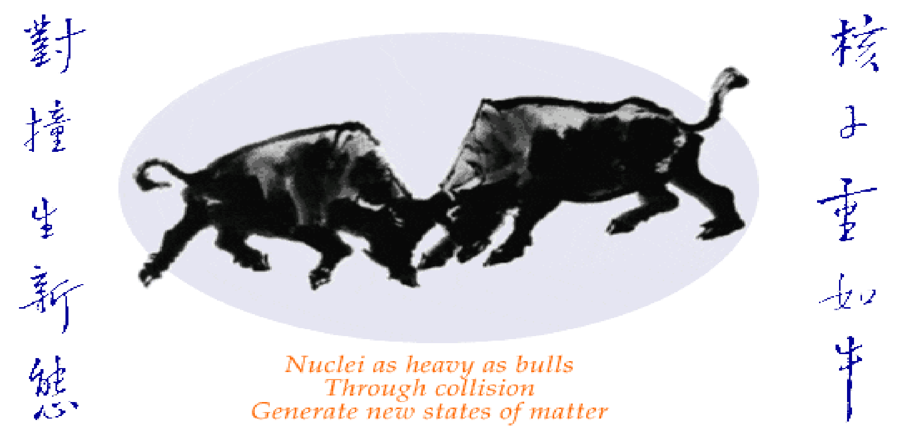
particle production

thermalization

QGP

hydrodynamic expansion

Freezeout

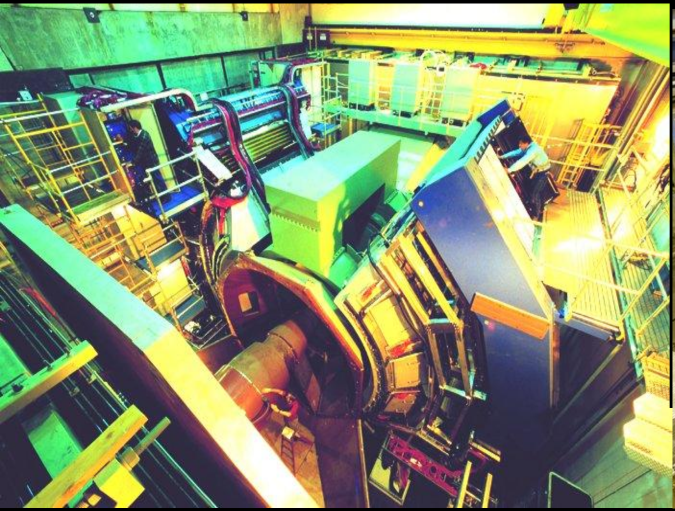


by T.D.Lee

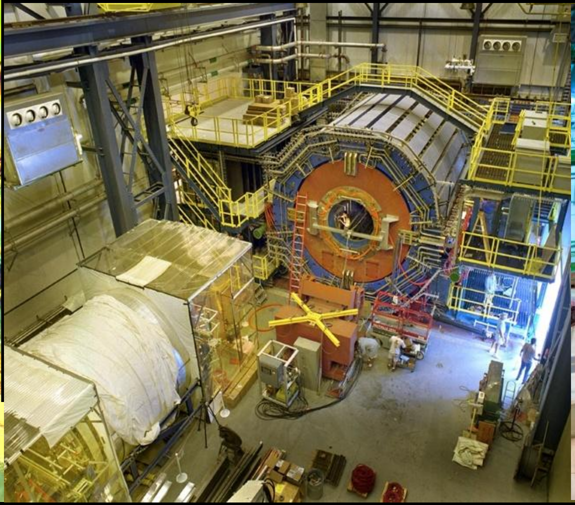
Tsinghua University, Beijing (photo by G. Baym)

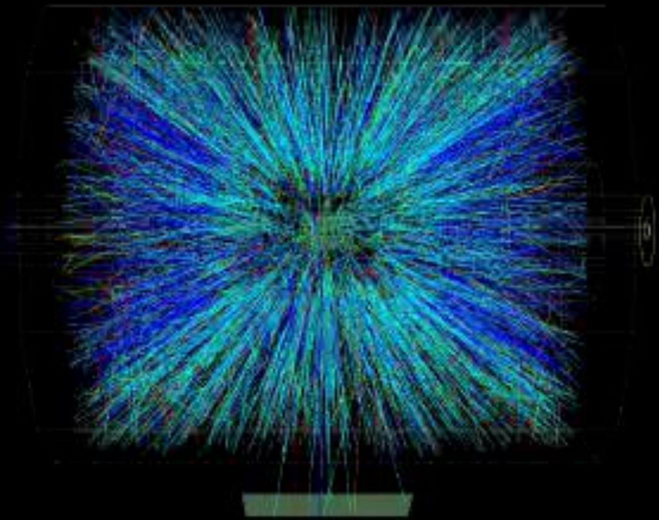
RHIC & LHC

PHENIX@RHIC

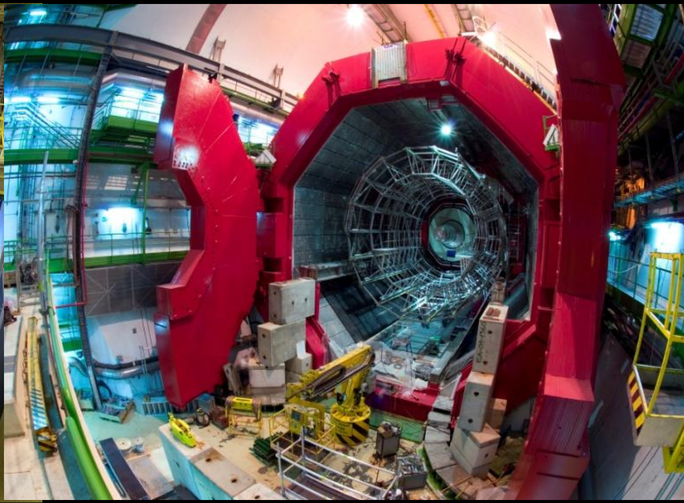


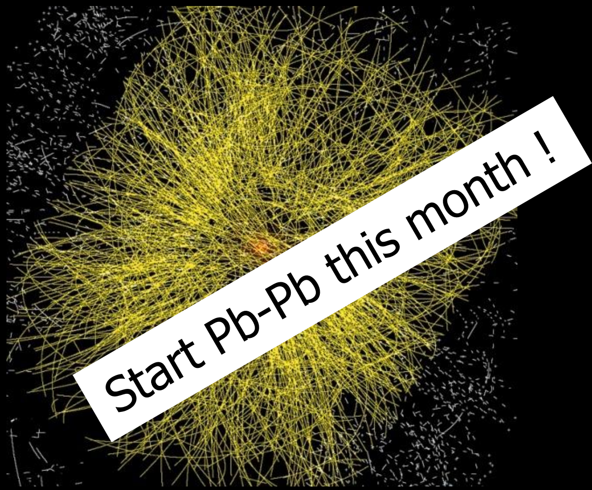
STAR@RHIC





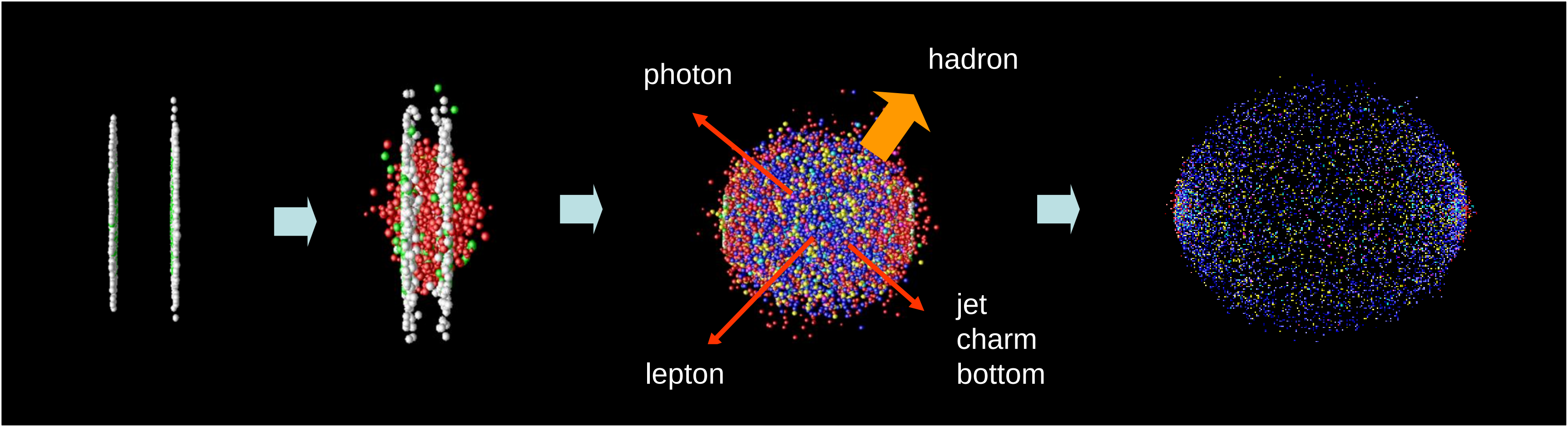
ALICE@LHC





Start Pb-Pb this month !

Hot QCD confronts RHIC/LHC experiments



Theoretical Inputs

for equation of state
for local correlations

- Lattice QCD
- pQCD
- AdS/QCD

Relativistic hydrodynamics

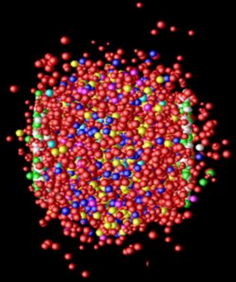
for space-time evolution

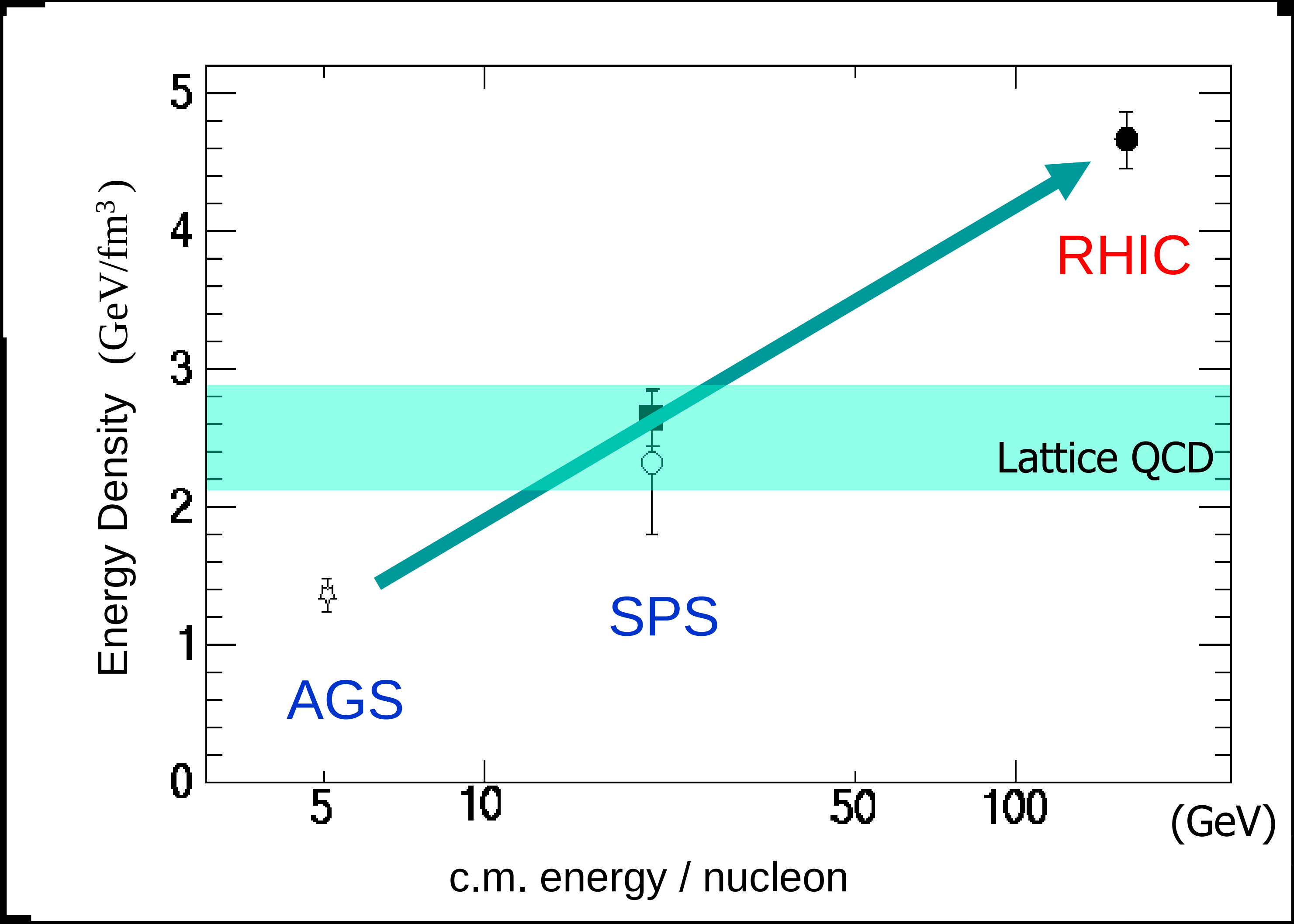
$$\partial_\mu T^{\mu\nu} = 0$$

Laudau (1953), Namiki & Iso (1957)

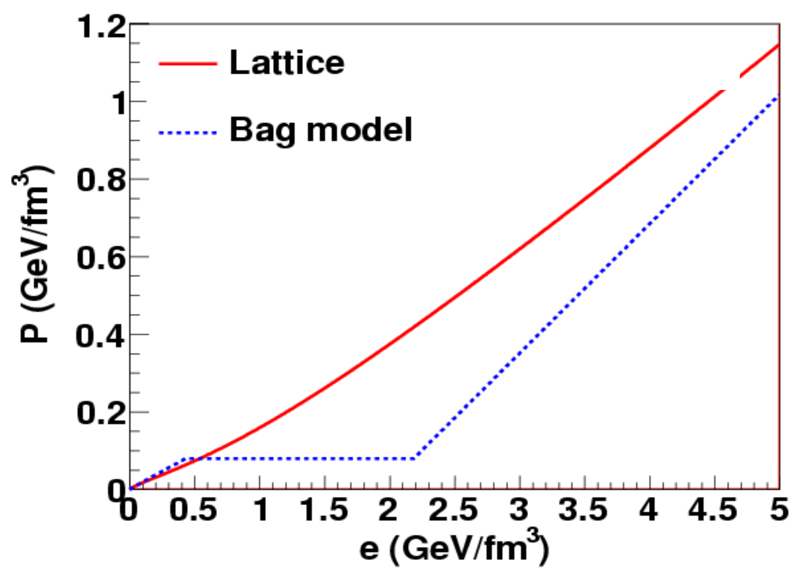
- 1D hydro by Bjorken (1983)
- 3D hydrodynamics by
  Hirano, Muroya, Nonaka (2000-)

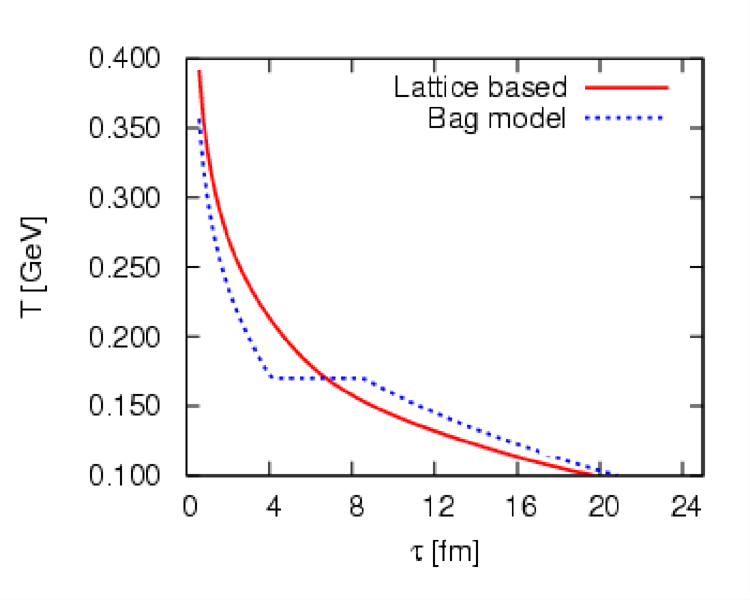
Initial energy density





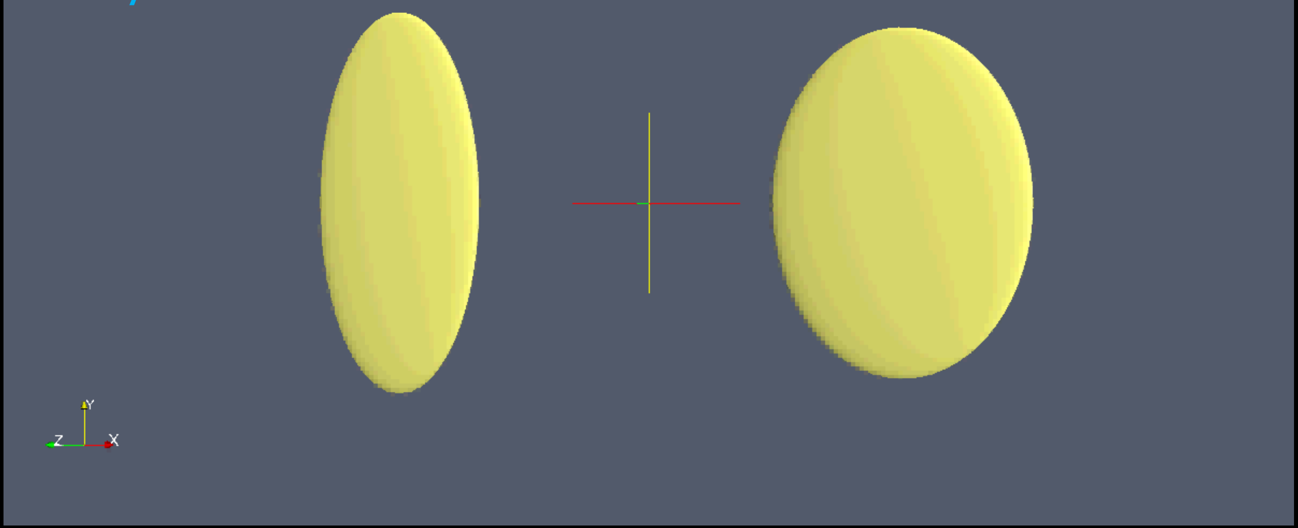
3D hydrodynamics with lattice EOS



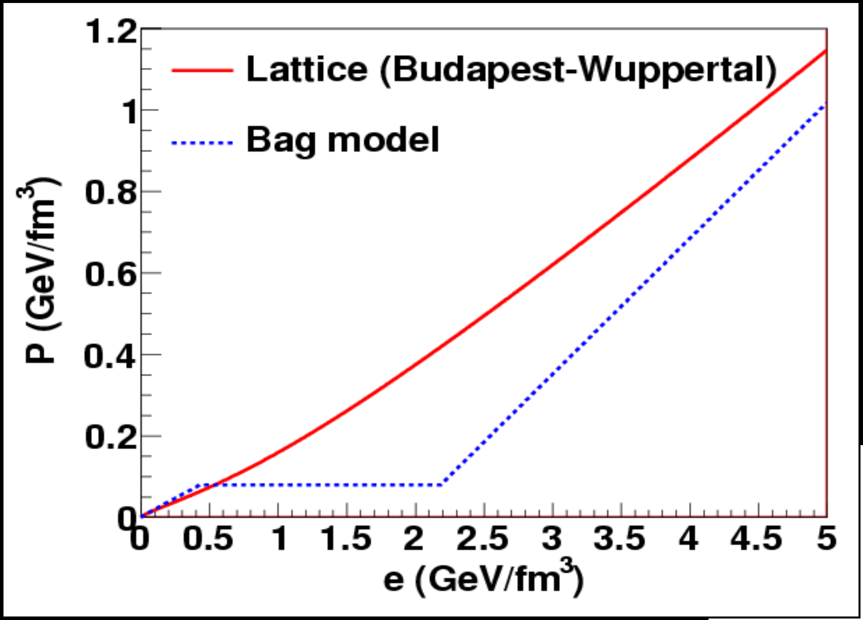


http://www.youtube.com/watch?v=p8_2TczsxjM

by T. Hirano

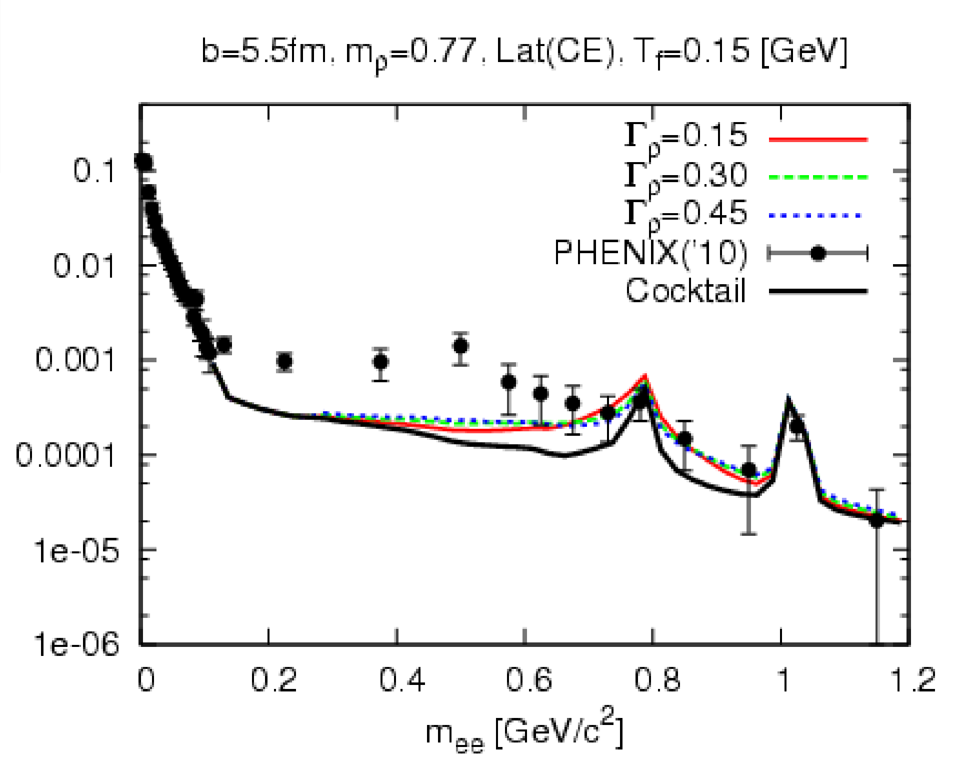


Dilepton emission with lattice EOS

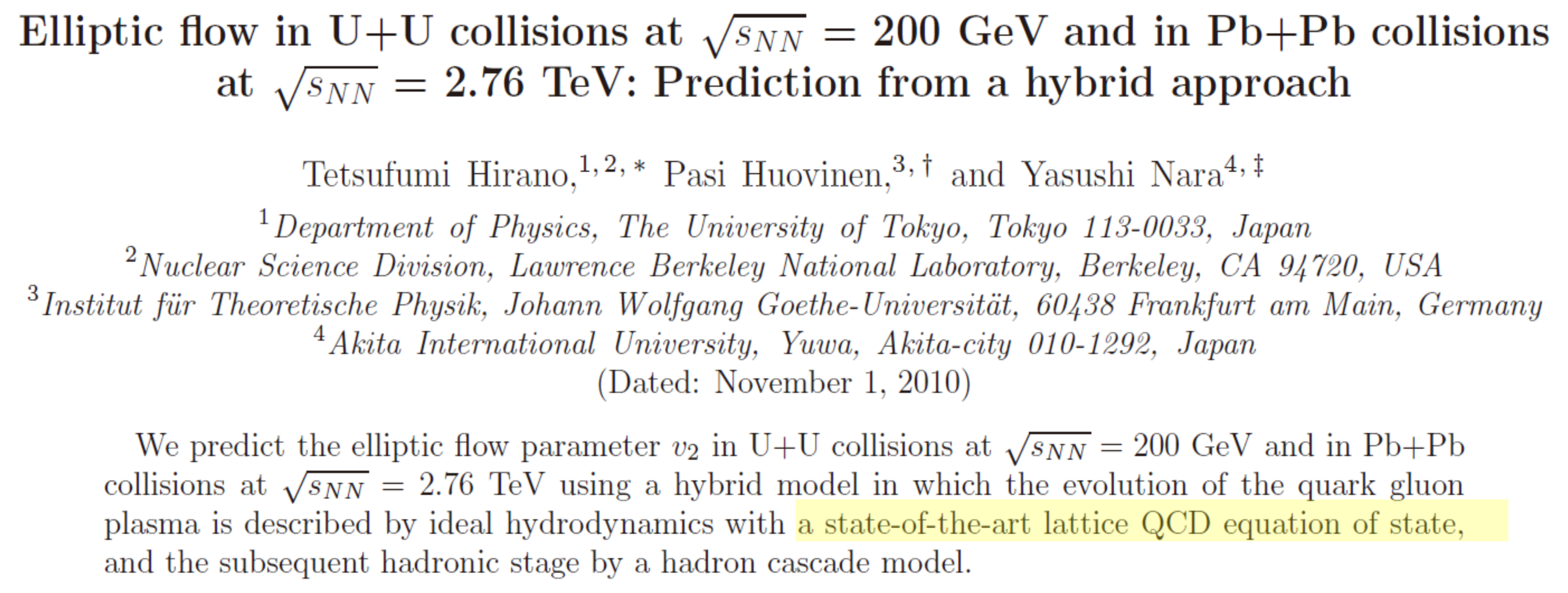


- State-of-the-art lattice EOS
- Full 3D hydrodynamics
- Experimental SPF at T=0

Akamatsu, Hatsuda, Hamagaki, & Hirano (2010)



Predictions for LHC



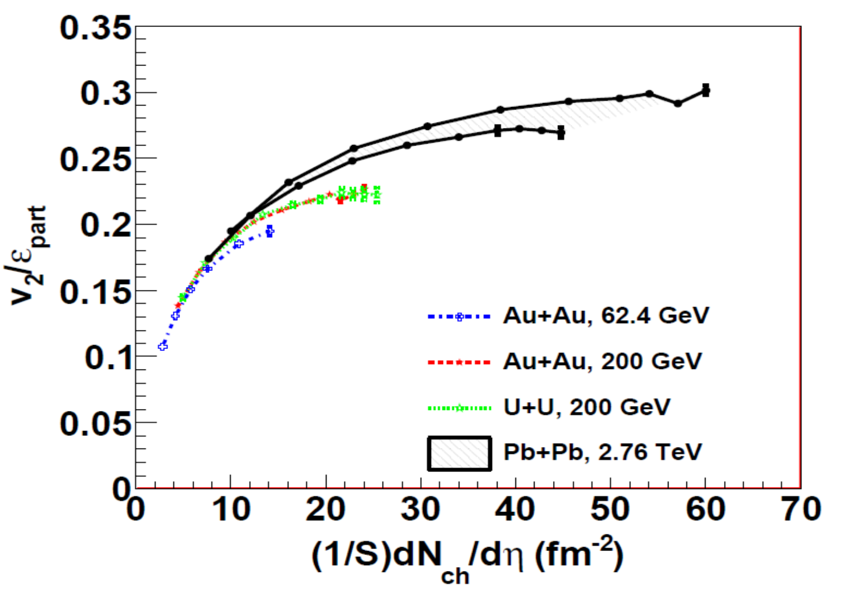
Elliptic flow in U+U collisions at $\sqrt{s_{NN}} = 200$ GeV and in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV: Prediction from a hybrid approach

Tetsufumi Hirano,[1,2,*] Pasi Huovinen,[3,†] and Yasushi Nara[4,‡]

[1] *Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*
[2] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
[3] *Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, 60438 Frankfurt am Main, Germany*
[4] *Akita International University, Yuwa, Akita-city 010-1292, Japan*

(Dated: November 1, 2010)

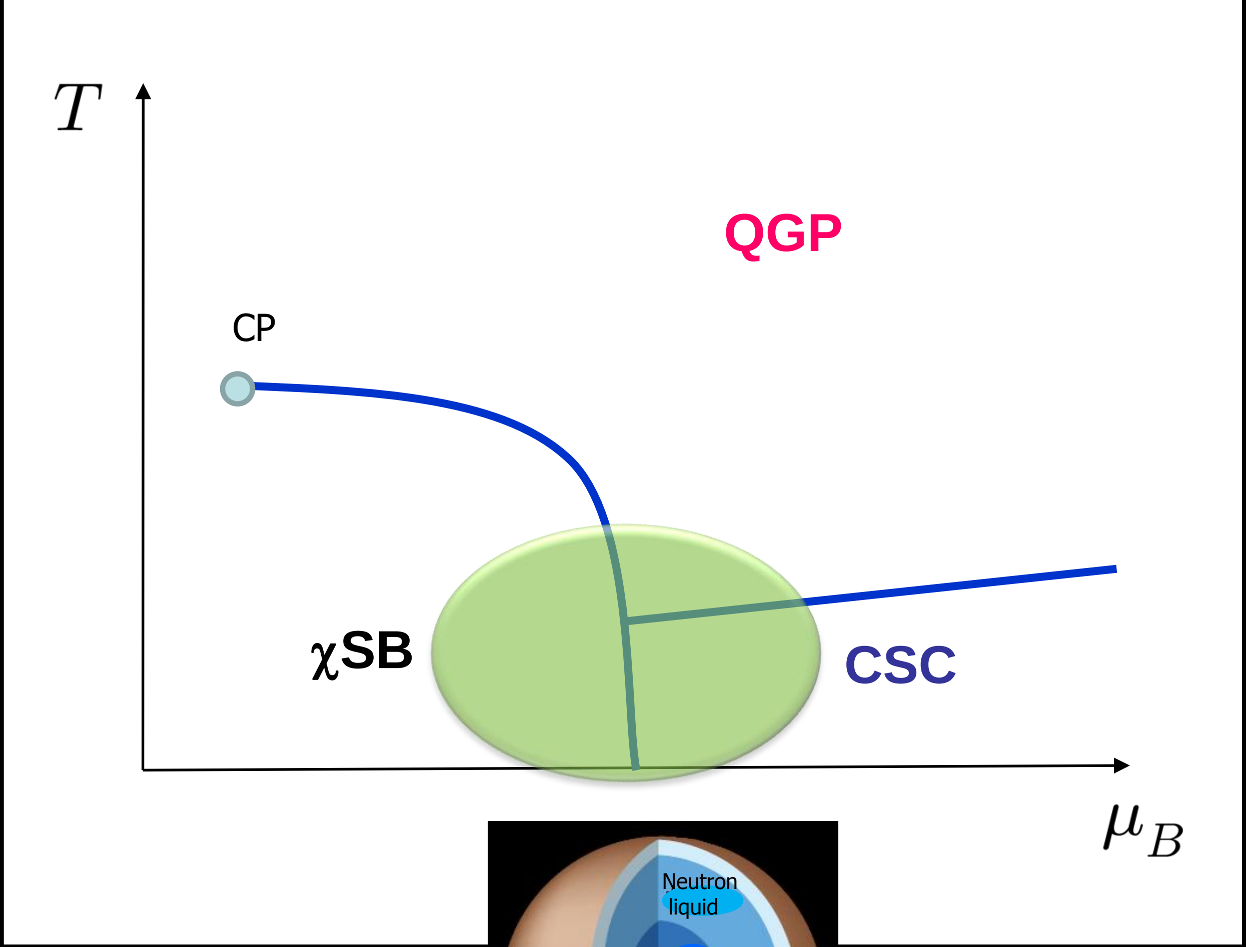
We predict the elliptic flow parameter v_2 in U+U collisions at $\sqrt{s_{NN}} = 200$ GeV and in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV using a hybrid model in which the evolution of the quark gluon plasma is described by ideal hydrodynamics with a state-of-the-art lattice QCD equation of state, and the subsequent hadronic stage by a hadron cascade model.

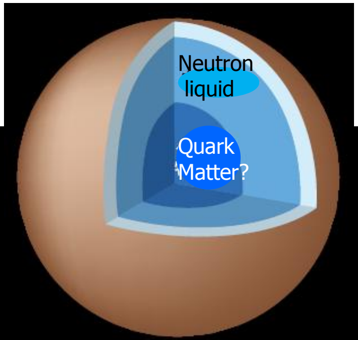


- State-of-the-art lattice EOS given by HOT QCD Coll.
- Full 3D hydrodynamics + hadron cascade

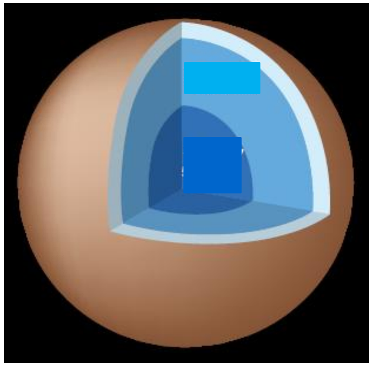
Hirano, Huovinen and Nara,
arXiv:10106222 [nucl-th] Nov.1 (2010)

Transition at finite μ

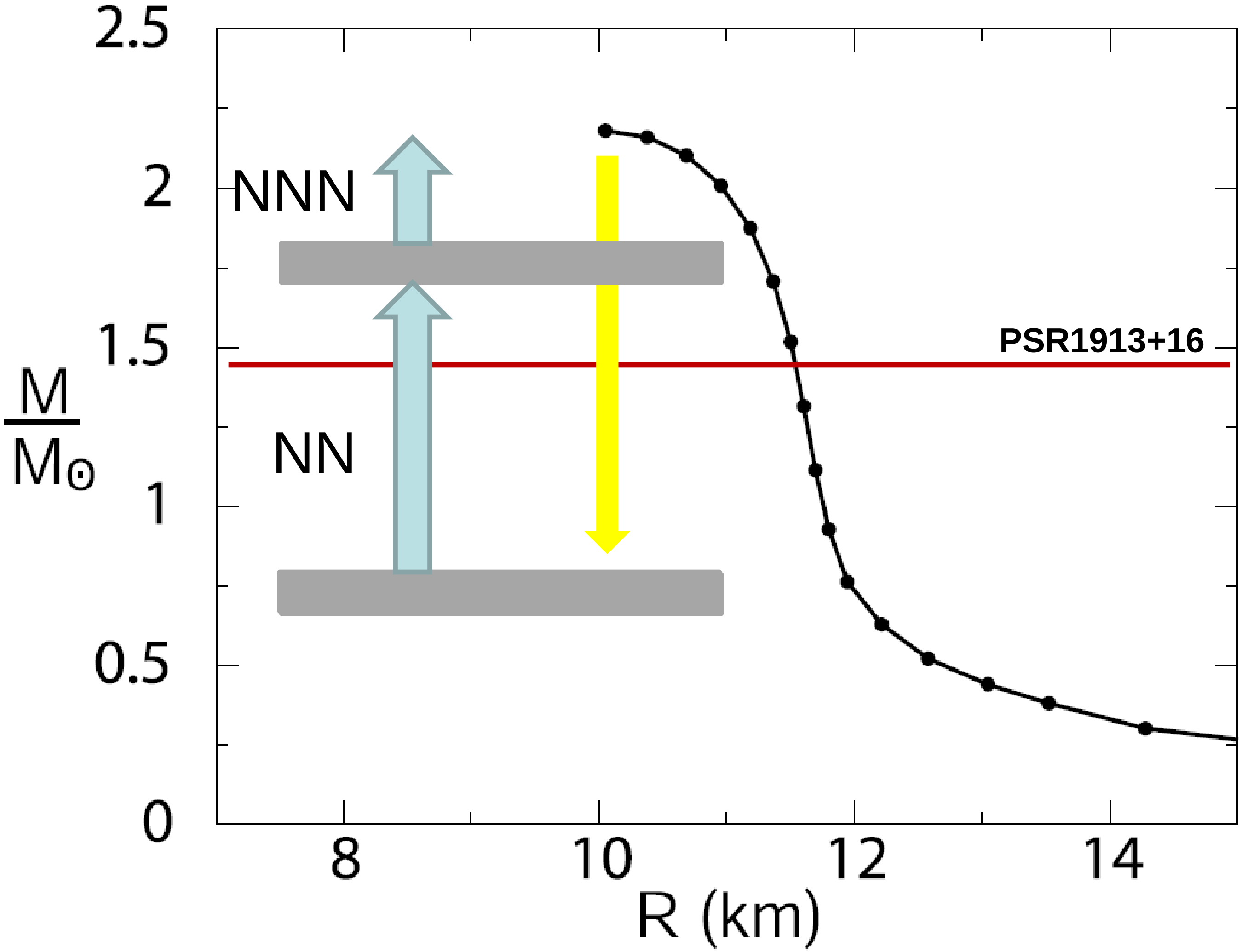




Nuclear Force and Neutron Star

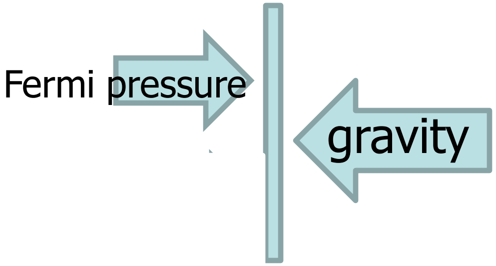


$(\rho_{max} \sim 6\rho_0)$



Neutron star binary

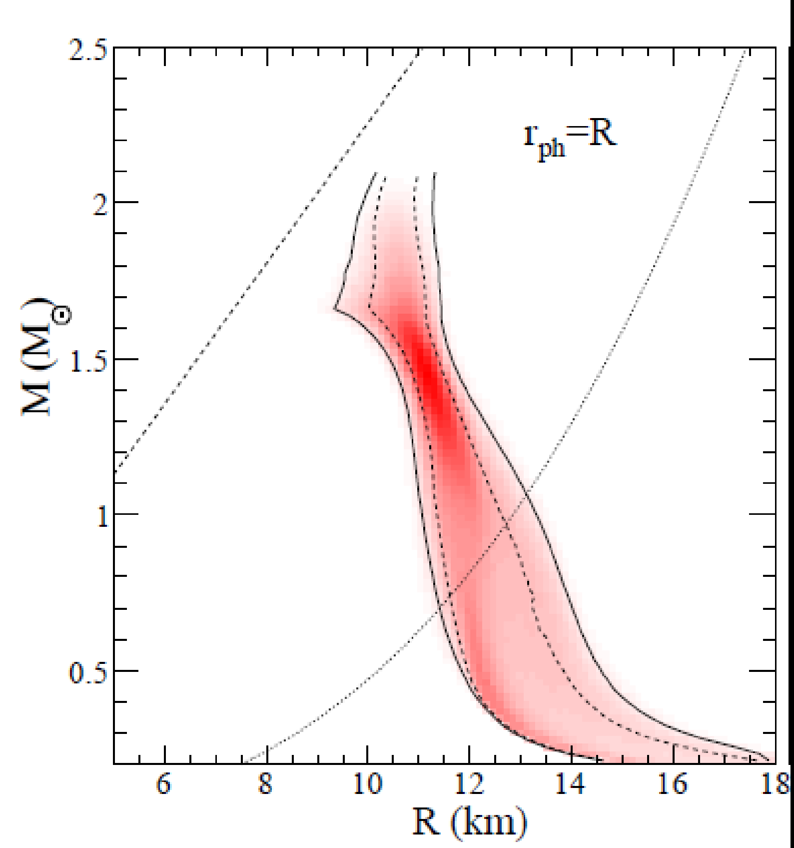
Pressure balance

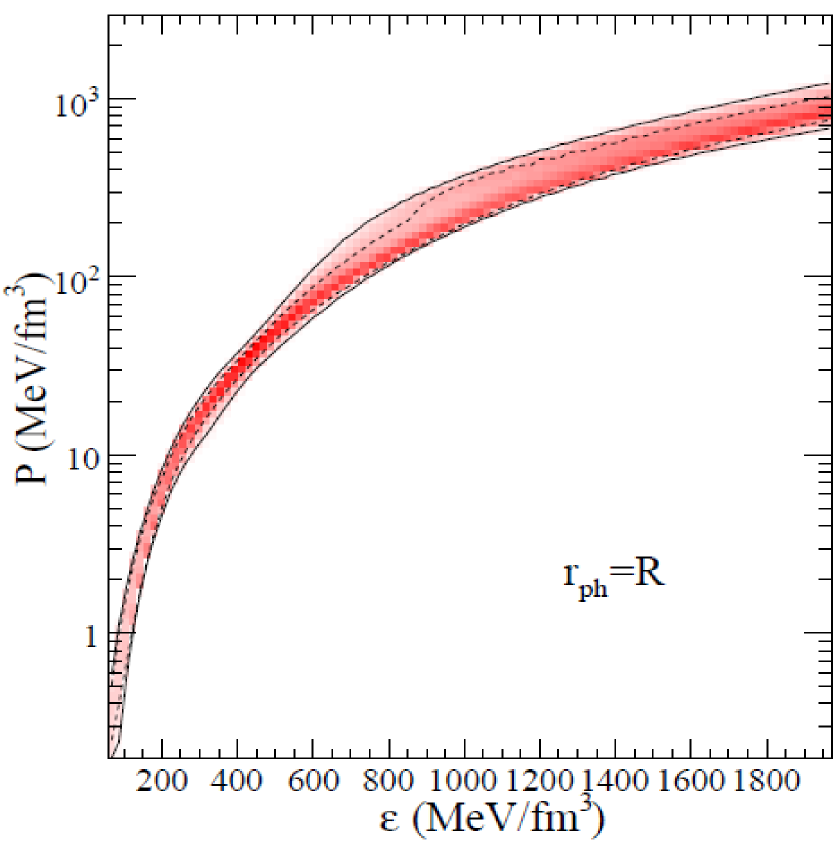


From TOV to EOS

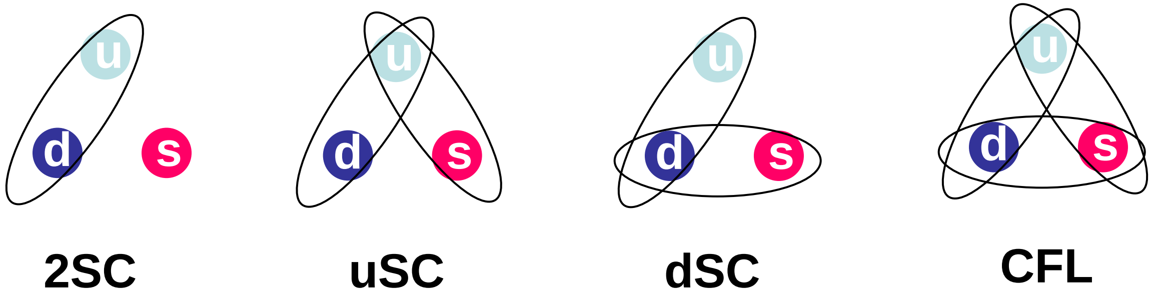
Ozel, Baym & Guver, arXiv: 1002.3153 [astro-ph.HE]
Steiner, Luttimer & Brown, arXiv: 1005.0811 [astro-ph.HE]

M-R relation from
thermonuclear Burst in X-ray Binaries





Color superconductivity



major differences from the standard BCS superconductor

1. Relativistic fermi system
   color-magnetic int. dominant

$$|d| \sim \varepsilon_{\mathrm{F}} \; e^{-c/\sqrt{\alpha_s}}$$

High T_c : $T_c/\varepsilon_F \sim 0.1$

Compact pair : r~ 1-10 fm

2. Color-flavor entanglement

$$d_{ia}$$

Various phases (c.f. Ice, ^{3}He)

2SC, uSC, dSC, CFL etc

GL analysis for chiral-super interplay in QCD (N_f=3)

Symmetry: $SU(3)_C \times [SU(3)_L \times SU(3)_R] \times U(1)_B \times Z(6)_A$

Chiral field:

$$\Phi_{ij} \sim (\bar{q}_R)^j_a (q_L)^i_a$$

$$\Phi \to \mathrm{e}^{-2i\alpha_A} V_L \Phi V_R^\dagger$$

diquark field:

$$(d_L)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_L)^j_b C (q_L)^k_c$$

$$d_L \to e^{2i\alpha_A} e^{2i\alpha_B} V_L d_L V_C^{\mathrm{T}}$$

$$\mathcal{V}(\Phi, d) = \mathcal{V}_\chi(\Phi) + \mathcal{V}_d(d_L, d_R) + \mathcal{V}_{\chi d}(\Phi, d_L, d_R)$$

Pisarski & Wilczek, PRD29 ('84)

Iida & Baym, PRD63 ('01)

Yamamoto, Tachibana,
Baym & Hatsuda, PRL97 ('06)

Complete classification of the GL potential ($m_{u,d,s}$=0)

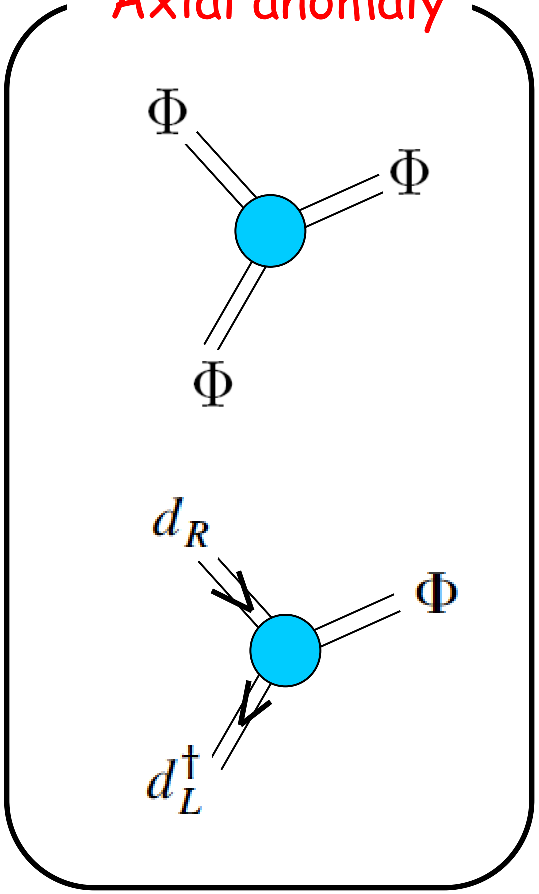
Yamamoto, Tachibana,
Baym & Hatsuda,
PRL 97 ('06), PRD 76 ('07)

$$
\begin{aligned}
\mathcal{V}_\chi &= \frac{a_0}{2}\mathrm{tr}\ \Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\ \Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 \\
&\quad \boxed{-\frac{c_0}{2}\left(\mathrm{det}\Phi + \mathrm{det}\Phi^\dagger\right)},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_d = &\ \alpha_0\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger] \\
&+\ \beta_1\left([\mathrm{tr}(d_L d_L^\dagger)]^2 + [\mathrm{tr}(d_R d_R^\dagger)]^2\right) \\
&+\ \beta_2\left(\mathrm{tr}[(d_L d_L^\dagger)^2] + \mathrm{tr}[(d_R d_R^\dagger)^2]\right) \\
&+\ \beta_3\ \mathrm{tr}[(d_R d_L^\dagger)(d_L d_R^\dagger)] + \beta_4\ \mathrm{tr}(d_L d_L^\dagger)\mathrm{tr}(d_R d_R^\dagger)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_{\chi d} = &\boxed{\gamma_1\ \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger]} \\
&+\lambda_1\ \mathrm{tr}[(d_L d_L^\dagger)\Phi\Phi^\dagger + (d_R d_R^\dagger)\Phi^\dagger\Phi] \\
&+\lambda_2\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]\cdot\mathrm{tr}[\Phi^\dagger\Phi] \\
&+\lambda_3\left(\mathrm{det}\Phi\cdot\mathrm{tr}[(d_L d_R^\dagger)\Phi^{-1}] + h.c\right)
\end{aligned}
$$

Axial anomaly

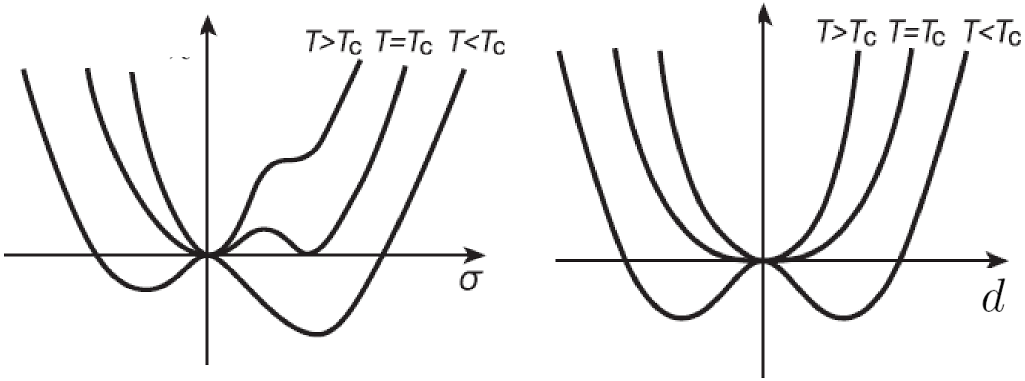


$Z(2)_L \times Z(2)_R$
broken explicitly

Chiral-CFL interplay in N_f=3

$$\Phi = \begin{pmatrix} \sigma & & \\ & \sigma & \\ & & \sigma \end{pmatrix} \qquad d_L = -d_R = \begin{pmatrix} d & & \\ & d & \\ & & d \end{pmatrix}$$

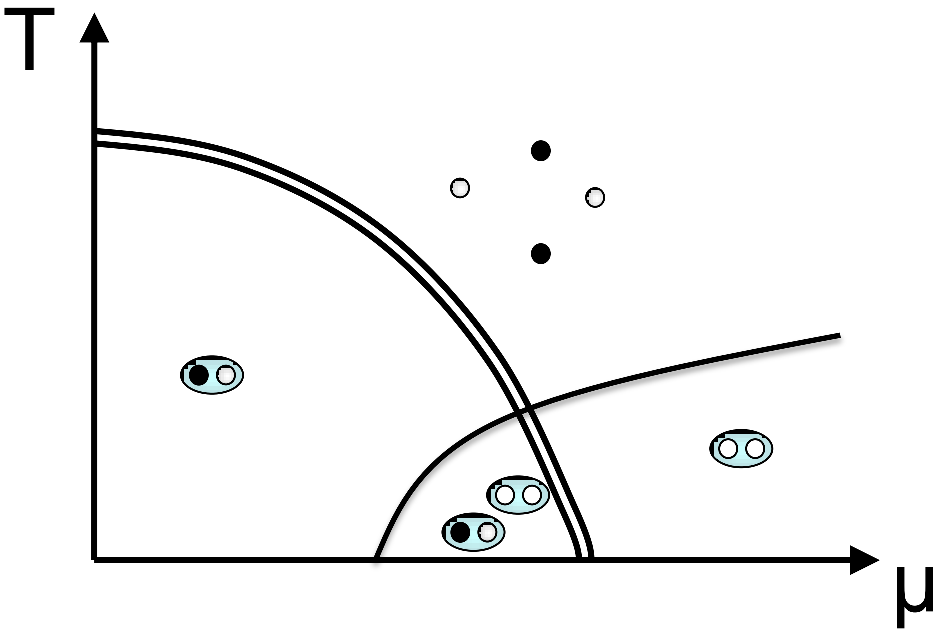
$$\boxed{\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2\sigma + \lambda d^2\sigma^2}$$



Natural parameter relations
$$\begin{cases} \beta > 0,\ b > 0 \\ \gamma \sim c > 0 \\ 1 \gg \lambda/\beta > 0 \end{cases}$$

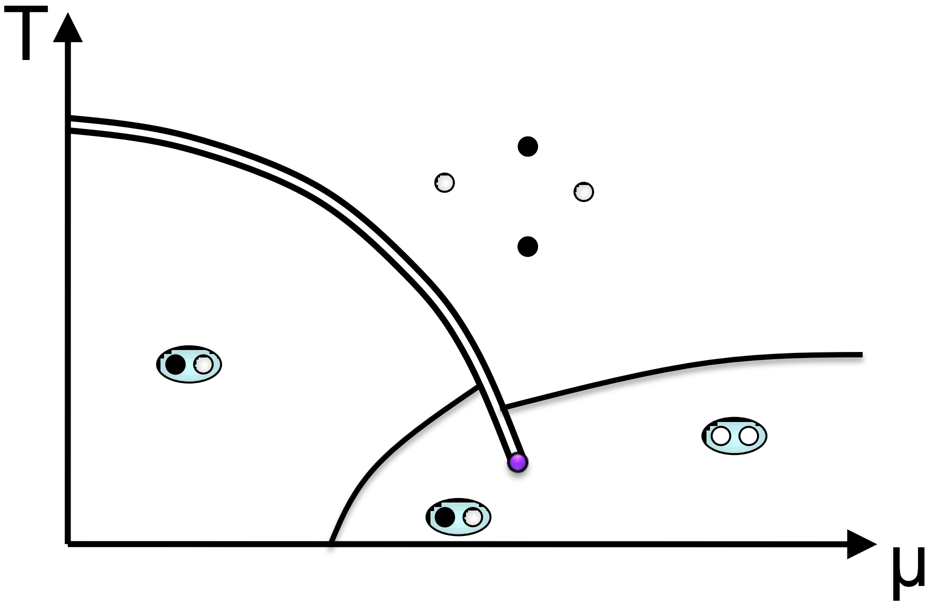
phase diagram (without d-σ coupling)

$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \xcancel{\gamma d^2}\sigma$$



phase diagram (with d-σ coupling)

$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) \quad -\gamma d^2\sigma$$

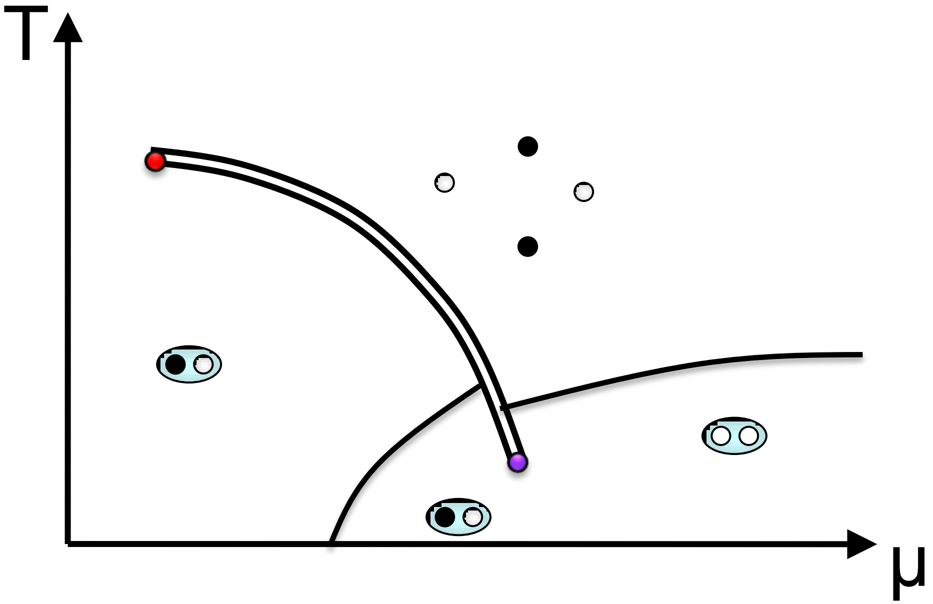


Low T critical point driven by the axial anomaly

phase diagram (with d-σ coupling & quark mass)

$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) \boxed{-\gamma d^2 \sigma}$$

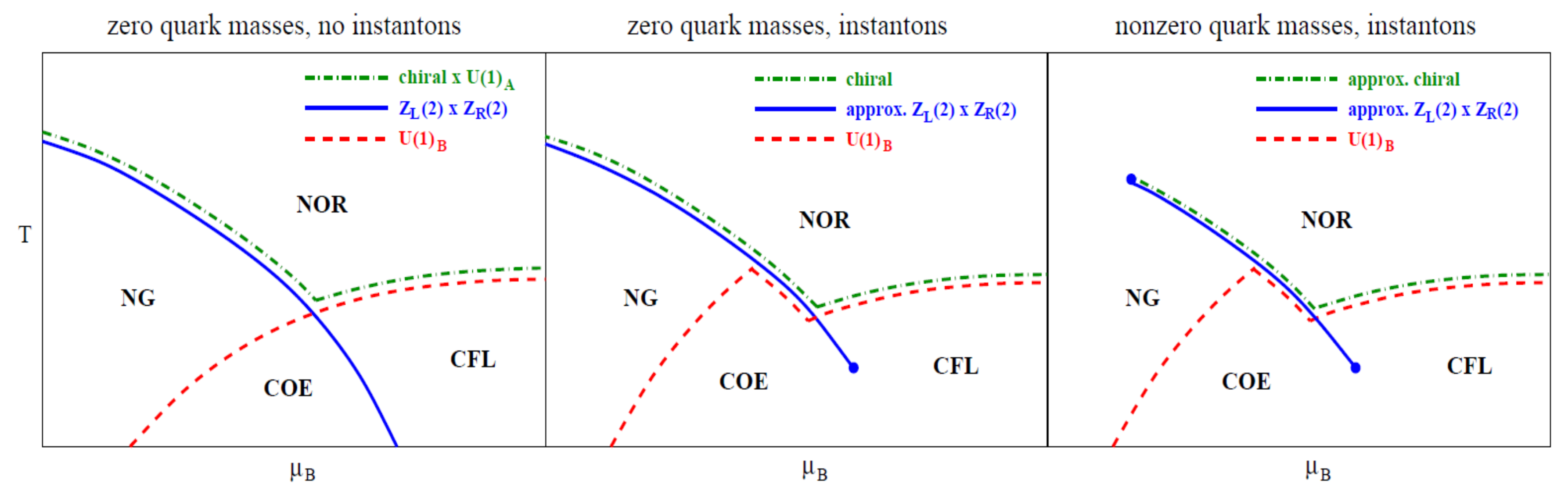
$$\boxed{-h\sigma}$$



Low T critical point driven by the axial anomaly

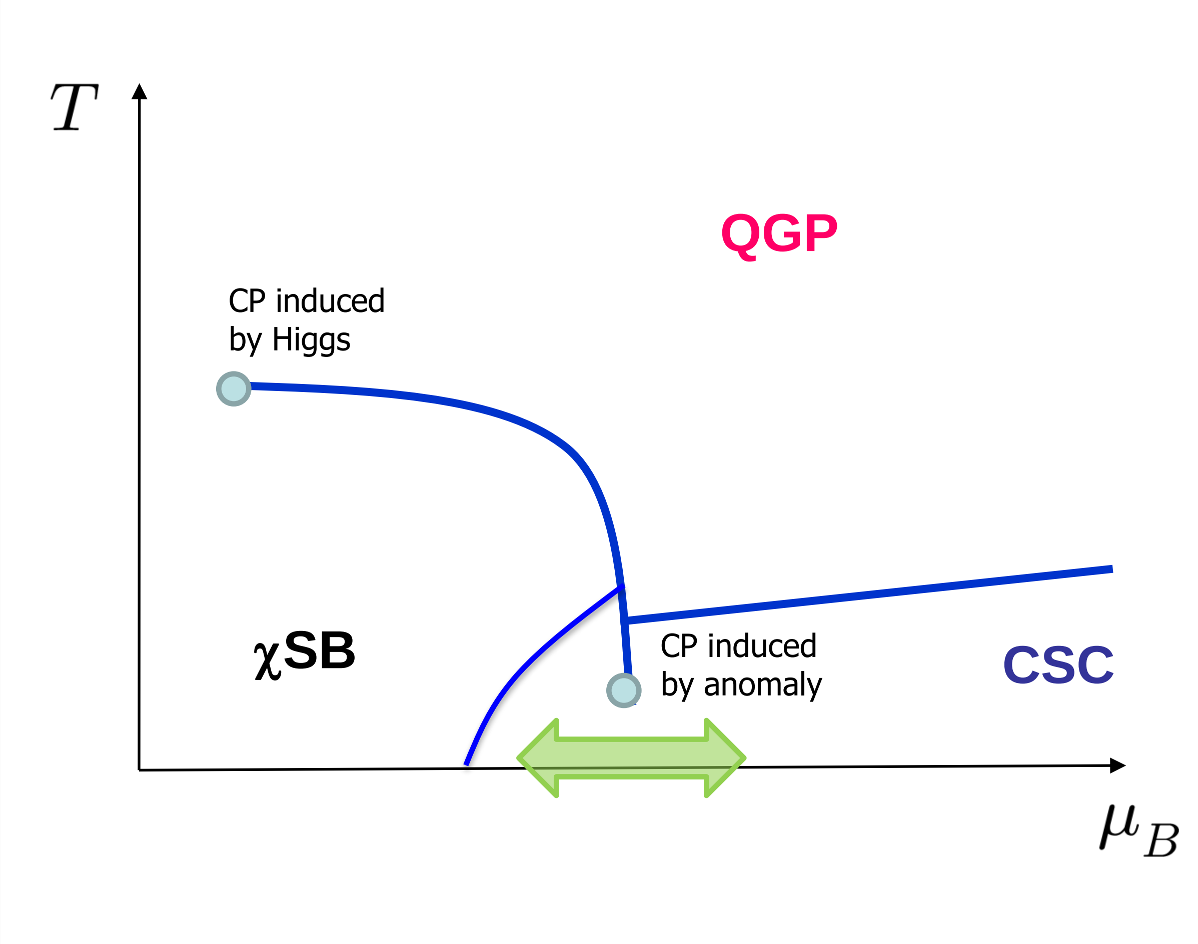
High T critical point driven by Higgs condensate

Critical points from symmetry point of view

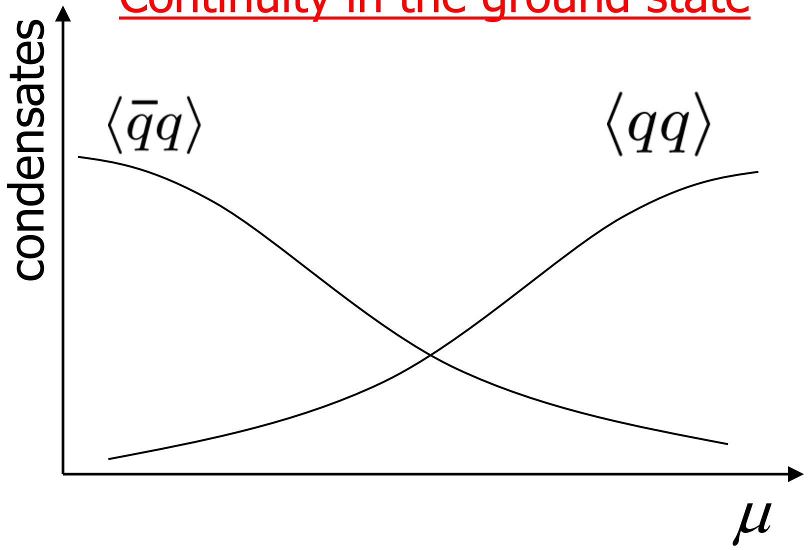


Original idea :
  Hatsuda and Alford (2010)
Figure taken from
  Schmitt, Stetina & Tachibana, arXiv: 10104243 [hep-ph]

Spectral continuity at finite μ



Continuity in the ground state



Continuity in excitation modes

○ Generalized Gell-Mann-Oakes-Renner relation :

$$f_\pi^2 m_\pi^2 = m_q(-\langle \bar{q}q \rangle + \Gamma \langle qq \rangle^2).$$

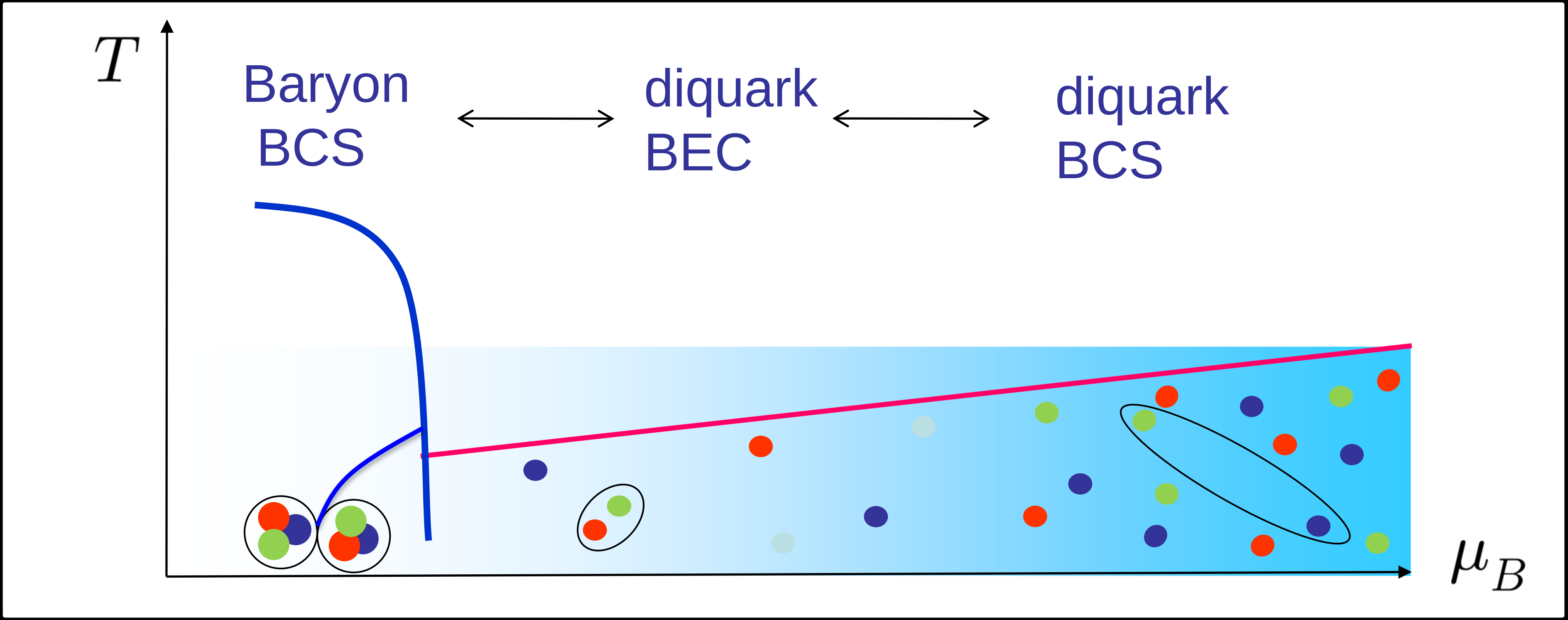
Yamamoto, Tachibana,
Baym & T.H., PR D76 ('07)

○ Vector meson – CFL gluon continuity (QCD sum rules):

$$\left(m_{\rm V}^{(8)}\right)^2 \simeq \frac{56\pi^3\alpha_s}{81\mu^4}\left(\langle \bar{q}q \rangle^2 + \frac{15}{7}\langle qq \rangle^2\right)$$

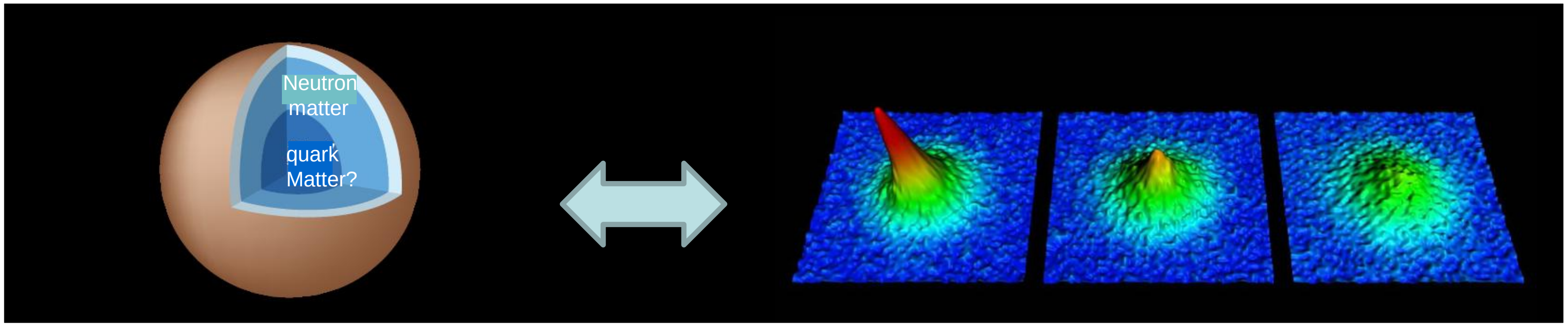
Tachibana, Yamamoto and T.H.,
PRD78 (2008)

Dense QCD confronts UCA experiments?



"Simulating dense QCD matter with Ultracold atoms"

Maeda, Baym and Hatsuda, Phys. Rev. Lett. 103 (2009) 085301



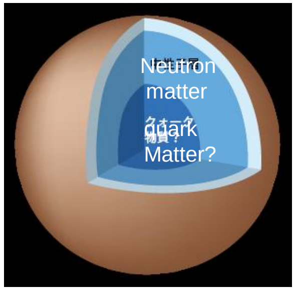
T[K]

10^{12} Quark-gluon plasma

Superfluid neutron matter

10^{9}

10^{7} Center of sun



Boiling water

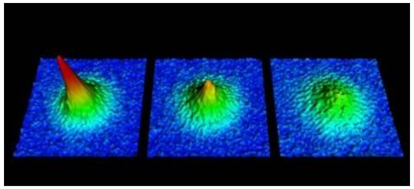
10^{2} Freezing water

10 Liquid nitrogen

1 Superfluid, superconductor

10^{-3} Superfluid of ^{3}He

10^{-7} **Ultracold atoms**



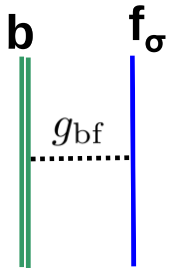
Ultracold Atoms (UCA)

- T ~ 10^{-7} K
- *hyperfine states magnetically controllable*
- density 10^{14} - 10^{15} cm^{-3} (cf. Air ~ 10^{19}cm^{-3})

Atomic Boson-Fermion mixture

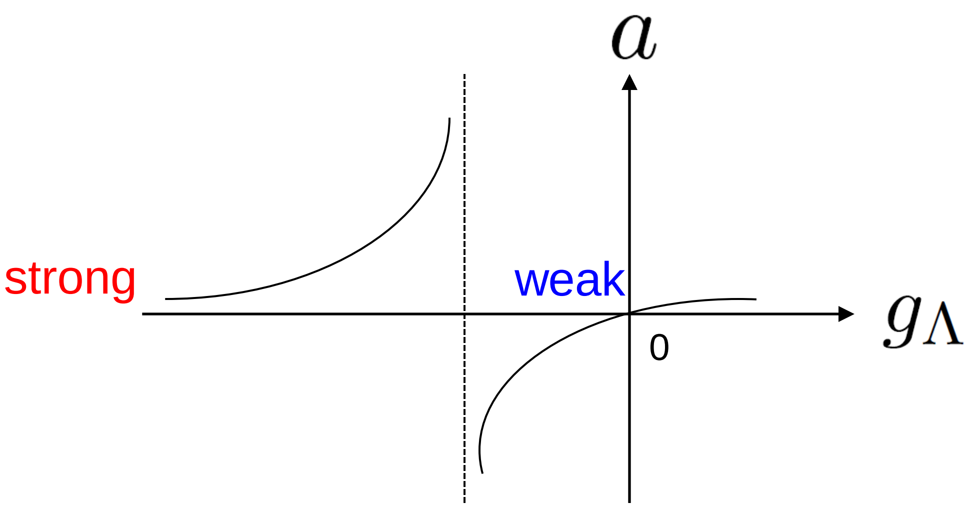
Maeda, Baym and Hatsuda,
Phys. Rev. Lett. 103 (2009) 085301

$$\begin{aligned}\mathcal{H} &= \phi^*(x)\left(-\frac{\nabla^2}{2m_{\rm b}} - \mu_{\rm b}\right)\phi(x) + \frac{1}{2}g_{\rm bb}|\phi(x)|^4 \\ &\quad +\psi^\dagger_\sigma(x)\left(-\frac{\nabla^2}{2m_{\rm f}} - \mu_{\rm f}\right)\psi_\sigma(x) + \frac{1}{2}g_{\rm ff}\psi^\dagger_\sigma(x)\psi_\sigma(x)\psi^\dagger_{-\sigma}(x)\psi_{-\sigma}(x) \\ &\qquad\qquad +g_{\rm bf}|\phi(x)|^2\psi^\dagger_\sigma(x)\psi_\sigma(x)\end{aligned}$$

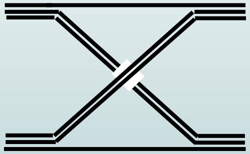


s-wave boson-fermion scattering length

$$\frac{2\pi\hbar^2}{m_{\rm R}}a = \frac{g_\Lambda}{1 + g_\Lambda\int^\Lambda \frac{d\mathbf{k}''}{(2\pi)^3}\frac{1}{\varepsilon(\mathbf{k}'')}}$$



Induced superfluidity of composite fermions
-- a novel analogy between cold atoms and QCD --

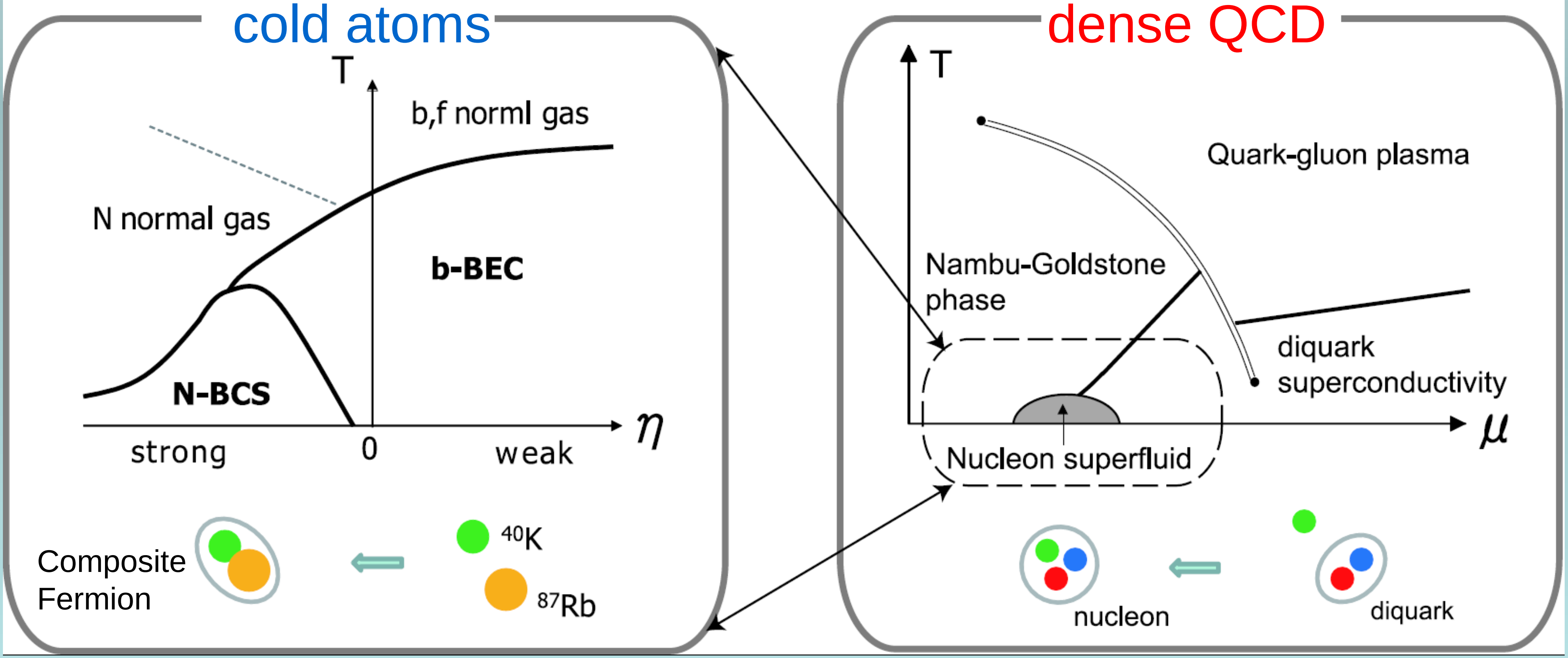


$$a_{\mathrm{NN}}^{\mathrm{Born}} = -\frac{m_{\mathrm{N}}}{2m_{\mathrm{R}}} a_{\mathrm{bf}}$$

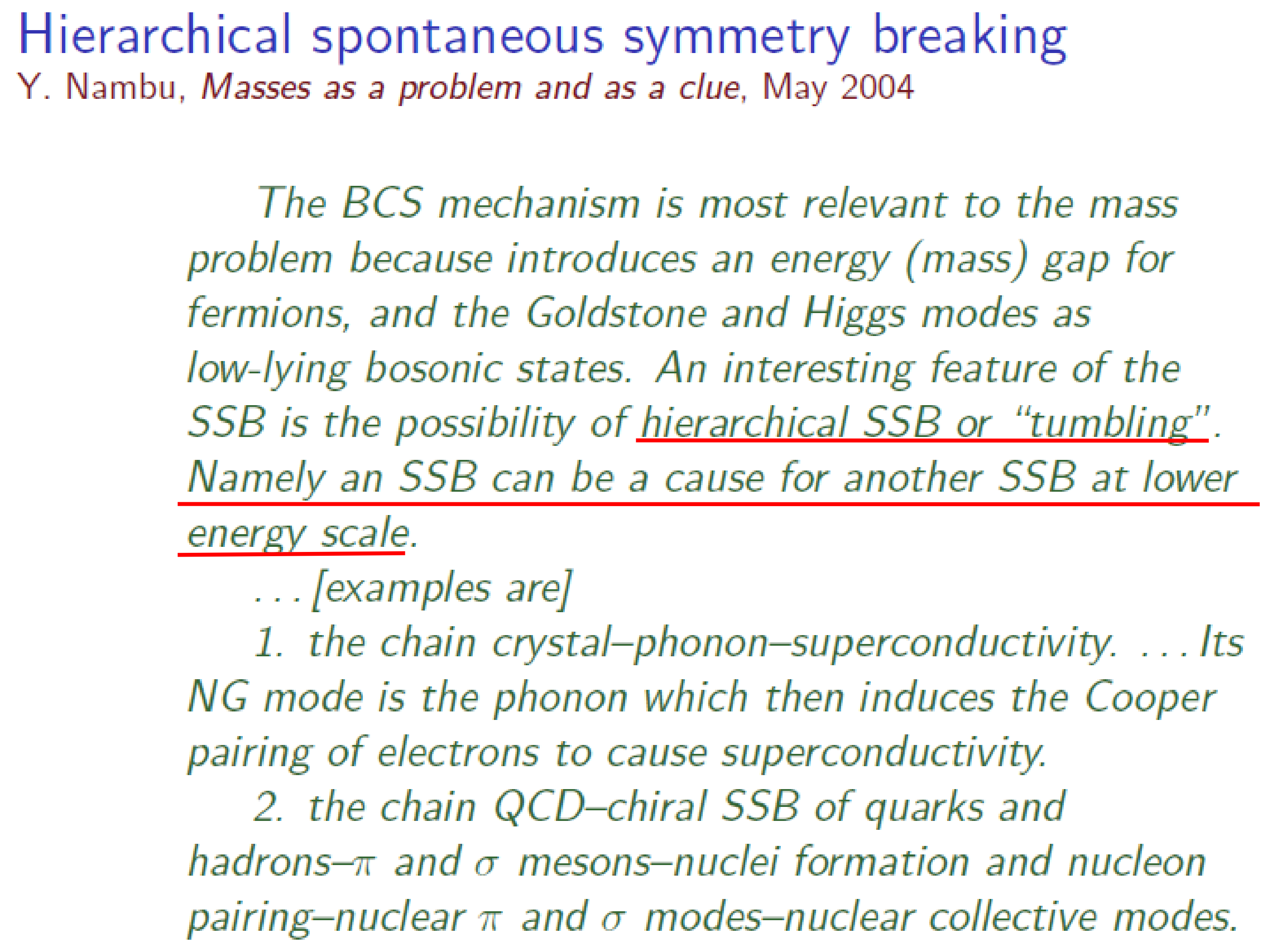
Maeda, Baym & Hatsuda,
Phys. Rev. Lett. 103 ('09)

weak (bf)-(bf) attraction ⇔ strong b-f attraction

baryon superfluidity ⇔ strong diqark correlation



Y. Nambu, Nobel Lecture (Dec.8, 2008), page 24/25



Hierarchical spontaneous symmetry breaking

Y. Nambu, *Masses as a problem and as a clue*, May 2004

> *The BCS mechanism is most relevant to the mass problem because introduces an energy (mass) gap for fermions, and the Goldstone and Higgs modes as low-lying bosonic states. An interesting feature of the SSB is the possibility of hierarchical SSB or "tumbling". Namely an SSB can be a cause for another SSB at lower energy scale.*
>
> *. . . [examples are]*
>
> *1. the chain crystal–phonon–superconductivity. . . . Its NG mode is the phonon which then induces the Cooper pairing of electrons to cause superconductivity.*
>
> *2. the chain QCD–chiral SSB of quarks and hadrons–π and σ mesons–nuclei formation and nucleon pairing–nuclear π and σ modes–nuclear collective modes.*

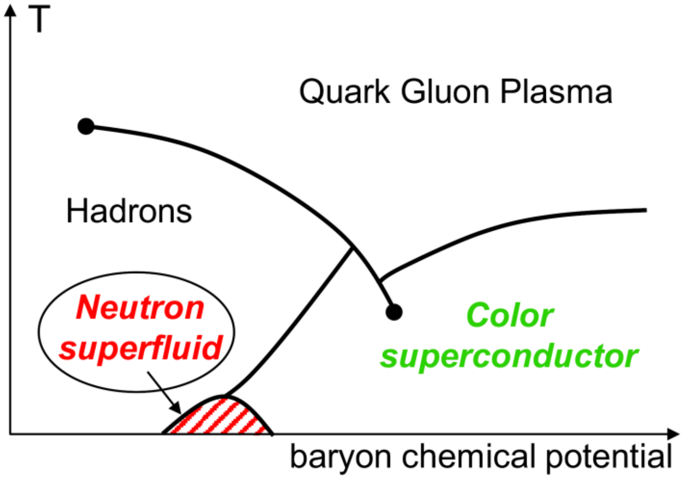
Summary and Future

1. QCD phase structure

- Three major phases in QCD: χSB, QGP and CSC
- Axial anomaly plays crucial roles everywhere
- Close similarity to high Tc supercond. & multi-comp. cold atoms

2. Chiral-super interplay driven by axial anomaly

- A new critical point at low T and high μ
- Continuity of χSB phase and CSC phase



3. Spectral continuity in high density QCD

- Pions are pions.
- Vector mesons are gluons.

4. Near future

- Hot QCD confront LHC experiments
- Dense QCD confronts neutron star observations
- Dense QCD confronts ultracold atomic experiments

Summary and Future

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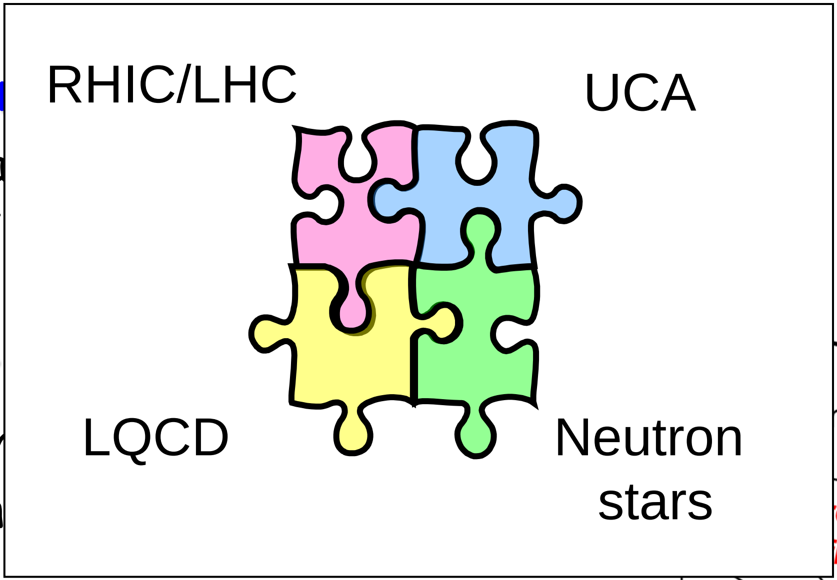

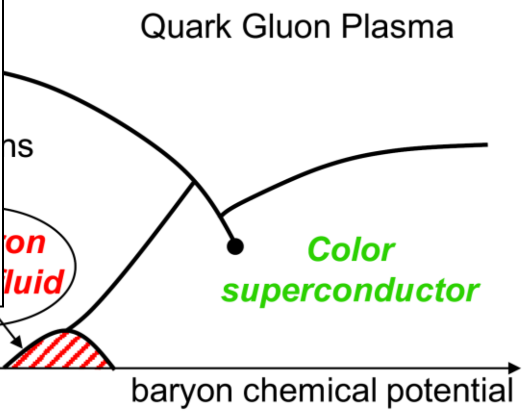

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