

Single flavor staggered fermions

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Japanese-German workshop

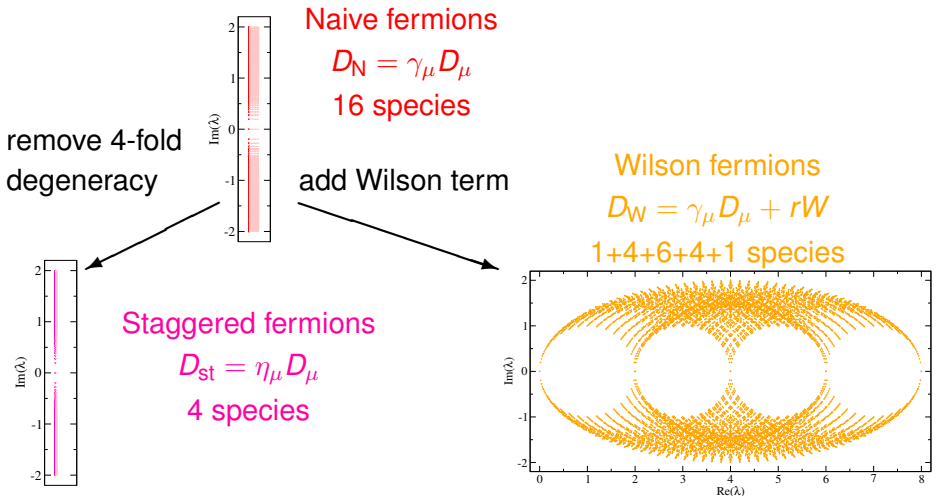
Mishima, Nov. 4, 2010

([arXiv:1009.5362](https://arxiv.org/abs/1009.5362))

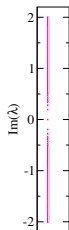


- 1 Introduction
- 2 Staggered Wilson
- 3 Symmetries
- 4 New operator
- 5 Tests in 2D
- 6 Outlook

Lattice fermions



Chiral symmetry



Staggered fermions

$$D_{\text{st}} = \eta_\mu D_\mu$$

4 species, no anomaly

$$U(1)_\epsilon : \{D_{\text{st}}, \epsilon\} = 0$$

$$\epsilon = (-1)^{x_1 + x_2 + x_3 + x_4}$$

$$\sim (\gamma_5 \otimes \xi_5)$$

Wilson fermions D_W
1+4+6+4+1 species

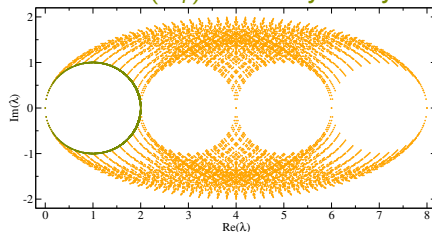


Overlap fermions

$$D_{\text{ov}} = \rho (1 + \gamma_5 \text{sign}(\gamma_5 (D_W - \rho)))$$

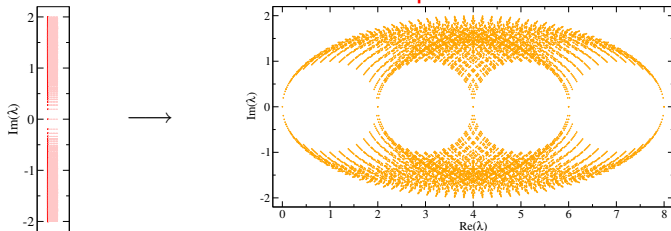
1 species, correct anomaly

Full $SU(N_f)$ chiral Symmetry



Wilson term

Add a **Wilson term** to **naive operator**:



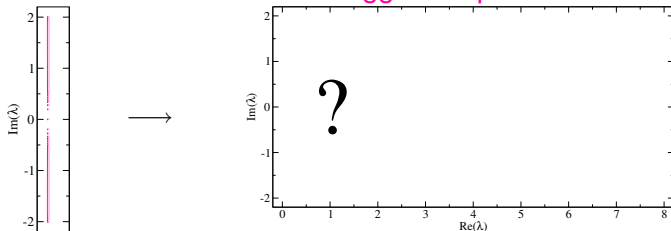
Momentum dependent mass (remove doublers)

Positivity of $\det(D_W)$:

- **Naive operator**: antihermitean, $\{D_N, \gamma_5\} = 0$
- **Wilson term W** : hermitean, $[W, \gamma_5] = 0$
- $D_W \gamma_5 = \gamma_5 D_W^\dagger$
- **eigenvalues real or in complex conjugate pairs** $\lambda_i = \lambda_{i^*}^*$

Staggered Wilson term

Add a Wilson term to staggered operator:



Momentum (taste) dependent mass (remove doublers)

Positivity of $\det(D_A)$:

- Staggered operator: antihermitean, $\{D_N, \epsilon\} = 0$

- Wilson term A : hermitean, $[A, \epsilon] = 0$

→ $D_A \epsilon = \epsilon D_A^\dagger$

→ eigenvalues real or in complex conjugate pairs $\lambda_i = \lambda_{i^*}^*$

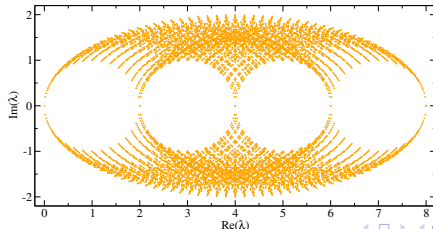
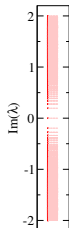
Wilson term construction

Usual Wilson term:

$$W = \sum_{\mu} (C_{\mu} + 1)$$

$$C_{\mu} := \frac{1}{2} (V_{\mu} + V_{\mu}^{\dagger}) \quad (V_{\mu})_{xy} := U_{\mu}(x) \delta_{x+\hat{\mu},y}$$

$$W^{\dagger} = W \quad \checkmark \quad [W, \gamma_5] = 0 \quad \checkmark$$



Staggered Wilson term construction

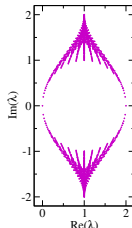
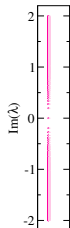
Staggered Wilson term: (Adams, 2010)

$$A = \epsilon \eta_5 (C_1 C_2 C_3 C_4)_{\text{sym}}$$

$$C_\mu := \frac{1}{2} (V_\mu + V_\mu^\dagger) \quad (V_\mu)_{xy} := U_\mu(x) \delta_{x+\hat{\mu}, y}$$

$$A^\dagger = A \quad \checkmark$$

$$[A, \epsilon] = 0 \quad \checkmark$$



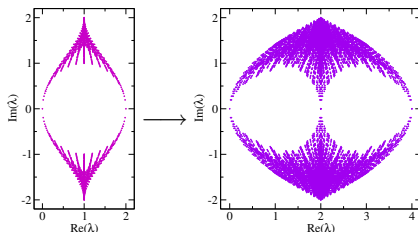
- $\eta_5 = \eta_1 \eta_2 \eta_3 \eta_4 = (-1)^{x_1 + x_3}$
- $\eta_\mu = (-1)^{\sum_{\nu < \mu} x_\nu} \sim (\gamma_\mu \otimes 1)$
- $\epsilon = (-1)^{x_1 + x_2 + x_3 + x_4} \sim (\gamma_5 \otimes \xi_5)$
- $\{C_\mu, \epsilon\} = 0$
- $A \sim (1 \otimes \xi_5) + O(a)$

Remnant flavor degeneracy

- Staggered flavor basis: $A \sim \xi_5 = \text{diag}(1, 1, -1, -1)$
- Twofold degeneracy left!
- Let us take

$$M_{\mu\nu} = i\epsilon_{\mu\nu}\eta_\mu\eta_\nu (C_\mu C_\nu)_{\text{sym}}$$

$$M_{\mu\nu}^\dagger = M_{\mu\nu} \quad \checkmark \quad [M_{\mu\nu}, \epsilon] = 0 \quad \checkmark$$



- $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu} = (-1)^{x_\mu + x_\nu}, \mu < \nu$
- $\eta_\mu = (-1)^{\sum_{\nu < \mu} x_\nu}$
- $M_{\mu\nu} \sim (1 \otimes \sigma_{\nu\mu}) + O(a)$
- $\sigma_{\nu\mu} = \text{diag}(1, -1, -1, 1)$

Discrete staggered symmetries

Remnants of Poincare symmetry:

	D_{st}	A	$M_{\mu\nu}$
shift (translation)	+	-	$\pm$
axis reversal	+	-	$\pm$
rotation	+	+	$M_{\alpha\beta}$

- Preserved by staggered operator
 - Preserved up to a sign flip by A
 - Rotation introduces new terms for $M_{\mu\nu}$
- **Bad:** new counterterms
- ! Search for more symmetric construction

Symmetrized staggered Wilson

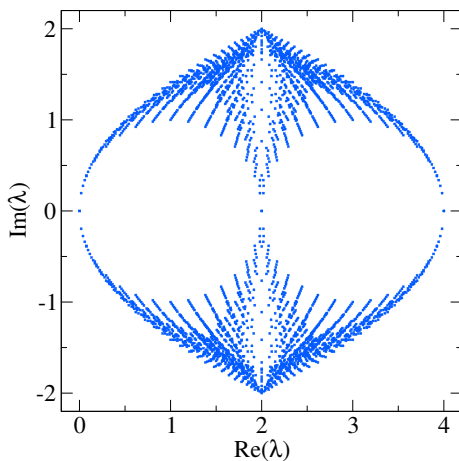
$$M_s = \frac{1}{\sqrt{3}} \left(\begin{aligned} &\text{green } s_{12} (\text{red } s_1 s_2 M_{12} + \text{red } s_3 s_4 M_{34}) \\ &+ \text{green } s_{13} (\text{red } s_1 s_3 M_{13} + \text{red } s_4 s_2 M_{42}) \\ &+ \text{green } s_{14} (\text{red } s_1 s_4 M_{14} + \text{red } s_2 s_3 M_{23}) \end{aligned} \right)$$

- Shift or axis reversal in ρ : $s_\rho \rightarrow -s_\rho$

rotation (ρ, σ)				sign flip
(1,4)	(2,3)	(3,1)	(2,4)	$s_{12} \rightarrow -s_{12}$
(1,2)	(3,4)	(4,1)	(3,2)	$s_{13} \rightarrow -s_{13}$
(1,3)	(4,2)	(2,1)	(4,3)	$s_{14} \rightarrow -s_{14}$

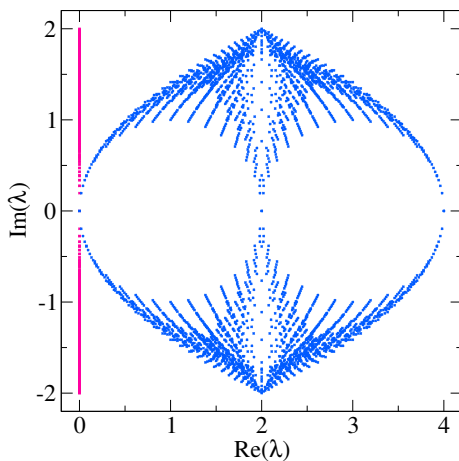
- $M_s \sim (1 \otimes \xi^{(s)}) + O(a) \quad \xi^{(s)} = \text{diag}(0, 0, 2, -2)$

Single flavor staggered operator



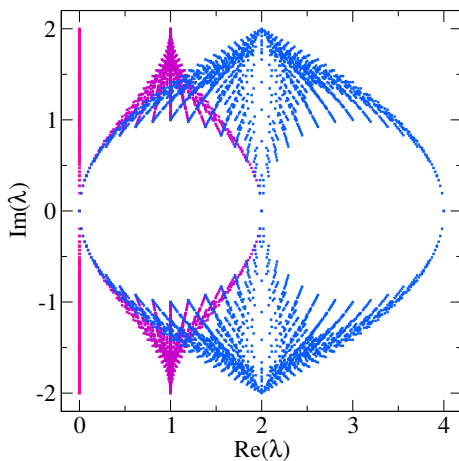
- D_{st}
- $D_{\text{st}} + A$
- $D_{\text{st}} + A + M_{\mu\nu}$
- $D_s = D_{\text{st}} + (2 + M_s)$
- $D_1 = 1 + \epsilon \text{sign}(\epsilon(D_s - 1))$

Single flavor staggered operator



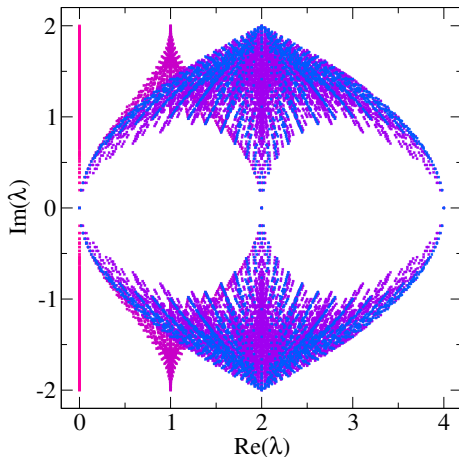
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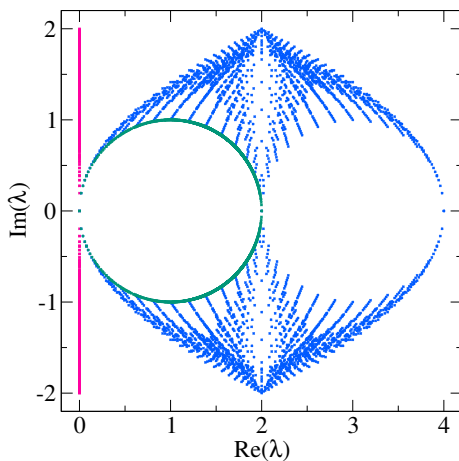
- D_{st}
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- $D_{\text{st}} + A + M_{\mu\nu}$
- $D_{\text{S}} = D_{\text{st}} + (2 + M_{\text{S}})$
- $D_1 = 1 + \epsilon \text{sign}(\epsilon(D_{\text{S}} - 1))$

Single flavor staggered operator



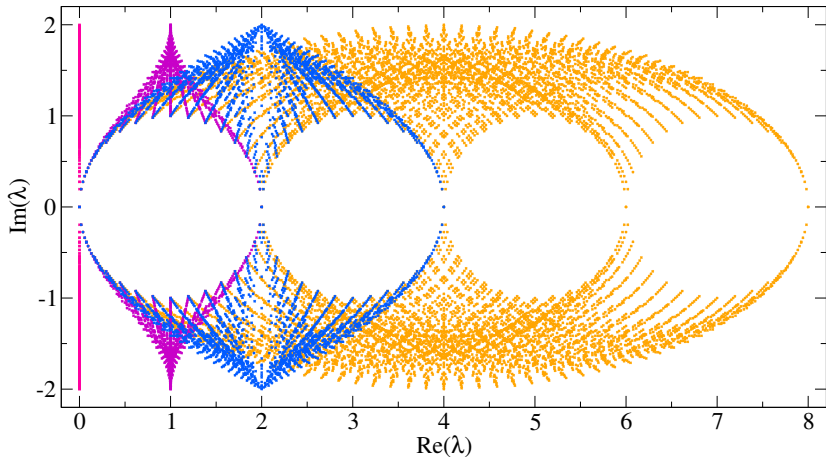
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Single flavor staggered operator

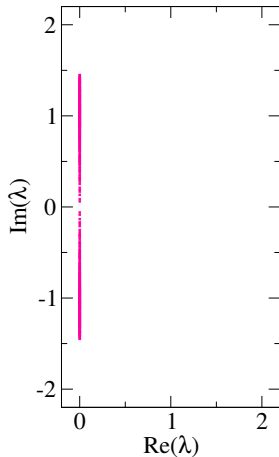


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Single flavor staggered operator



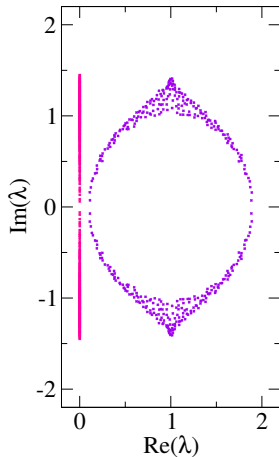
Some tests in 2D



- Only 2-fold degeneracy in 2D
- M_{12} uniquely lifts this degeneracy

- D_{st}
- $D_{\text{st}} + 1 + M_{12} \quad Q = 0$
- $1 + \epsilon \text{sign}(\epsilon(D_{\text{st}} + M_{12}))$

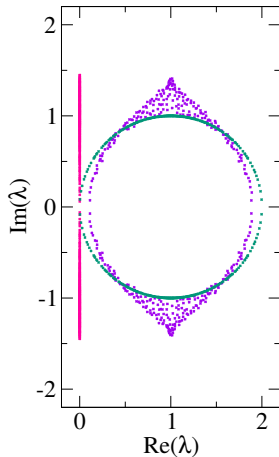
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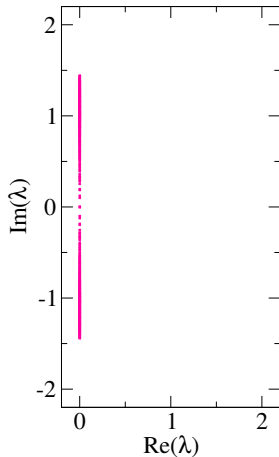
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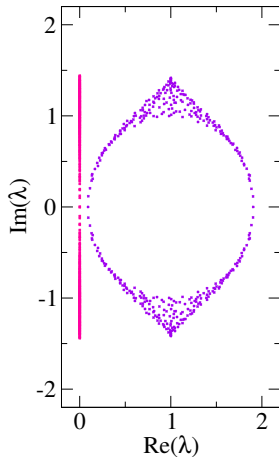
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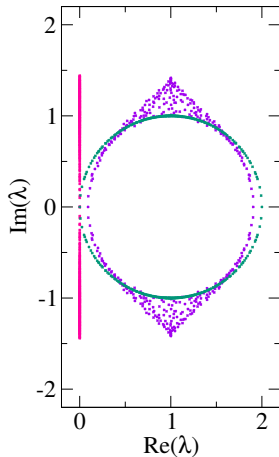
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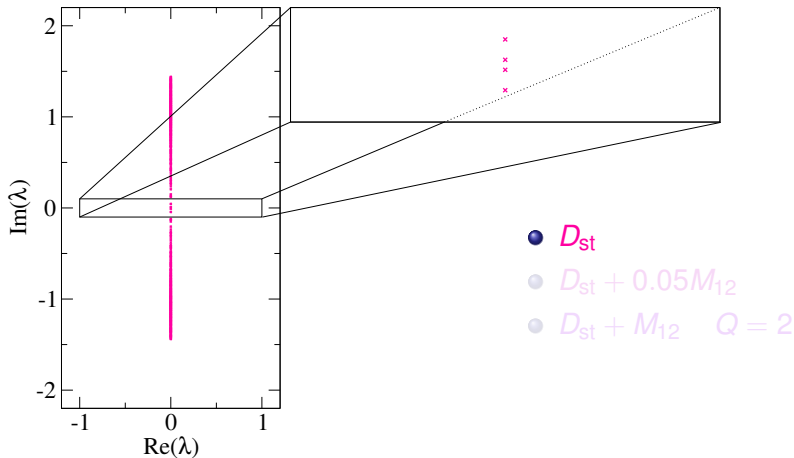
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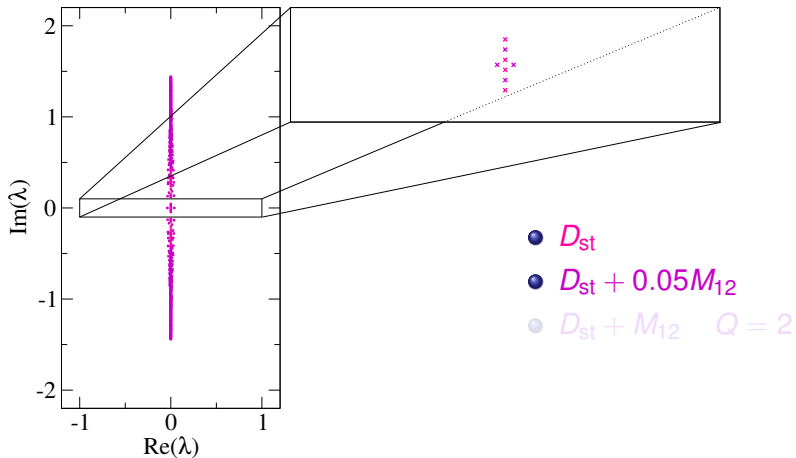


- D_{st}
- $D_{\text{st}} + 1 + M_{12} \quad Q = 1$
- $1 + \epsilon \text{sign}(\epsilon(D_{\text{st}} + M_{12}))$

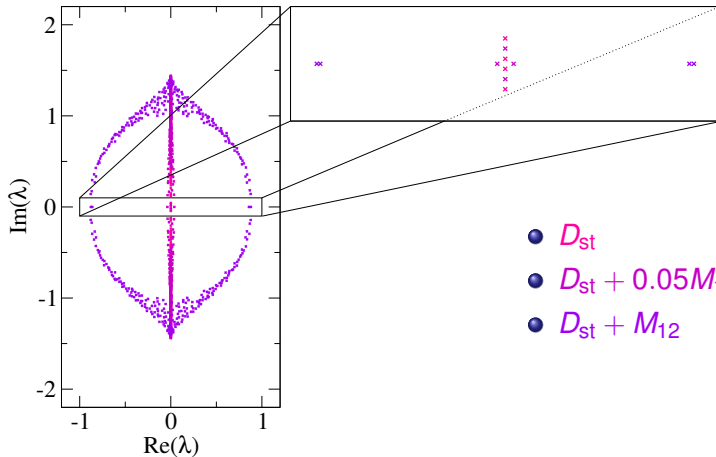
Some tests in 2D



Some tests in 2D



Some tests in 2D



- D_{st}
- $D_{\text{st}} + 0.05M_{12}$
- $D_{\text{st}} + M_{12} \quad Q = 2$

Conclusion

Single flavor staggered operator is possible

Wilson fermions without remnants of spurious naive degeneracy exist

But is it usefull?

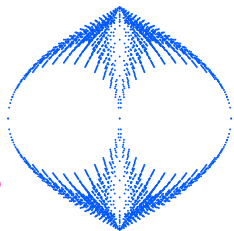
- ✓ Better condition number
- ✓ Smaller matrix

- ✗ Breaks flavor symmetry
- ✗ 2-hop Wilson term

Staggered Wilson

To do list:

- Find counterterm structure
- Construct mesons, baryons
- $O(a)$ improvement
 - Clover “for free” due to 2-hop Wilson term?
(some CPU, but no additional bandwidth)
- Optimize algorithms for the structure
- Check flavor breaking
- Study scaling
- Apply to real problem
 - Insensitive to flavor breaking
 - Ground states, bulk properties, spectral quantities
 - Hadron/quark masses? Thermodynamics?



Staggered overlap

To do list:

- Find counterterm structure
- Check locality
- Check flavor breaking
- Study scaling
- Apply to real problem
 - Insensitive to flavor breaking
 - Chiral symmetry essential
 - Spectral quantities?

