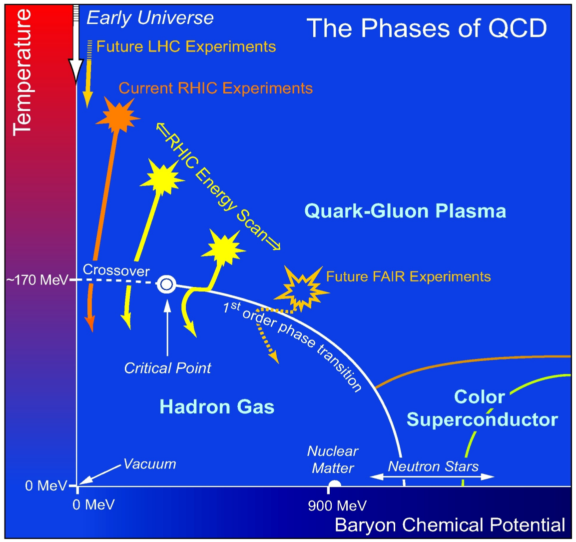
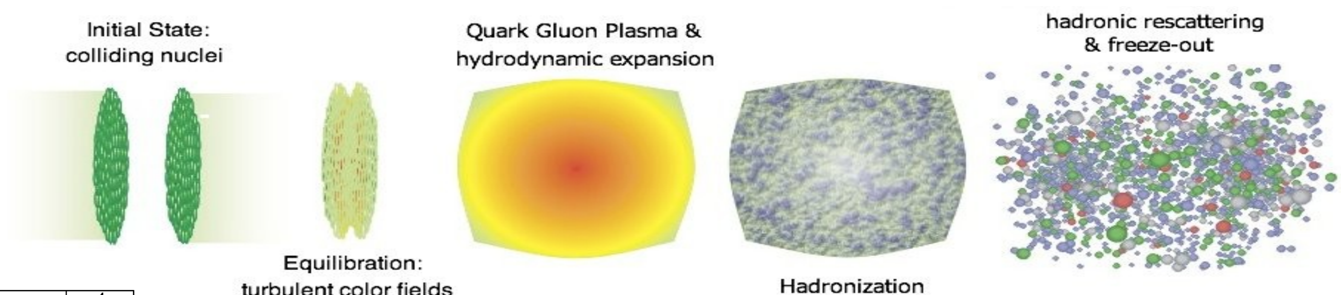
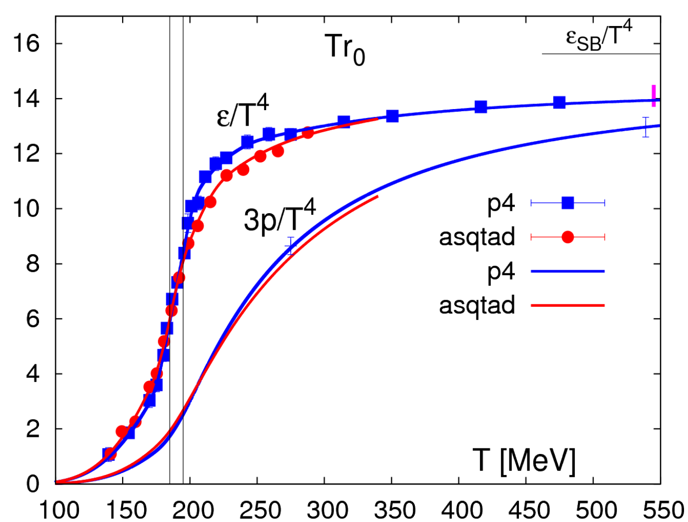
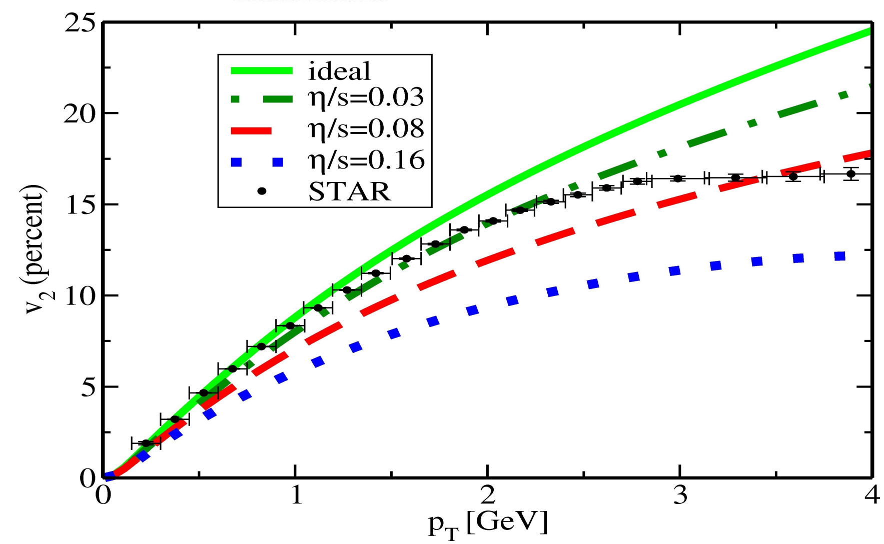
QCD Thermodynamics and Heavy Ion Collisions



Equation of State and transition temperature
Dilepton rates and transport properties
Charge fluctuations and freeze-out conditions
Universality and the phase diagram







Yamagata 1995

Expansion of hot & dense matter
Hydrodynamics and the EoS

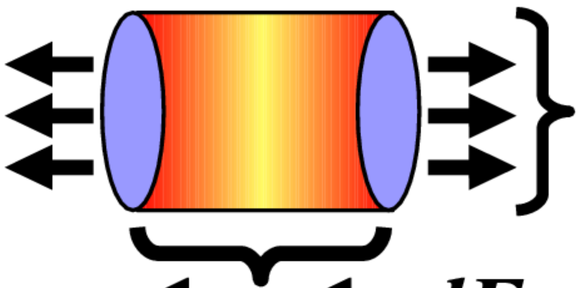
- EoS controls expansion of hot/dense matter created in a heavy ion collision;
- slower velocity of sound $\Rightarrow$ larger lifetime $\Rightarrow$ larger thermal rates of e.g. thermal dileptons, thermal photons,.....

Bjorken formula:

$$R = 1.2 A^{1/3} \mathrm{fm}$$

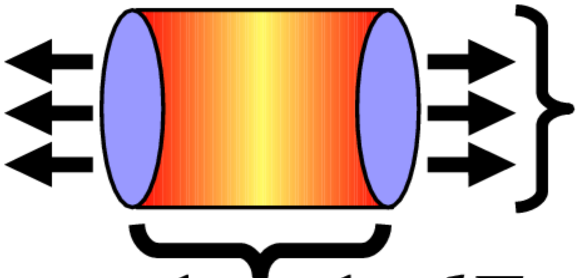
$$\frac{dE_T}{dy} \simeq \langle E_T \rangle \frac{dN}{dy}$$

τ_0 : equilibration time;
time after collision, at
which hydro description may start



$$\varepsilon_{Bj} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

Expansion of hot & dense matter
Hydrodynamics and the EoS



$$\varepsilon_{Bj} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

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time after collision, at
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RHIC:

$$\begin{aligned} R_{Au} &\simeq 7\ \mathrm{fm} \\ \langle E_T \rangle &\simeq 1\ \mathrm{GeV} \\ dN/dy &\simeq 600 \\ \tau_0 &\simeq (0.5-1)\ \mathrm{fm} \end{aligned}$$

$$\Rightarrow \epsilon_{Bj} \simeq (5-10)\ \mathrm{GeV/fm}^3$$

$$\Rightarrow T_0 \simeq (225-270) \mathrm{MeV}$$

LHC:

$$\tau_0 \simeq 0.2\ \mathrm{fm}$$

$$\epsilon_{Bj} \simeq 300\ \mathrm{GeV/fm}^3$$

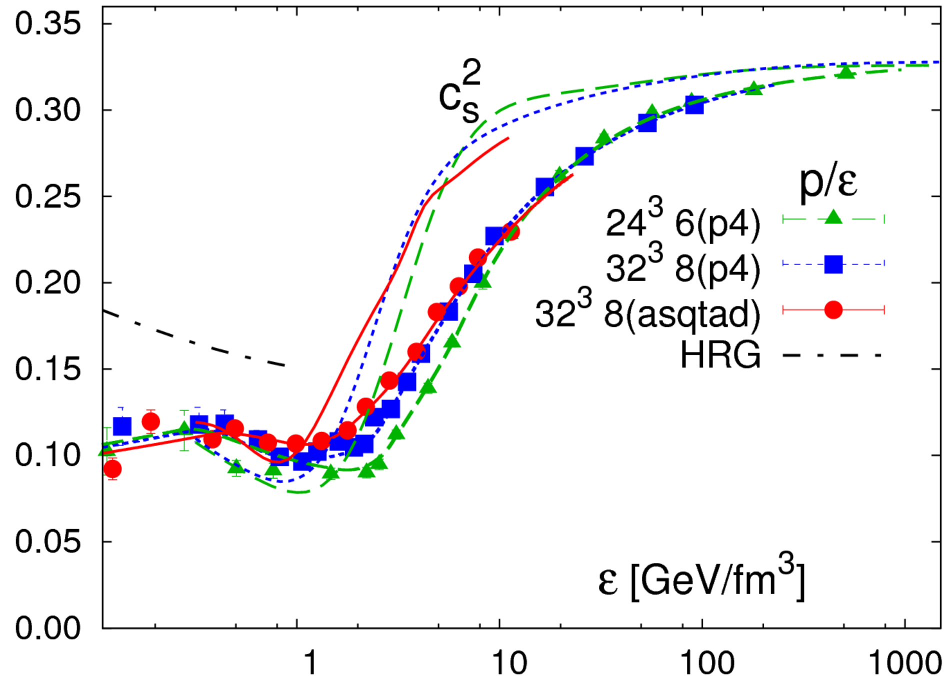
$$T_0 \simeq 700\ \mathrm{MeV}$$

Expansion of hot & dense matter
1-d hydro & lifetime of the QGP

- simple 1-d hydro: $\partial_\mu T_{\mu\nu} = 0 \Rightarrow \frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} = 0$

- ideal gas EoS: $p/\epsilon = 1/3$ $\Rightarrow$ $\frac{\epsilon(\tau)}{\epsilon(\tau_0)} = \left(\frac{\tau_0}{\tau}\right)^{4/3}$

$$\tau_c = \tau_0 \left(\frac{\epsilon(\tau_0)}{\epsilon(\tau_c)}\right)^{3/4}$$



M. Cheng et al. (hotQCD), arXiv:0903.4379

- Lattice EoS = (2+1)-f QCD EoS:

$$\frac{p}{\epsilon} \simeq \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\epsilon}\right)$$

Expansion of hot & dense matter
1-d hydro & lifetime of the QGP

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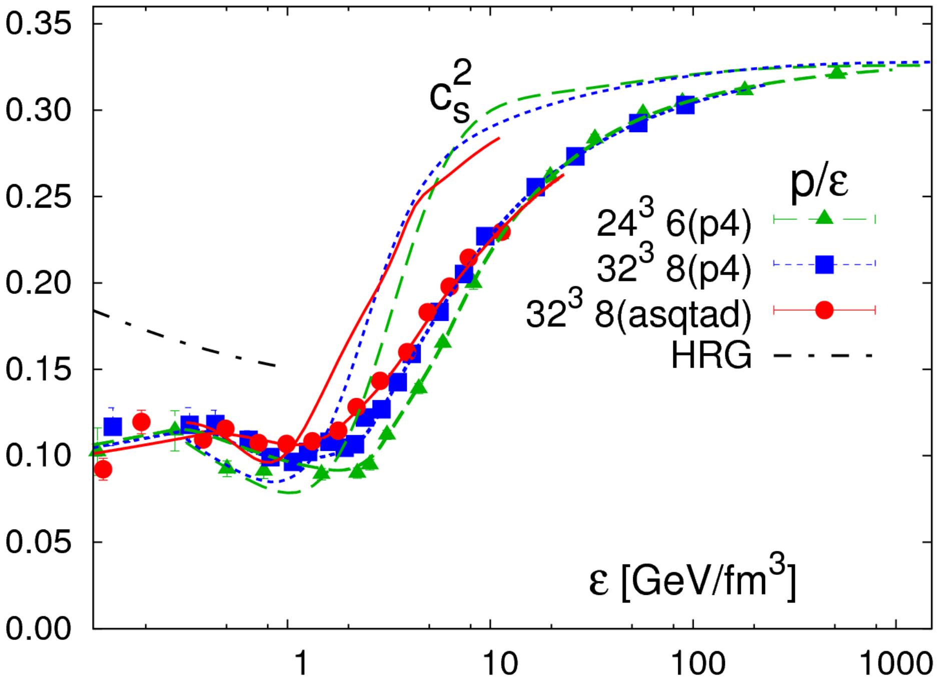
$$\tau_c = \tau_0 \left(\frac{\epsilon(\tau_0)}{\epsilon(\tau_c)}\right)^{3/4}$$



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- Lattice EoS = (2+1)-f QCD EoS:

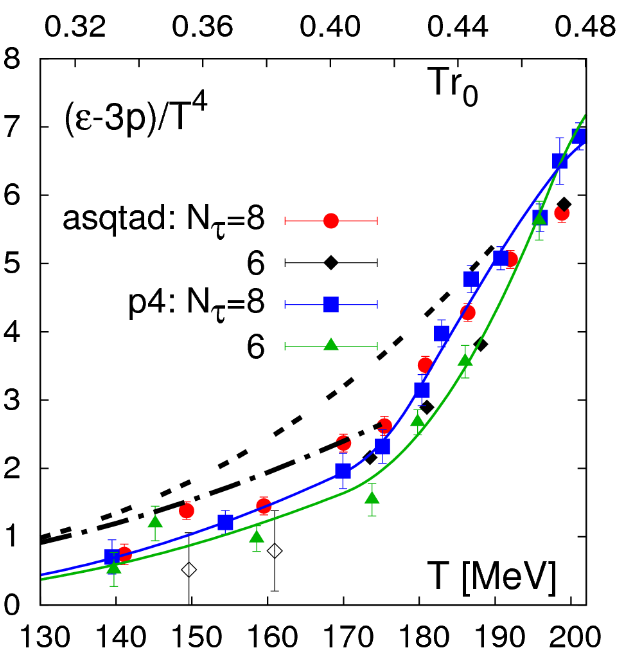
$$\frac{p}{\epsilon} \simeq \frac{1}{3}\left(1 - \frac{1.2}{1+0.5\epsilon}\right)$$

$$\epsilon(\tau_0) = 10\mathrm{GeV/fm}^3\ ,\ \epsilon(\tau_c) \simeq 1\mathrm{GeV/fm}^3$$
$$\Rightarrow \tau_c \simeq 5.5\mathrm{fm}\ (IG) \qquad \tau_c \simeq 7\ \mathrm{fm}\ (LGT)$$

Equation of State

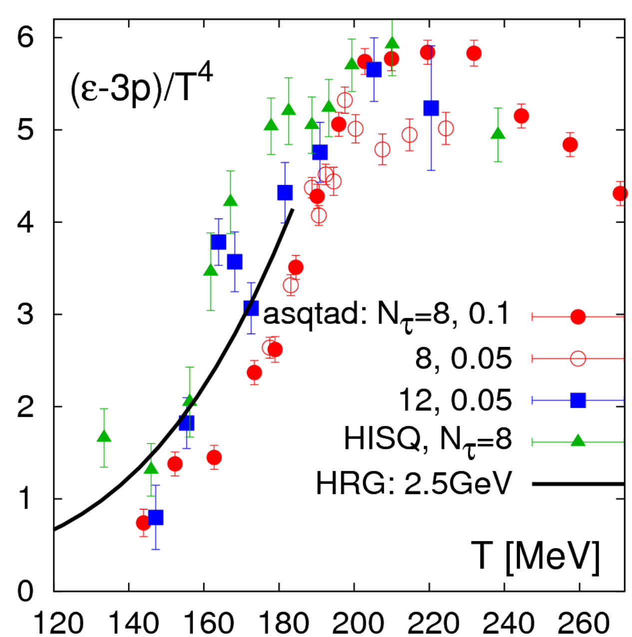
HotQCD ongoing research project

EoS in the transition region;
hadron resonance gas vs.
O(N) critical behavior



A. Bazavov et al (HotQCD collaboration),
Phys. Rev. D80, 014504 (2009)

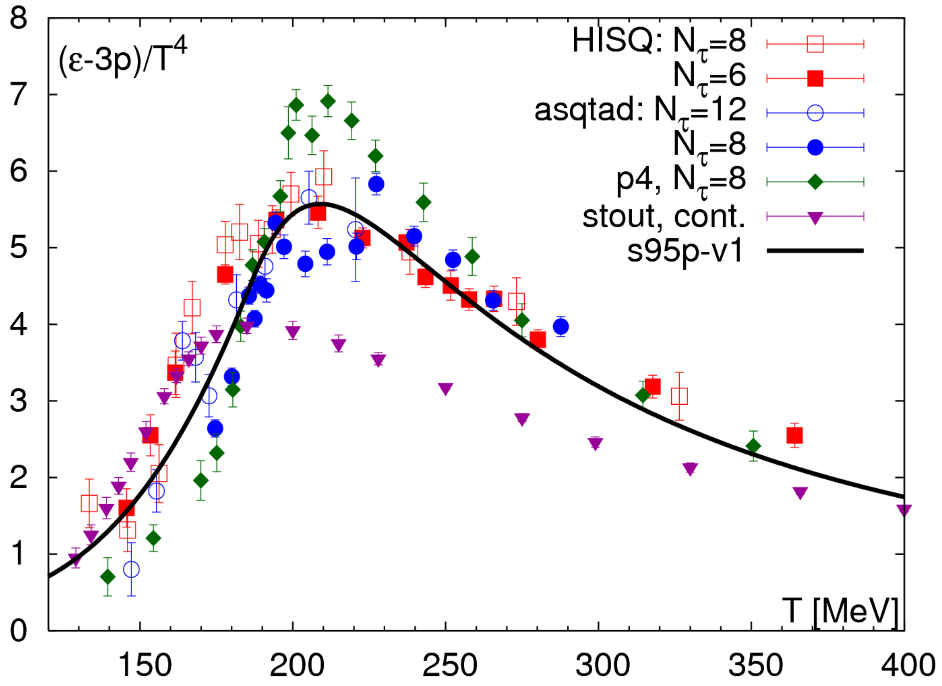
trace anomaly: NEW
asqtad Nt=12, hisq Nt=8



HotQCD preliminary

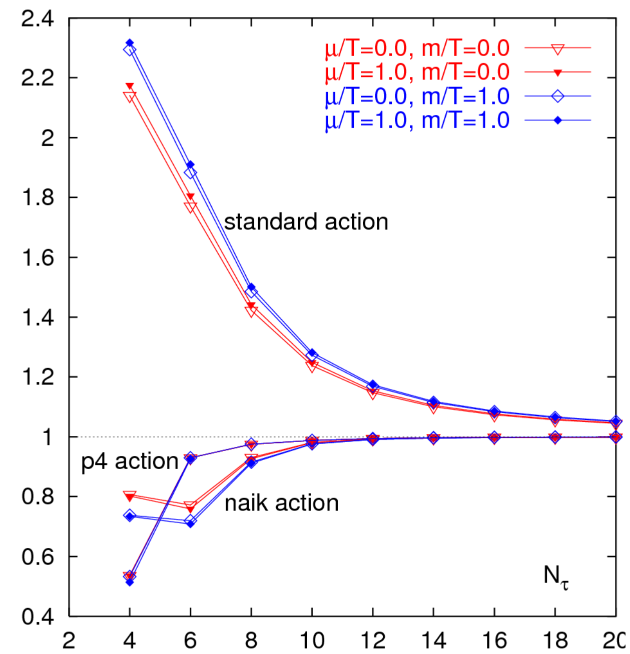
W, Soeldner, Lattice 2010, Viillasimius
A. Bazavov, xQCD 2010, Bad Honnef

Equation of State
(2+1)-flavor QCD



stout, cont: ((Nt=8)+(Nt=10))/2

$\mathcal{O}(a^2)$ discretization errors at high temperature



M. Cheng et al. (hotQCD), arXiv:0903.4379
+ hotQCD preliminary;
stout data: S. Borsanyi et al (Wuppertal-Budapest), arXiv: 1007.2580

Expansion of hot & dense matter
viscous hydrodynamics

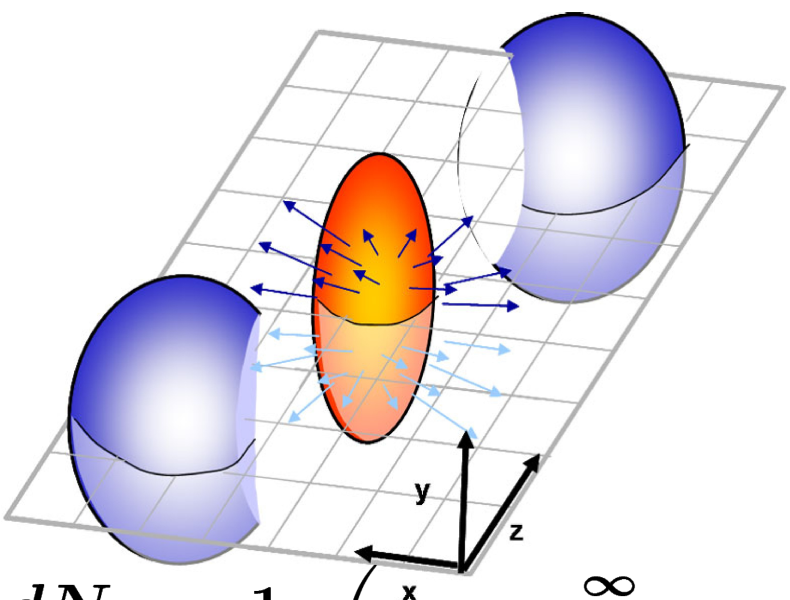
- need 3-d, viscous hydro to describe asymmetric pressure gradients

$$\partial_\mu T_{\mu\nu} = 0 \;\Rightarrow\; \frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} - \frac{1}{\tau^2}\left(\frac{4}{3}\eta + \Theta\right) = 0$$

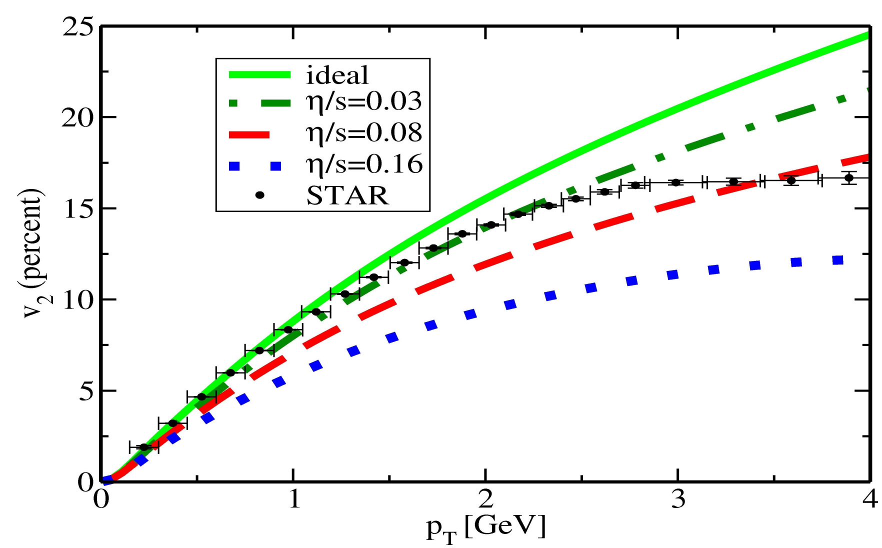
bulk and shear viscosity

- need small η / s to understand RHIC data

elliptic flow



$$\frac{dN}{d\phi} = \frac{1}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n cos(\phi)\right)$$



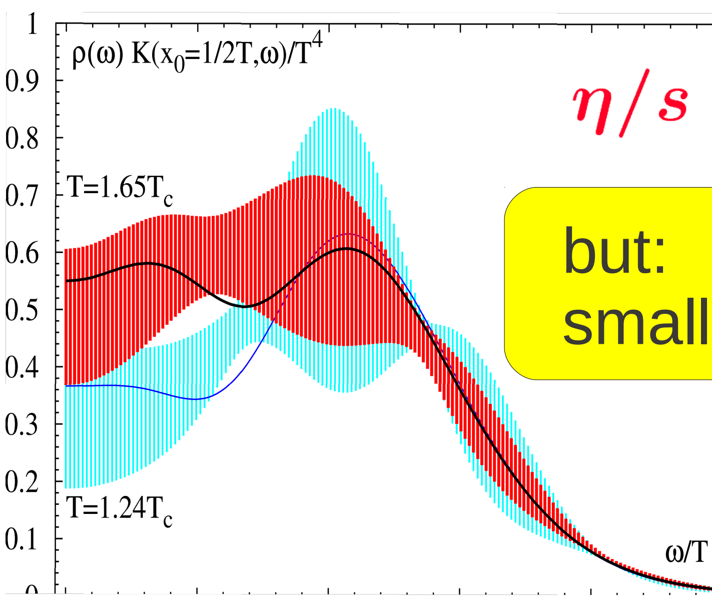
P.+U. Romatschke, PRL 99, 172301 (2007)

Expansion of hot & dense matter
viscous hydrodynamics

- spectral representation of the energy-momentum tensor

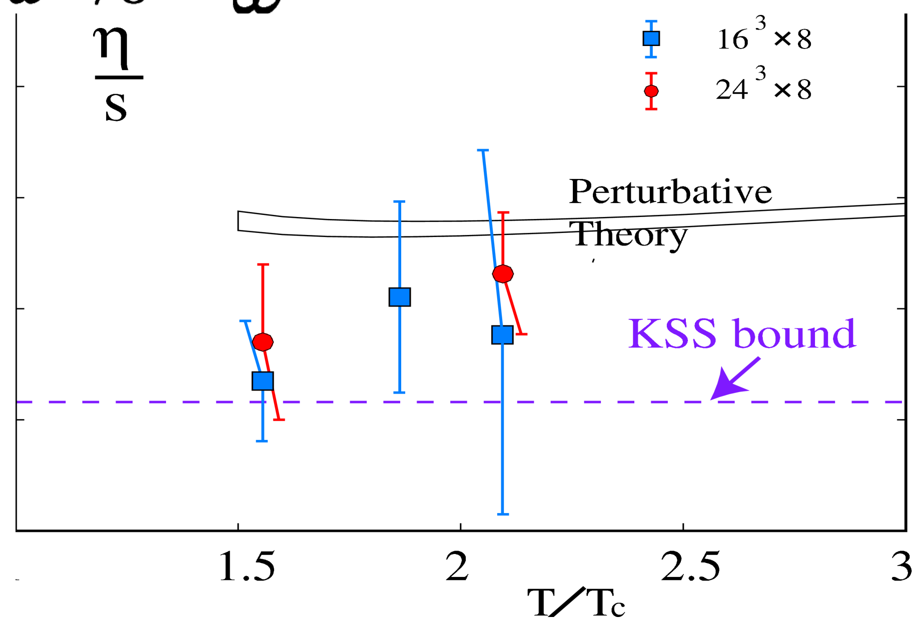
$$\langle T_{12}(0)T_{12}(t)\rangle = \int_0^\infty d\omega\, \rho(\omega)\, \frac{\cosh(\omega(t-1/2T))}{\sinh(\omega/2T)}$$

⇒ shear viscosity $\quad \eta = \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$



$\eta/s \simeq 0.1$

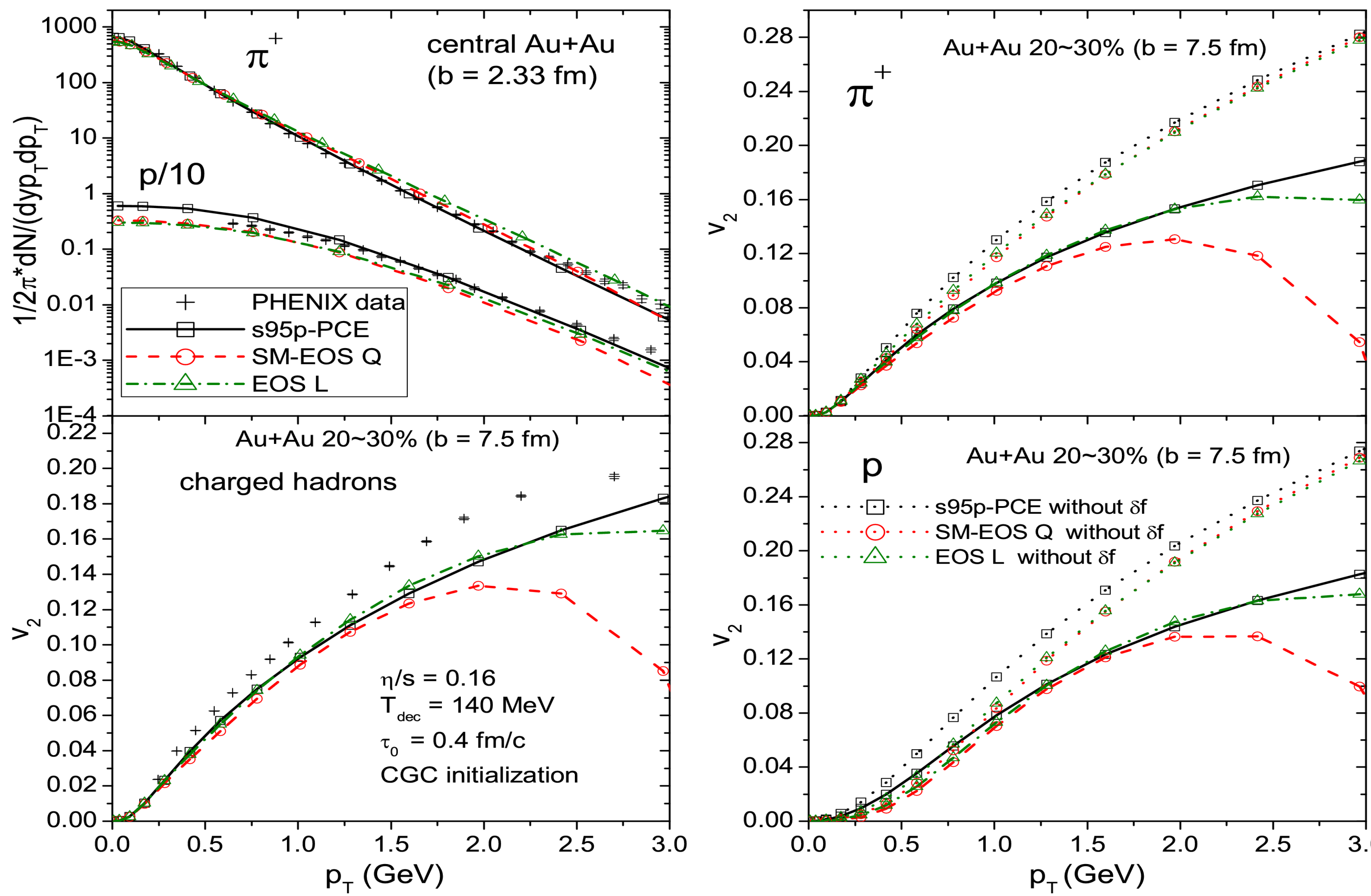
but:
small lattices



H. Meyer, PRD76, 101701 (2007)

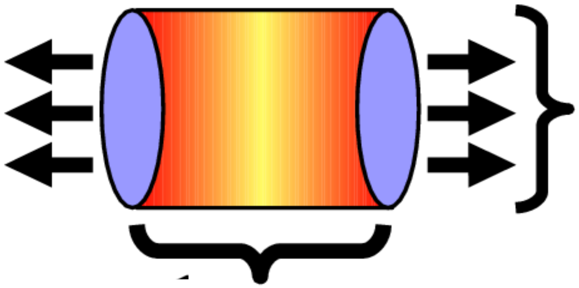
A. Nakamura, S. Sakai, PRL94, 072305 (2005)

3-d viscous hydro with EoS from lattice QCD



C. Shen, U. Heinz, P. Huovinen, H. Song, arXiv:1010.1856

Thermal Dileptons
probing the structure of the QGP



$$V = \pi R^2 \Delta y$$

thermal dilepton rate:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p \mathrm{d}^4x} = \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

thermal emission during expansion:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \pi R^2 \Delta y \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\int_{\tau_0}^{\tau_c} \mathrm{d}\tau \frac{\rho_V(\omega,\vec{p},T(\tau))}{\omega^2(\mathrm{e}^{\omega/T(\tau)}-1)}$$

$T(\tau)$ from EoS using Bjorken model or BETTER 3-hydro

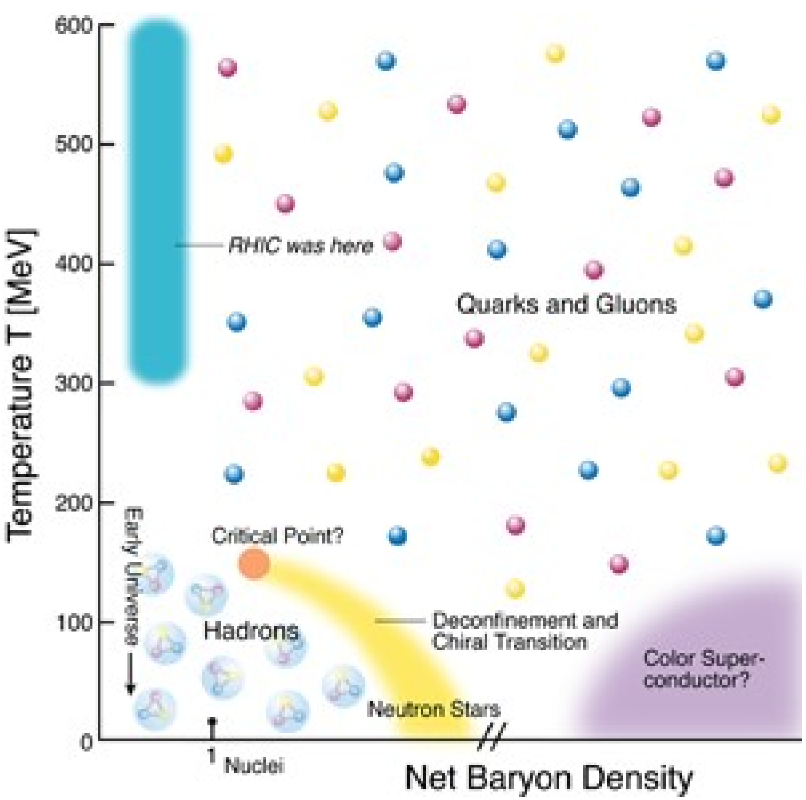
need more information on temperature and
momentum dependence of the vector spectral function

Thermal Dilepton & Photon Rates
Interplay with Hydro-Evolution

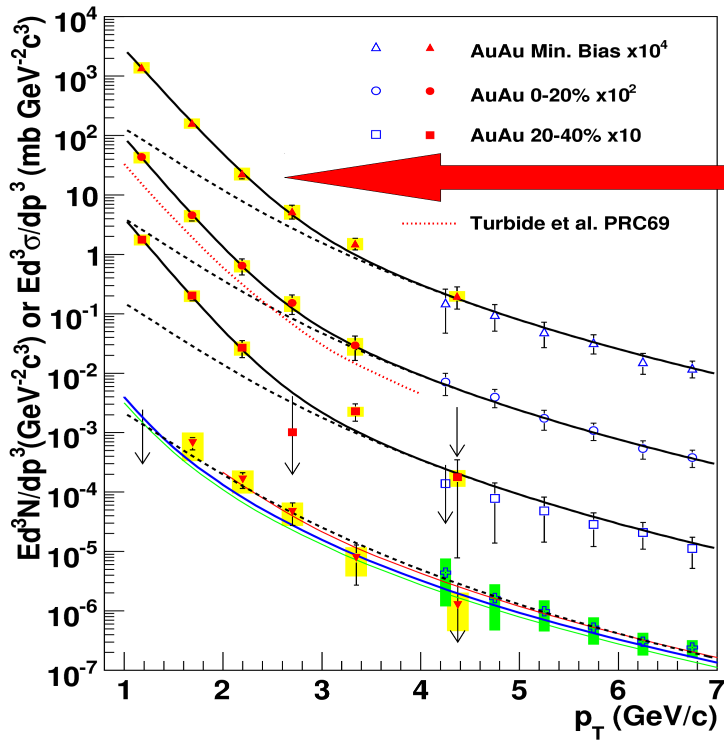
centrality	$dN/dy(p_T > 1\mathrm{GeV}/c)$	$T(\mathrm{MeV})$
0-20%	$\mathbf{1.50 \pm 0.23 \pm 0.35}$	$\mathbf{221 \pm 19 \pm 19}$
20-40%	$\mathbf{0.65 \pm 0.08 \pm 0.15}$	$\mathbf{217 \pm 18 \pm 16}$
Min. Bias	$\mathbf{0.49 \pm 0.05 \pm 0.11}$	$\mathbf{233 \pm 14 \pm 19}$

initial temperature:

$$T_{init} = (300 - 600)\ \mathrm{MeV}$$



C. Gale, Physics 3, 28 (2010)



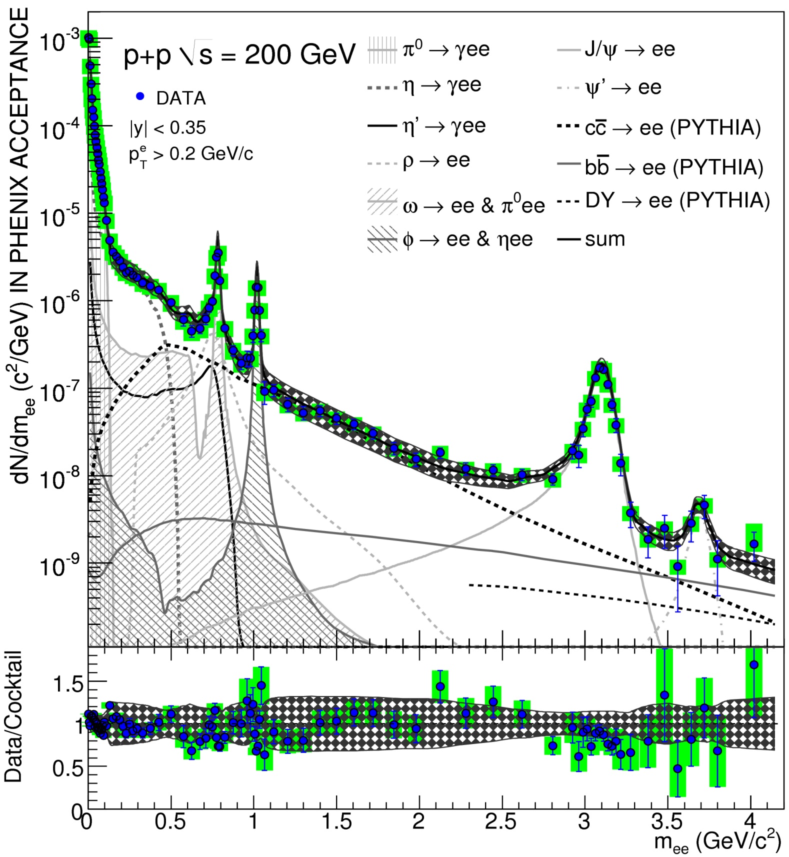
A. Adare et al. (PHENIX Collaboration), Phys. Rev. Lett. 104, 132301 (2010)

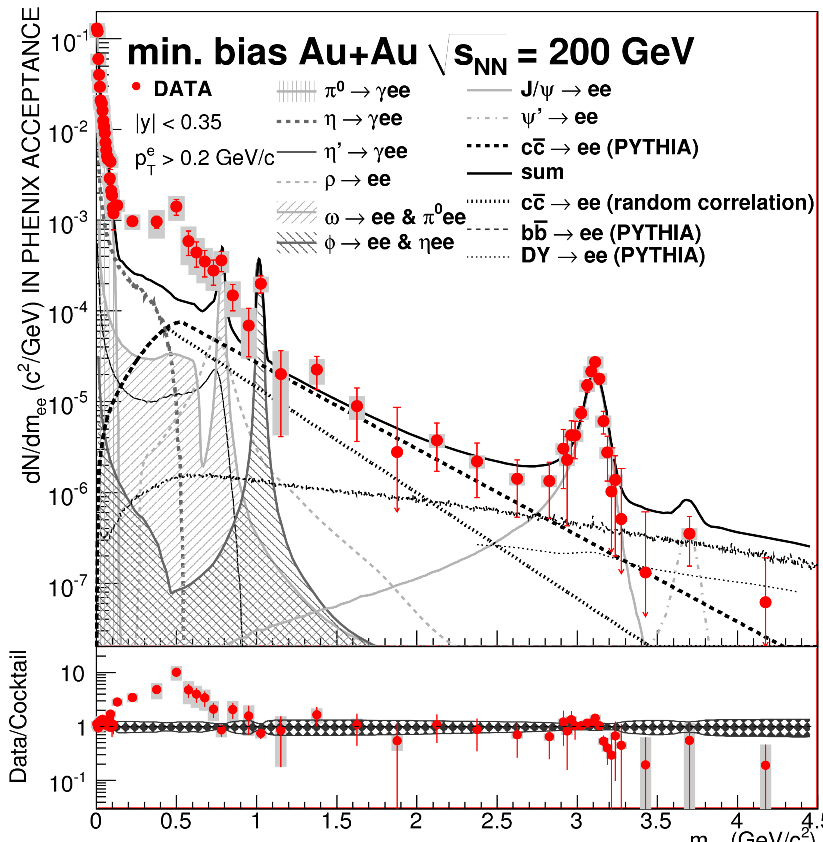
thermal photons?
need to integrate emission rate
over expansion history

Thermal Dilepton & Photon Rates
low mass dilepton enhancement

Thermal dilepton rate: vector spectral function

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$





arXiv:0912.0244v1 [

A. Adare et al. (PHENIX Collaboration), Phys. Rev. C81, 034911 (2010)

Thermal Dilepton & Photon Rates
low mass dilepton enhancement

Thermal dilepton rate:

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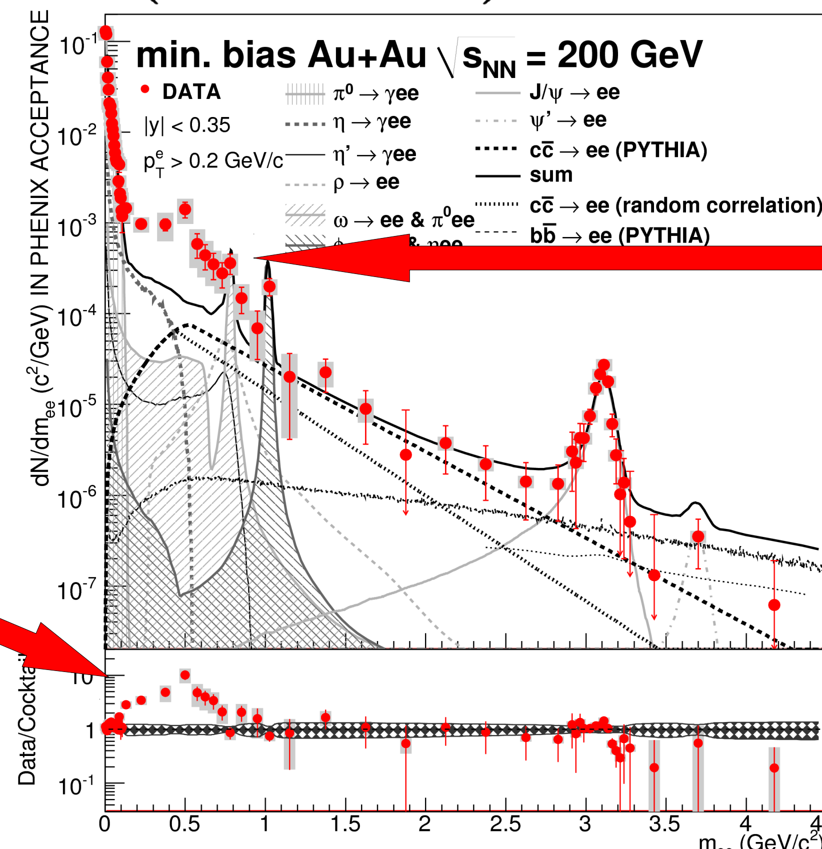
$$\frac{\mathrm{d}W}{\mathrm{d}\omega\mathrm{d}^3p} = \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

thermal dileptons?
need to integrate emission rate
over expansion history



arXiv:0912.0244v1 [

dilepton enhancement
for $\omega \sim 0.5\mathrm{GeV}$

$$T \sim (170 - 300)\ \mathrm{MeV}$$

$$\Rightarrow\ \omega/T \sim (1-3)$$

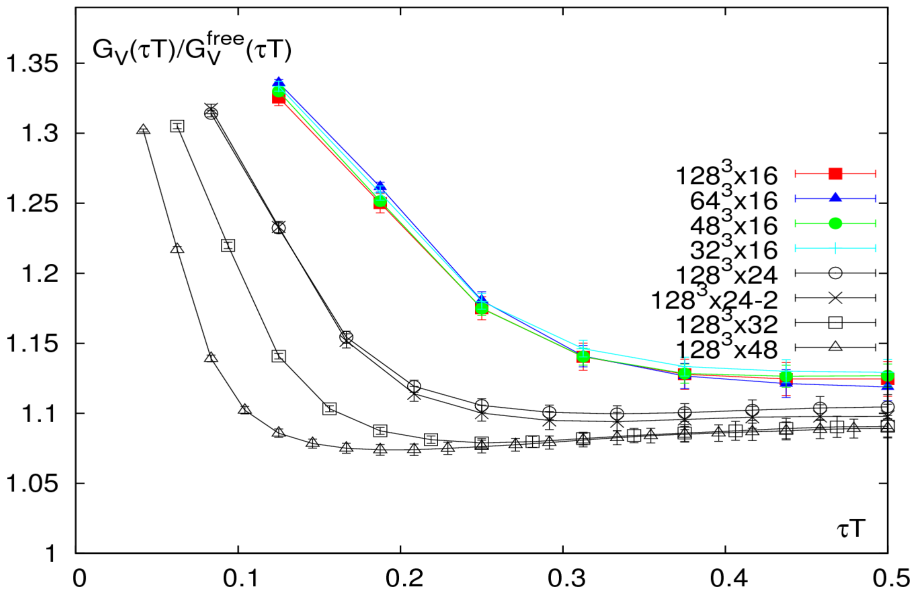
A. Adare et al. (PHENIX Collaboration), Phys. Rev. C81, 034911 (2010)

Vector Meson Spectral Function
Thermal Dilepton

Vector meson correlation function:

$$G_V(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \rho_V(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

$$\vec{p} = 0$$



BNL-Bielefeld, Vector current correlator
in quenched QCD, in preparation

quenched QCD, clover action:

volume dependence

$$N_\sigma^3 \times 16\ ,\ N_\sigma = 48 - 128$$

cut-off dependence

$$128^3 \times N_\tau\ ,\ N_\tau = 16 - 48$$

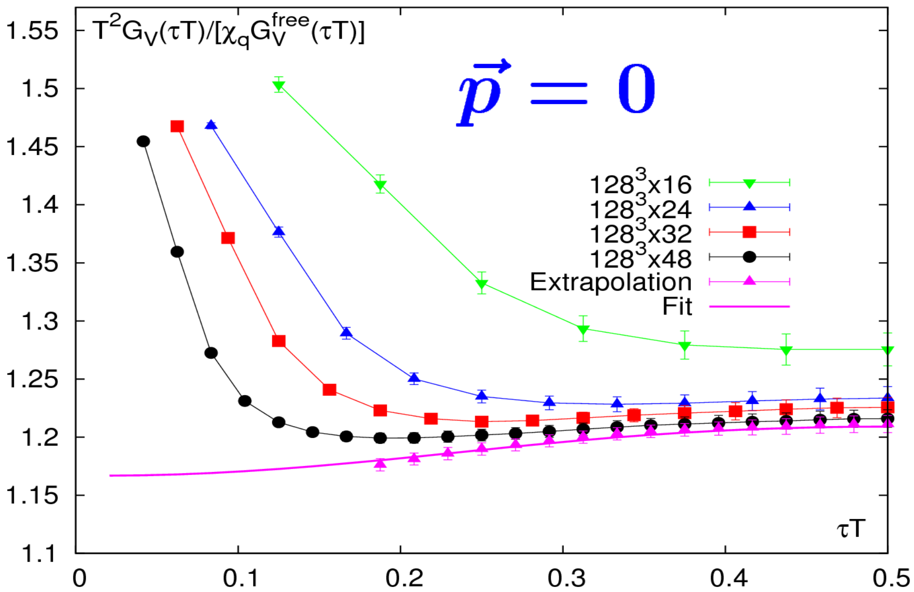
quark mass dependence

$$m_q^{\bar{M}S}/T = 0.02,\ 0.11$$

Vector Meson Spectral Function
Thermal Dilepton

Vector meson correlation function:

$$G_V(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_V(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$



BNL-Bielefeld, Vector current correlator in quenched QCD, in preparation

$$\rho_V(\omega) = -\chi_q \omega \delta(\omega) + \rho_{ii}(\omega)$$

ansatz: BW+continuum

$$\rho_{ii}(\omega,T) = c_{BW} \frac{\omega\Gamma}{\omega^2 + (\Gamma/2)^2}$$

$$(1+k)\frac{3}{2\pi}\omega^2 \tanh(\omega/4T)$$

$$*\Theta(\omega_0,\Delta_\omega)$$

Thermal Dilepton & Photon Rates
Interplay with Hydro-Evolution

Thermal dilepton rate:

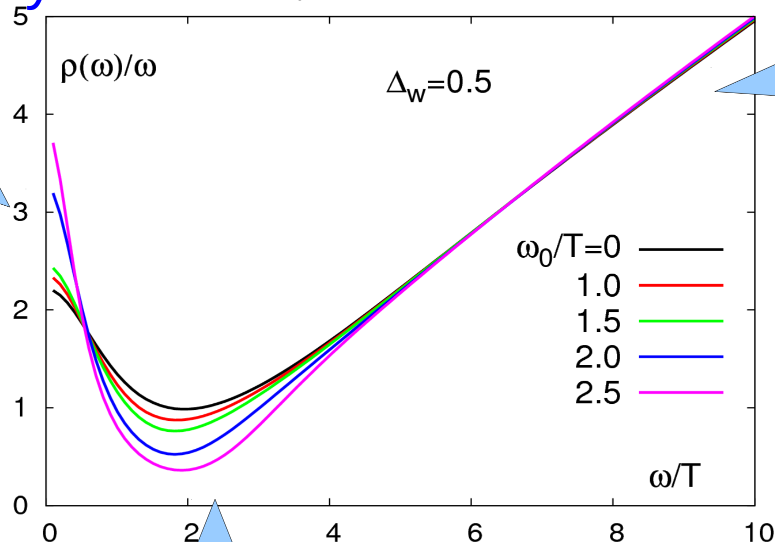
vector spectral function

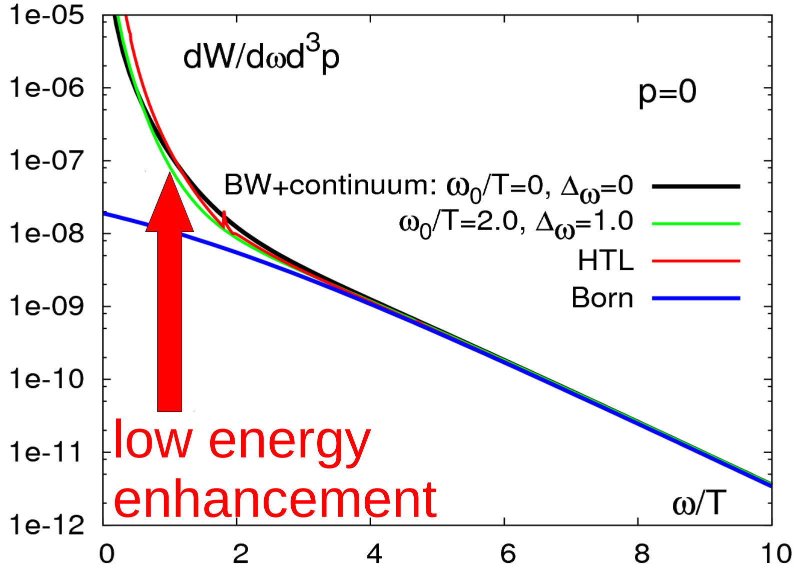
$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5}{9} \frac{\alpha_{em}^2}{6\pi^3} \frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

electrical conductivity

$$\sigma \sim \lim_{\omega\to 0} \rho_{ii}(\omega)/\omega$$

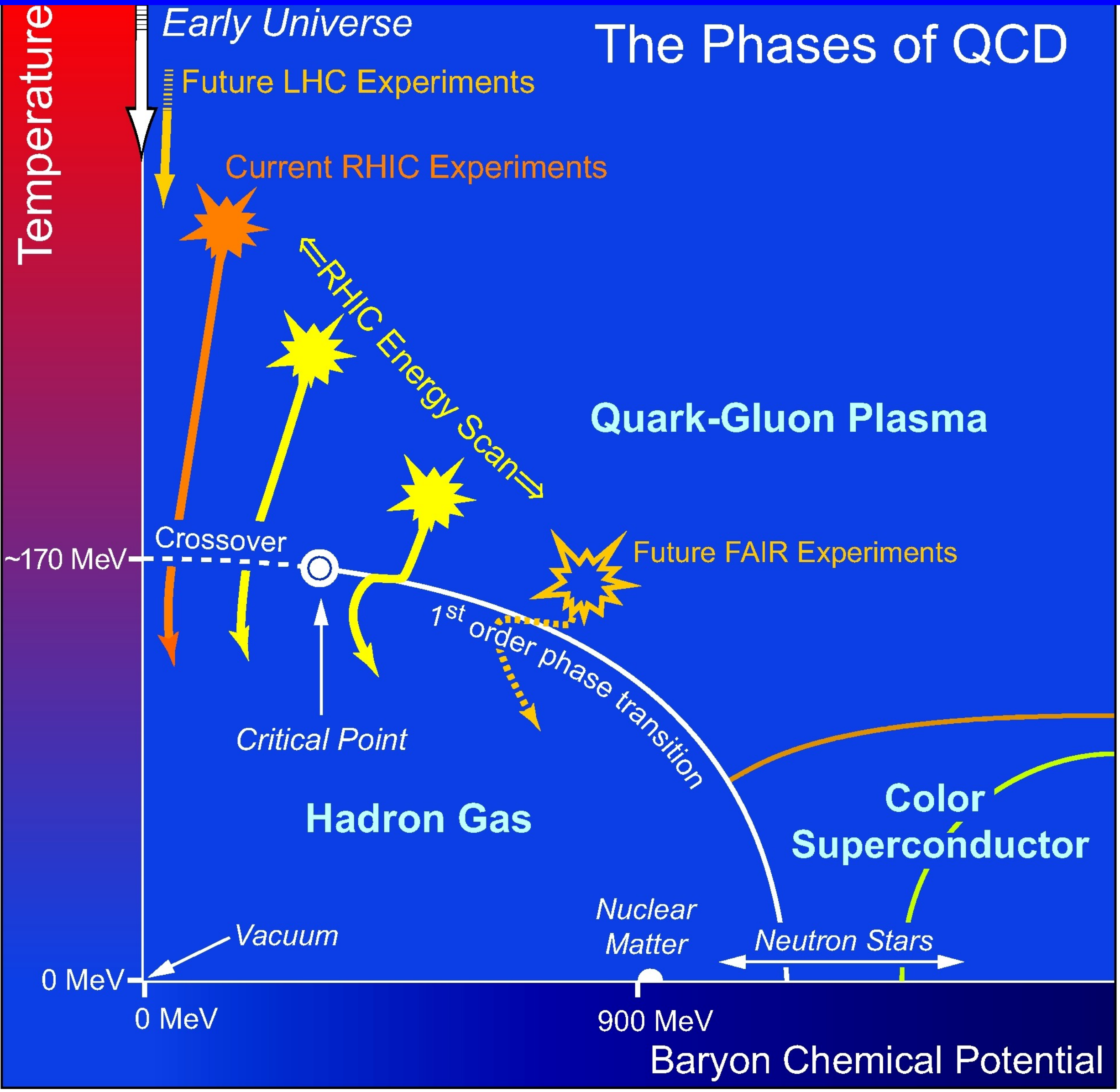
HTL resummation, perturbation theory





Breit-Wigner ansatz

QCD phase diagram
(close to the chiral limit)



RHIC low energy runs:

$$\sqrt{s} = (9 - 200)\mathrm{GeV}/A$$

- charge fluctuations along the freeze-out line
- higher moments of charge fluctuations, e.g.

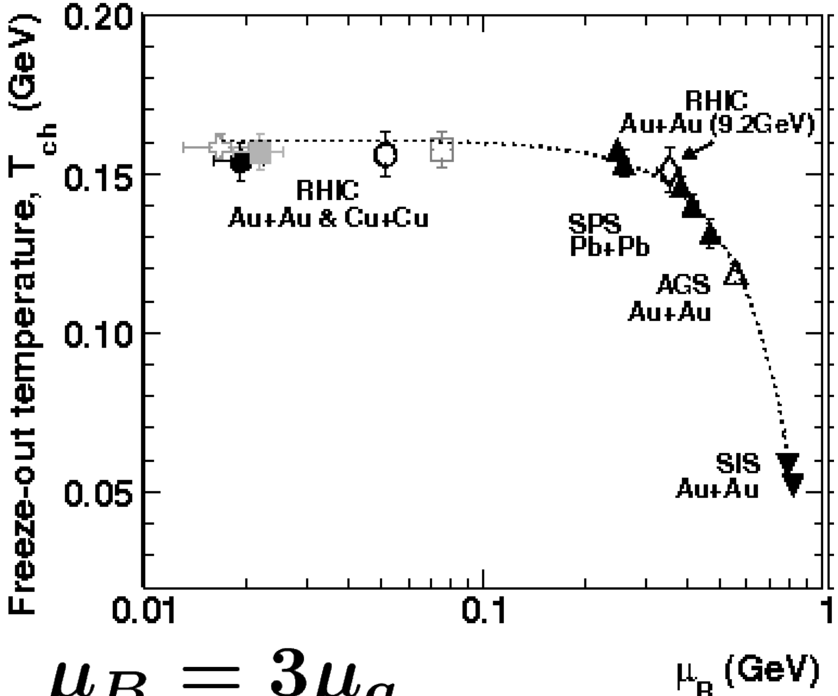
  Skewness

$$S_q \equiv \langle (\delta N_q)^3 \rangle / \sigma_q^3$$

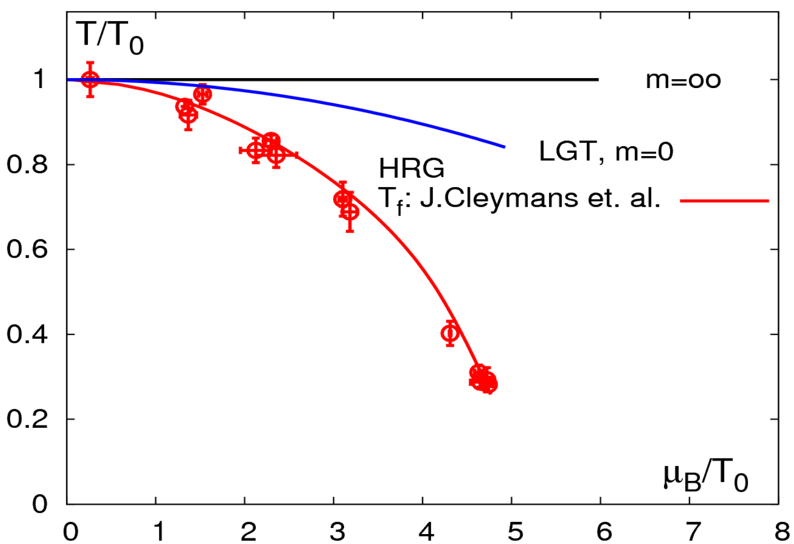
  Kurtosis

$$\kappa_q \equiv \langle (\delta N_q)^4 \rangle / \sigma_q^4 - 3$$

The RHIC low energy runs

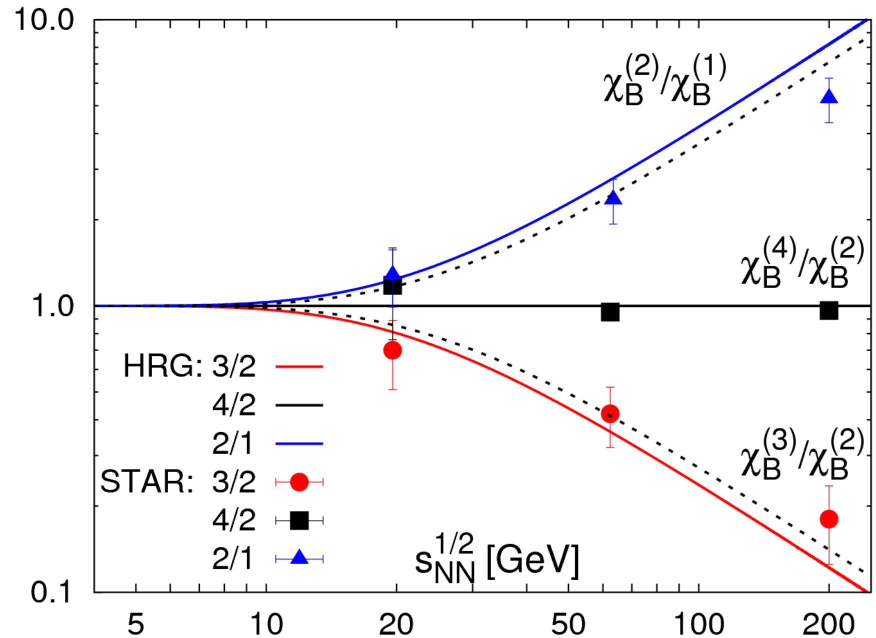


$$\mu_B = 3\mu_q$$



Moments of charge fluctuations

talk by C. Schmidt



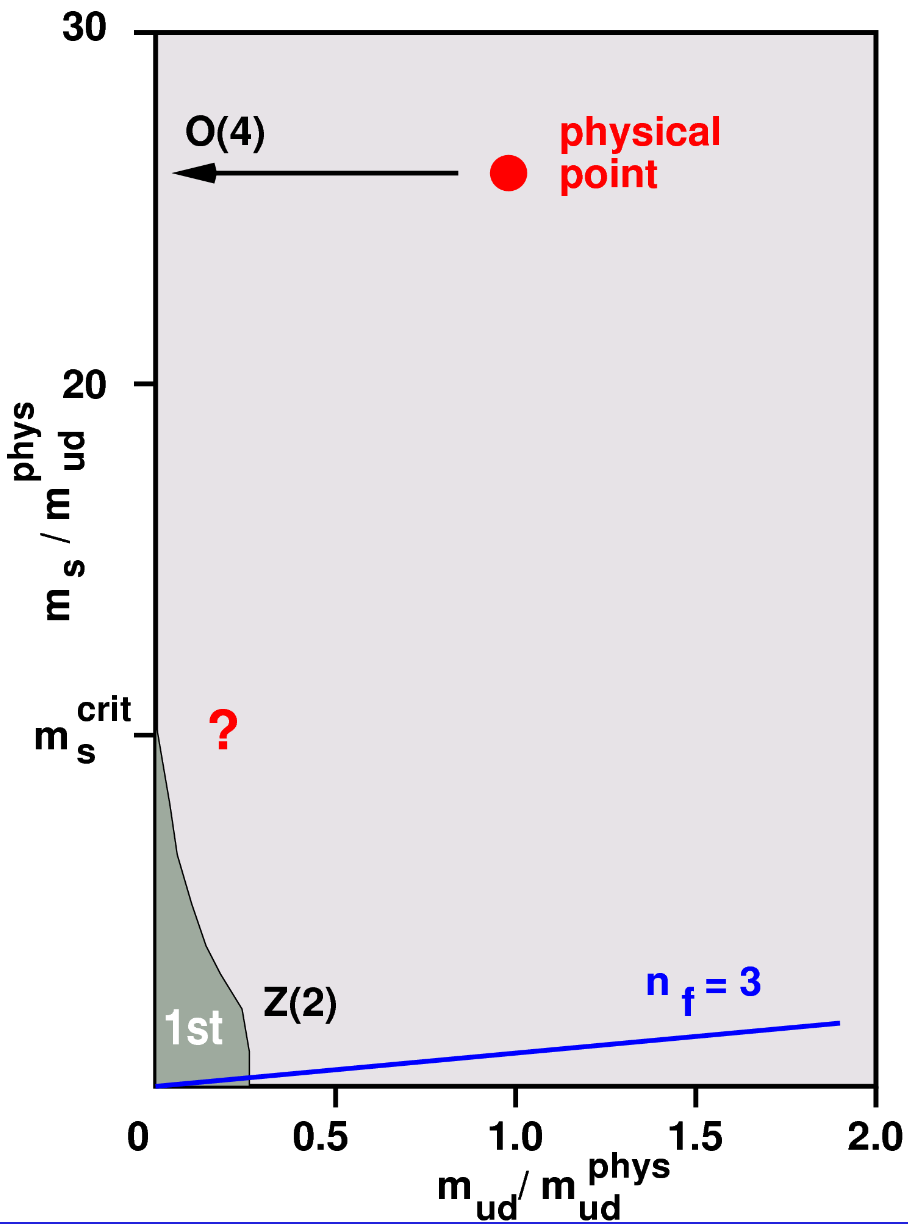
FK, K Redlich, arXiv:1007.2581
data from STAR: arXiv:1004.4959

charge fluctuations at freeze-out agree with HRG model predictions

freeze-out line and chiral phase transition

BNL-Bielefeld-GSI, in preparation

Phase diagram for $\mu_B = 0$



- drawn to scale

Is physics at the physical quark mass point sensitive to (universal) properties of the chiral phase transition?

physical point may be above $\mathrm{m_s^{crit}}$

$$n_f = 3: \ \mathrm{m_s^{crit}} \lesssim 70\ \mathrm{MeV}$$

first order region starts below physical pion mass value

O(N) scaling and the chiral transition

- thermodynamics in the vicinity of a critical point:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T) = h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) + f_r(V,T)$$

with scaling fields $\quad t \equiv \frac{1}{t_0}\left(\frac{T}{T_c} - 1\right) \quad , \quad h \equiv \frac{1}{h_0}\frac{m_l}{m_s}$

- In the vicinity of (t,h)=(0,0) the chiral order parameter and its susceptibility are given in terms of scaling functions

$$M = h^{1/\delta} f_G(z) \quad , \quad \chi_M = \partial M/\partial h = h^{1/\delta - 1} f_\chi(z)$$

$$\chi_t = \partial M/\partial T = \frac{1}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'(z)$$

$$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z)\right)$$ known from 3d O(N) spin model

Scaling analysis in (2+1)-flavor QCD

RBC-Bielefeld-GSI arXiv:0909.5122; hotQCD in preparation

QCD with 2 light and a "physical" strange quark mass:

- improved staggered fermions; most detailed: p4-action
  extended to asqtad and hisq

- calculations have been performed on $N_\sigma^3 \times N_\tau$ lattices

$$N_\sigma = 32,\ 48\ ,\ N_\tau = 4,\ 6,\ 8$$

- calculations with p4-action cover a wide quark mass range:

$$N_\tau = 4: \quad 1/80 \le m_l/m_s \le 2/5$$

$$\Rightarrow \quad 75\ \mathrm{MeV} \le m_\pi \le 320\ \mathrm{MeV}$$

→ evidence for O(N) scaling

expect O(2) rather than O(4) scaling with staggered fermions at non-zero lattice spacing

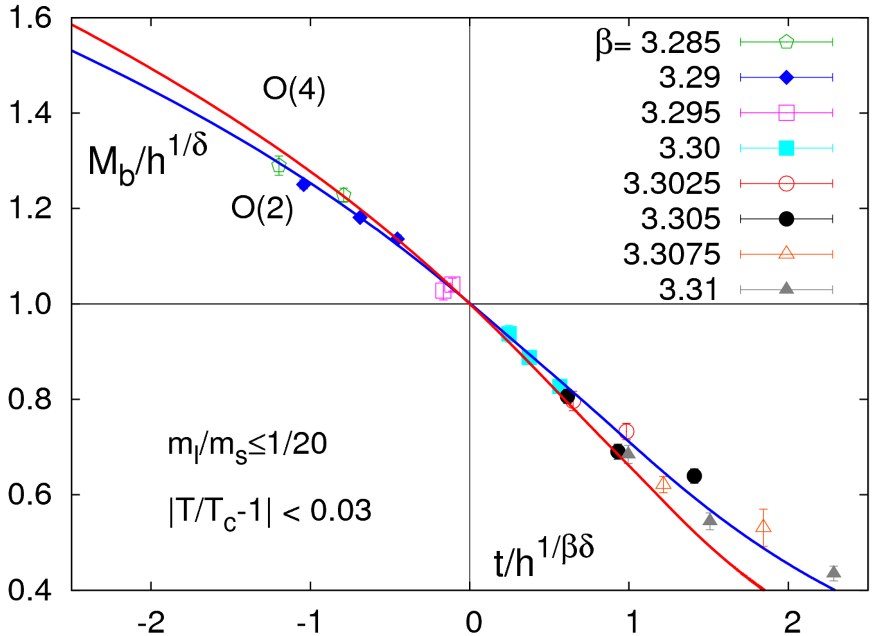
Magnetic Equation of State
(2+1)-flavor QCD

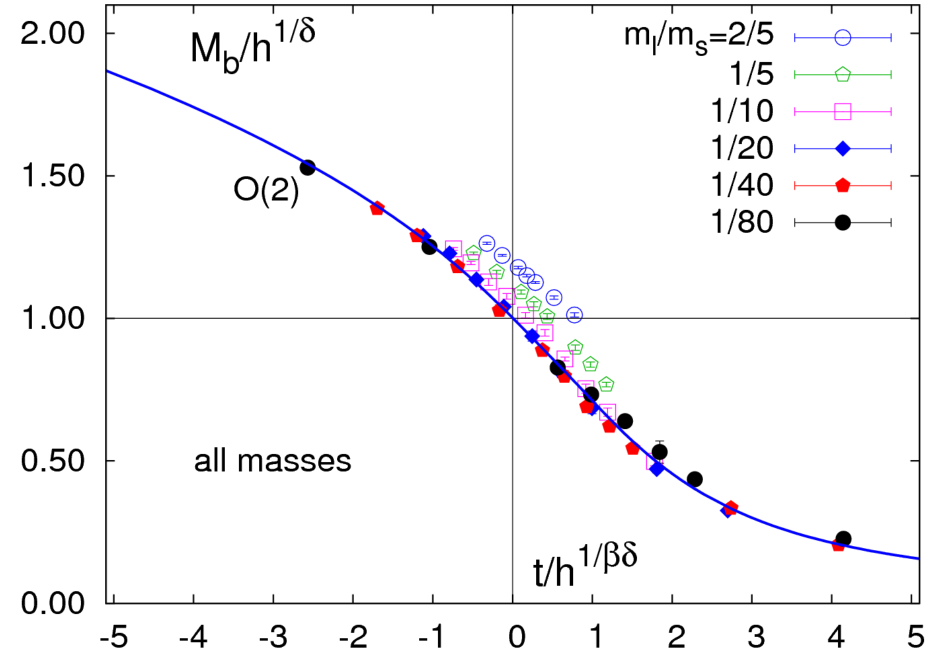
chiral order parameter:

$$M_b \equiv \frac{m_s \langle \bar{\psi}\psi \rangle}{T^4} \quad z \equiv t/h^{1/\beta\delta}$$

O(2) vs. O(4)

z → 1.2z





p4-action: $N_\sigma^3 \times 4$, $N_\sigma = 16,\ 32$

S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

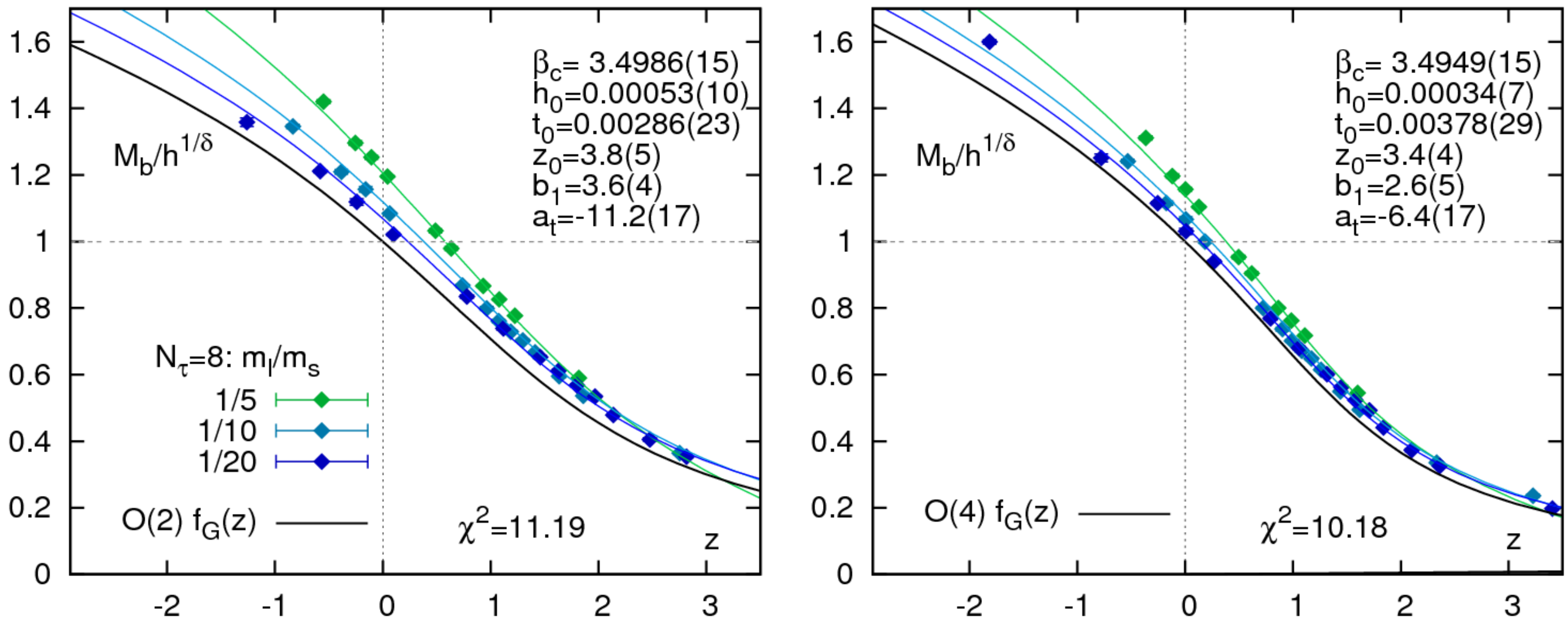
Scaling analysis in (2+1)-flavor QCD

O(2) vs O(4) fits

$N_\tau = 8$

$$M_b \equiv \frac{m_s \langle \bar\psi \psi \rangle}{T^4} \qquad z \equiv t/h^{1/\beta\delta}$$

hotQCD preliminary



$$\Rightarrow \quad T_c,\ t_0,\ h_0 \rightarrow z_0 = h_0^{1/\beta\delta}/t_0$$

W. Unger, PhD thesis, Bielefeld, September 2010

chiral phase transition

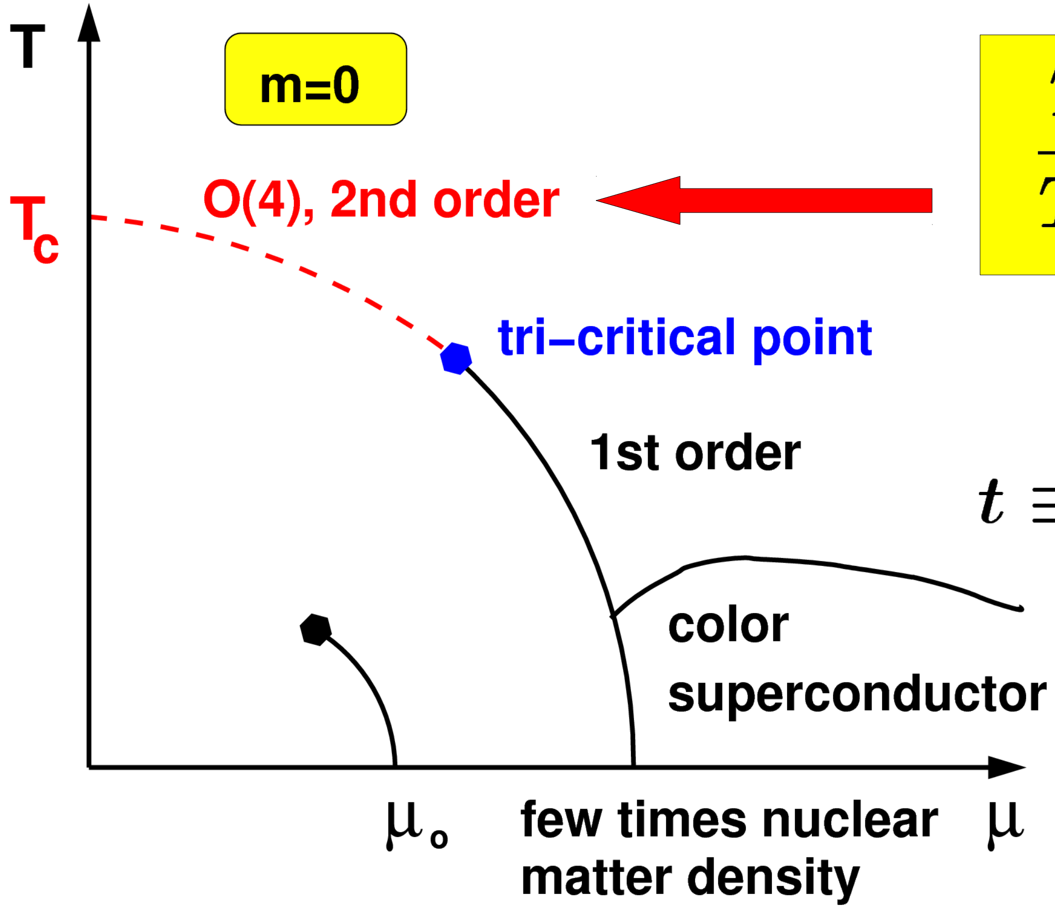
controls $T_c(m_l/m_s)$

The curvature of the critical line

BNL-Bielefeld-GSI in preparation

- QCD, chiral limit (u,d quarks only)

$$\mu_u = \mu_d > 0,\ \mu_Q = \mu_s = 0$$



$$\frac{T}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2$$

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2\right)$$

scaling laws control curvature of chiral transition line for small μ_q/T

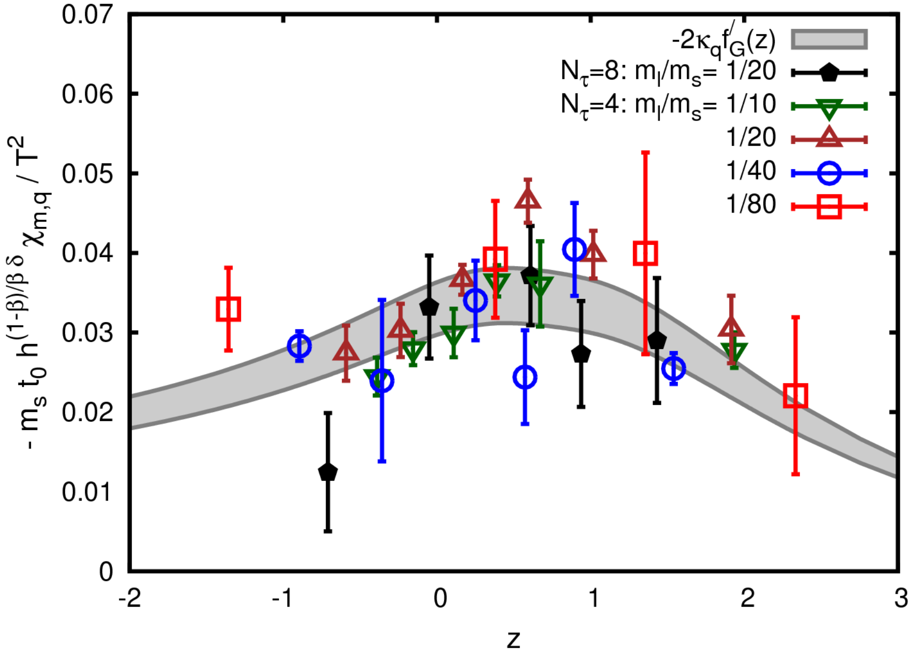
The curvature of the critical line

BNL-Bielefeld-GSI in preparation

- "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right)-\kappa_q\left(\frac{\mu_q}{T}\right)^2\right)\ ,\ z = t/h^{1/\beta\delta}$$

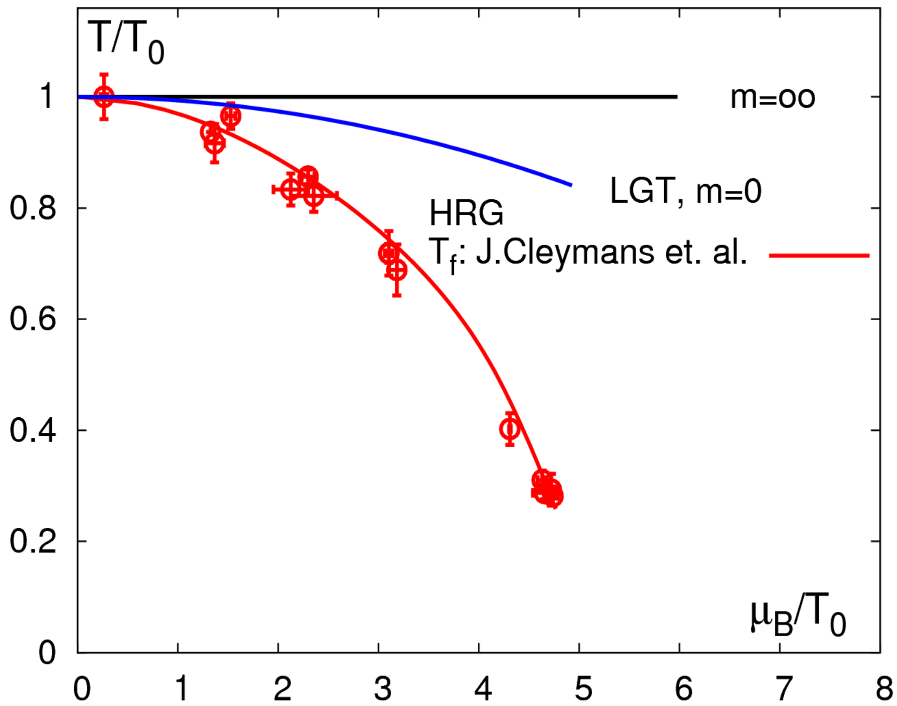
$$M_b = h^{1/\delta} f_G(z)\ ,\ \chi_t = \partial^2 M_b/\partial(\mu_q/T)^2 = \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'(z)$$



$$\kappa_q = 0.059 \pm 0.006$$

compare to freeze-out curve:

Chiral Transition and Freeze-out



chiral phase transition curve:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

open issues:
- continuum limit
- strangeness conservation
- non zero charge

freeze-out curve in heavy ion collisions:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - c\left(\frac{\mu_B}{T}\right)^4$$

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

J. Cleymans et al.,
Phys.Rev. C73, 034905 (2006)

Chiral Phase Transition and Pseudo-critical temperature

- thermodynamics in the vicinity of a critical point:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T) = h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) + f_r(V,T)$$

- critical behavior controlled by two relevant fields: t , h
  - all couplings that do not explicitly break chiral symmetry contribute in leading order only to 't'

$$t = \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2 - \kappa_s \left(\frac{\mu_s}{T}\right)^2 - \kappa_{qs} \left(\frac{\mu_q \mu_s}{T}\right)^2\right)$$

$$h = \frac{1}{h_0} \frac{m_l}{m_s}$$

Pseudo-critical temperature

- select observables that are sensitive to critical behavior in the chiral limit

  (magnetic) response functions:

- chiral susceptibility: $\langle\bar\psi\psi\rangle_l = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$

$$\chi_{m,l} = \chi_{l,disc} + 2\chi_{l,con}$$

$$\frac{\chi_{m,l}}{T^2} = 2\frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1}f_\chi(z) + a_t\Delta T + b_1\right)$$

$$\frac{\chi_{m,l}}{T^2} \sim m_l^{-1/2} \;\text{ for }\; T \le T_c$$

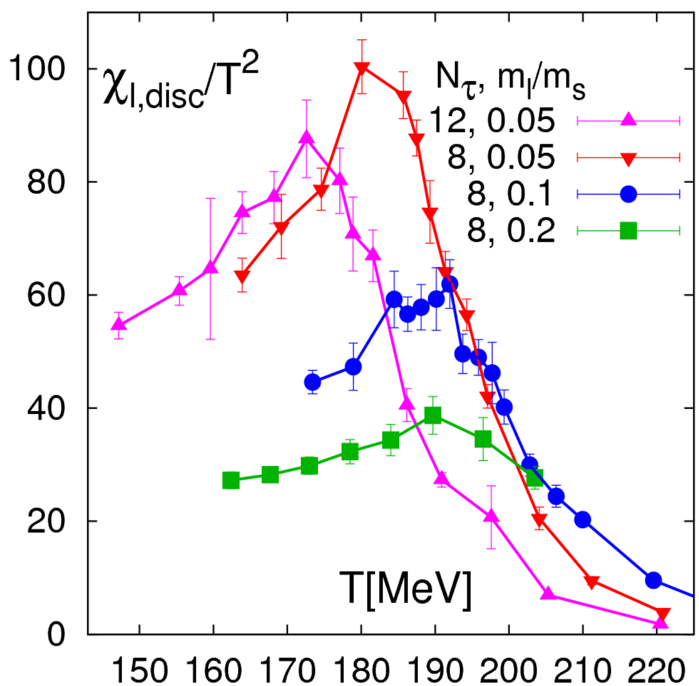
$$\frac{\chi_{m,l}}{T^2} \sim m_l^{1/\delta-1} \;\text{ for }\; T = T_c \;\text{ or }\; T = T_{pc}$$

Chiral susceptibility
hotQCD preliminary

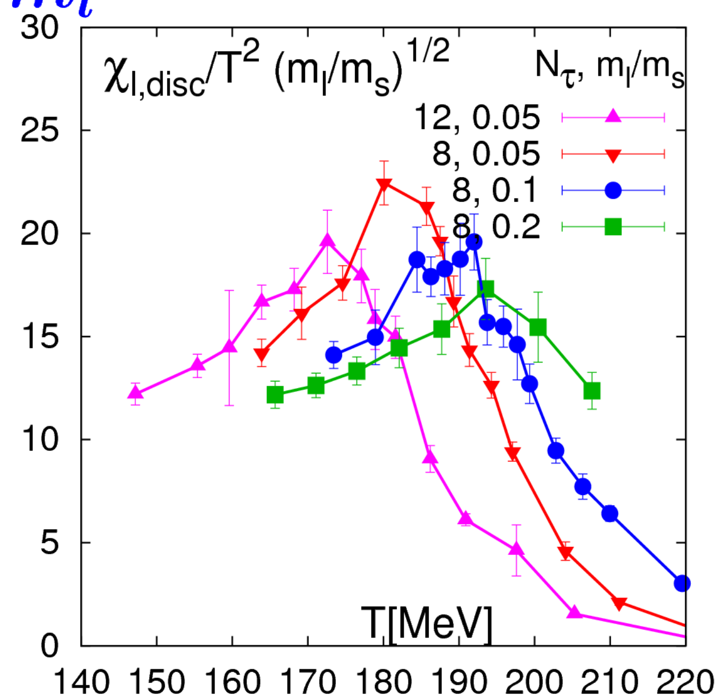
- select observables that are sensitive to critical behavior in the chiral limit

(magnetic) response functions:

- chiral susceptibility: $\langle \bar{\psi}\psi \rangle_l = \frac{T}{V} \frac{\partial \ln Z}{\partial m_l}$

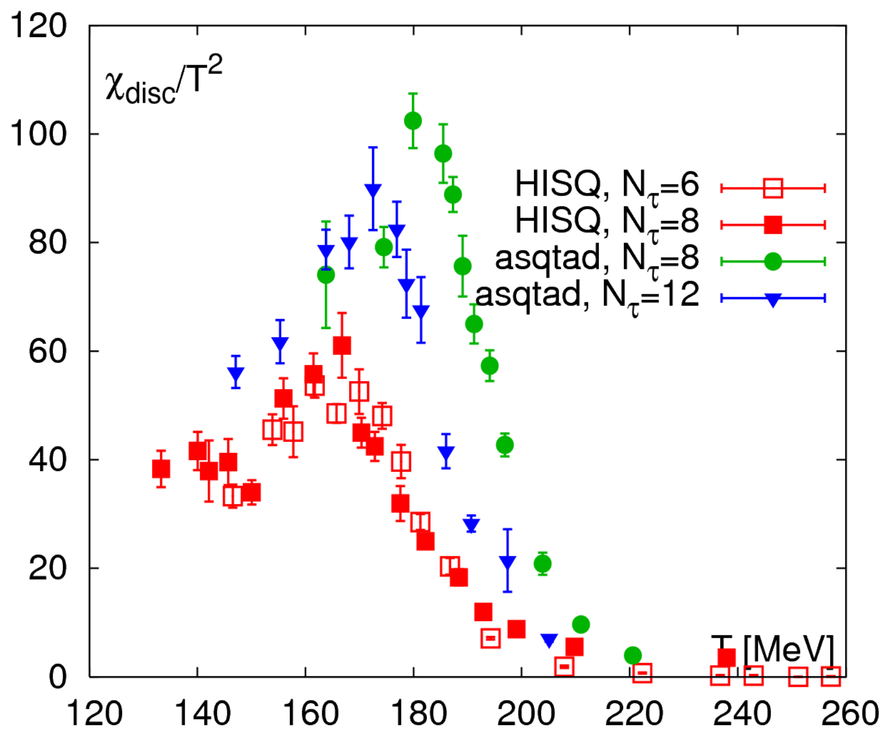


asqtad



Chiral susceptibility
hotQCD preliminary

- closer to the continuum limit with the asqtad staggered fermion action:
- reduced taste symmetry violation with the highly improved staggered action (hisq):



(2+1)-flavor QCD:

$$m_l/m_s = 1/27$$

$$T_{pc} = (164 \pm 6)\ \mathrm{MeV}$$

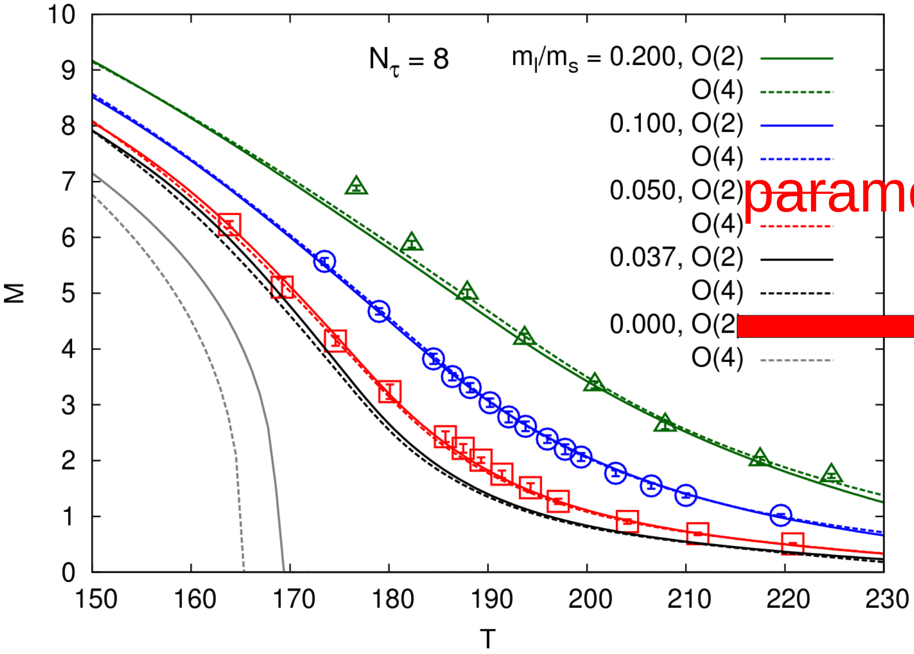
hotQCD preliminary
(Lattice 2010)

Chiral Phase Transition and Pseudo-critical temperature

- scaling ansatz can be used to extract transition temperatures from chiral condensates;

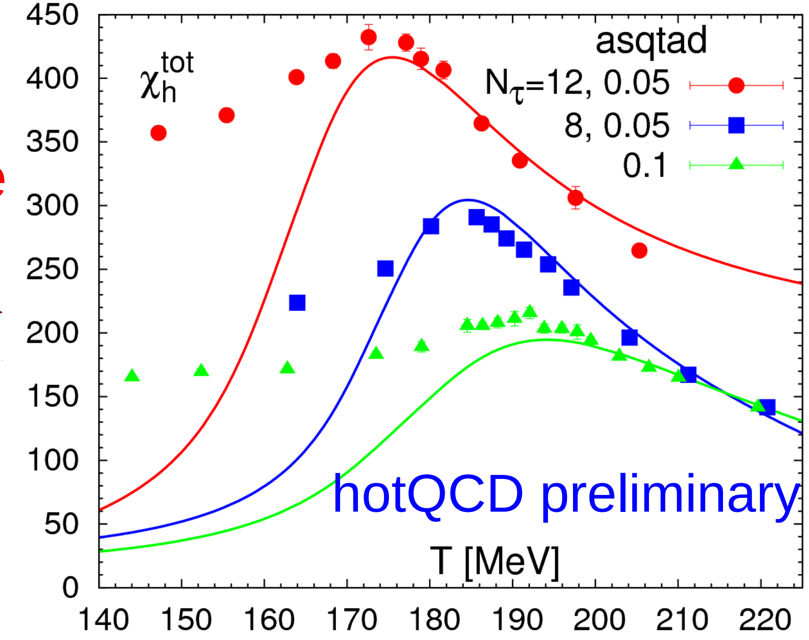
asqtad action

chiral condensate



parameter free

chiral susceptibility



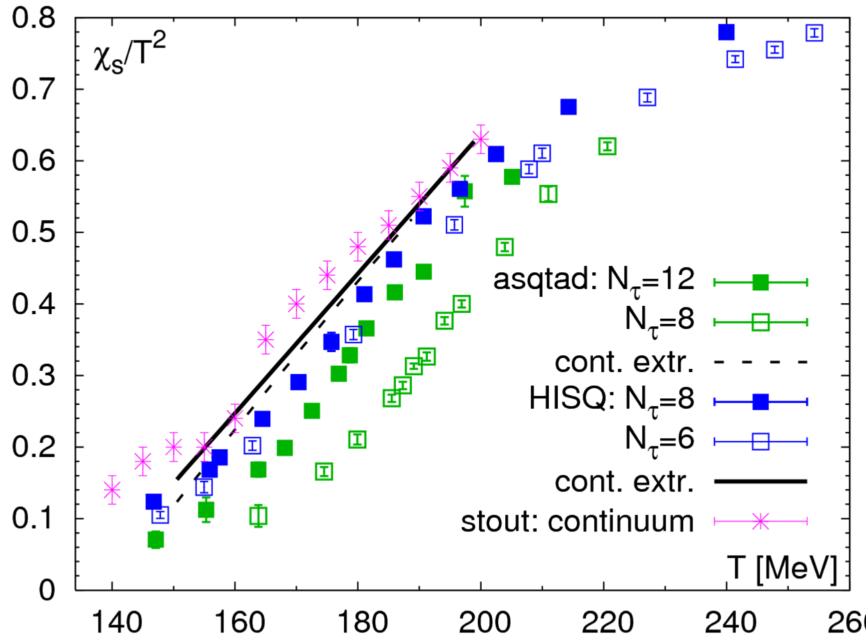
consistent with Tc determination from susceptibility peaks

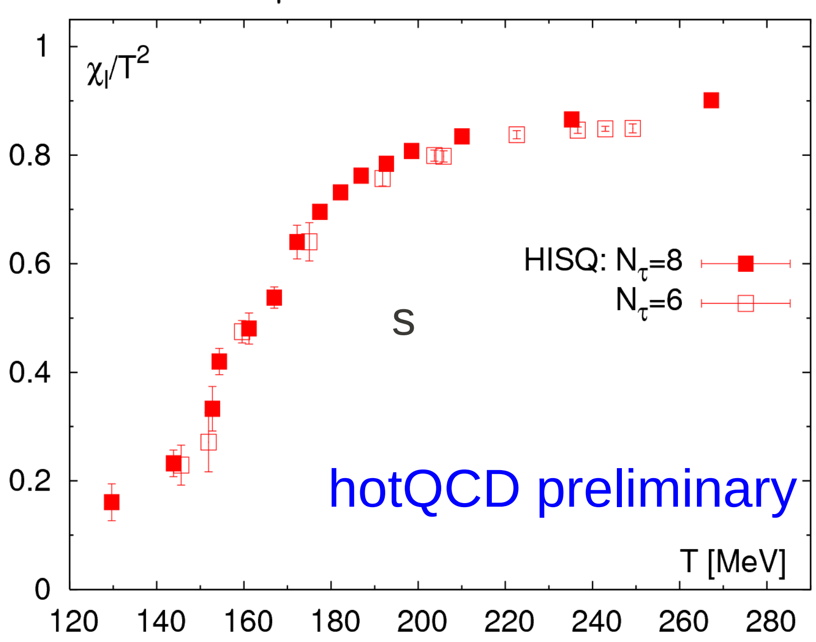
Quark number susceptibility
hotQCD preliminary

- not a response function, dominated by regular contributions

$$\frac{\chi_q}{T^2} = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_q/T)^2} \; , \quad q = l,\; s \qquad (T_c \equiv T_c(m_l = 0))$$

$$\frac{\chi_q}{T^2} \sim c_0 + c_1(T - T_c) + A_\pm \left| \frac{T - T_c}{T_c} \right|^{1-\alpha} \; , \quad \alpha < 0$$



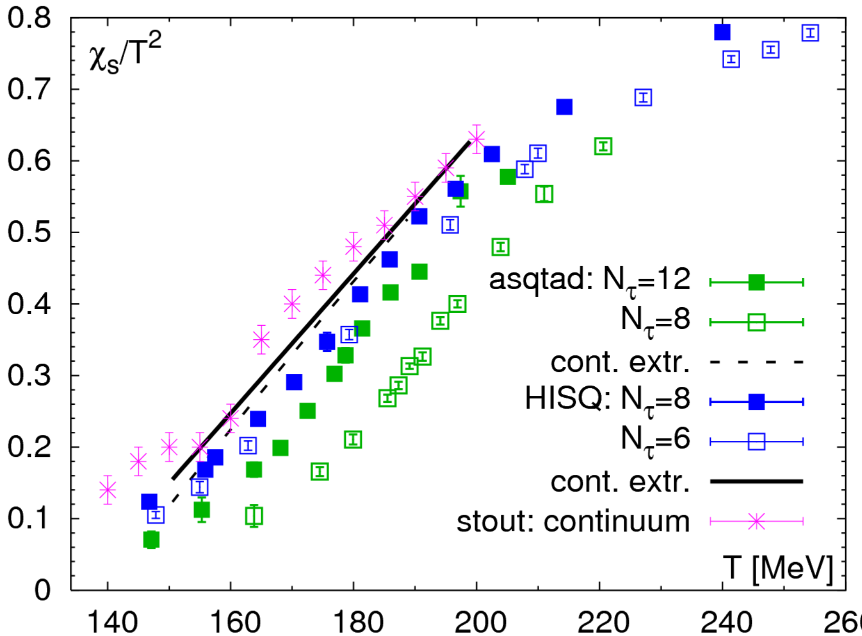


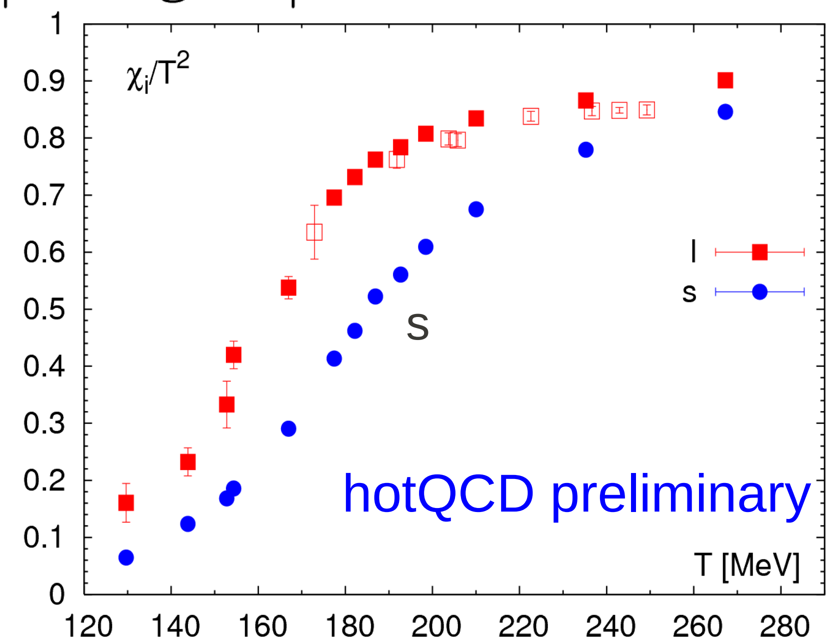
Quark number susceptibility
hotQCD preliminary

- not a response function, dominated by regular contributions

$$\frac{\chi_q}{T^2} = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_q/T)^2}\ , \quad q = l,\ s \qquad (T_c \equiv T_c(m_l = 0))$$

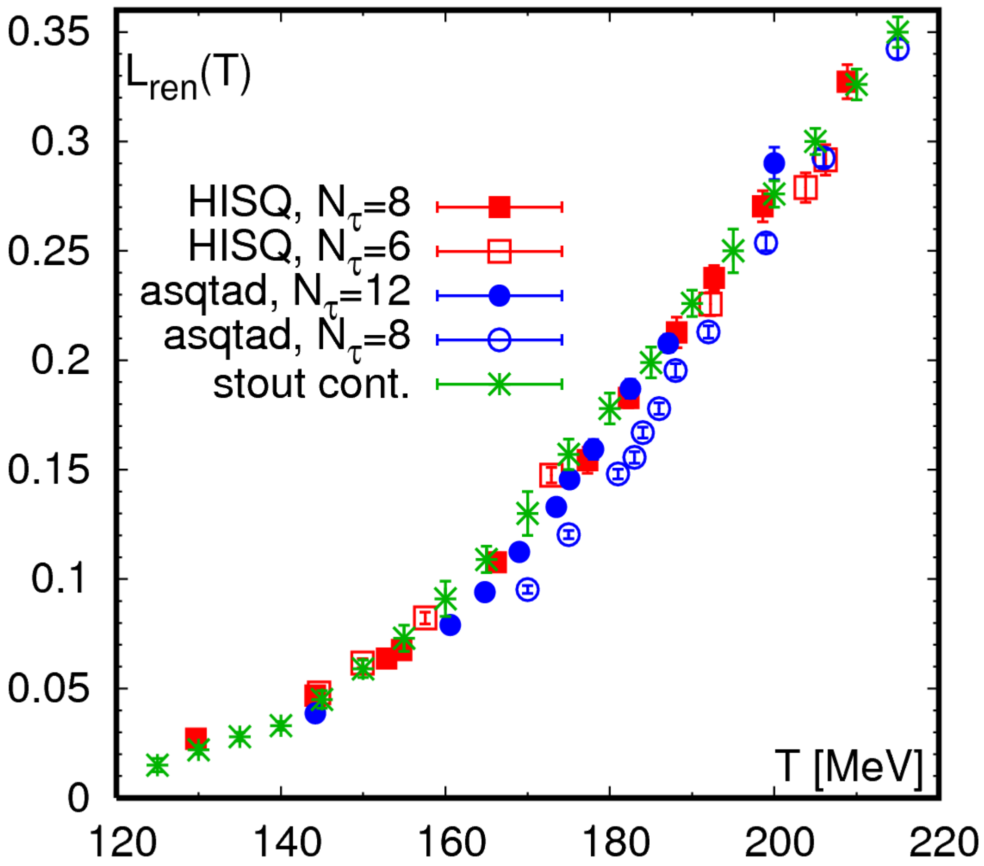
$$\frac{\chi_q}{T^2} \sim c_0 + c_1(T - T_c) + A_\pm \left|\frac{T - T_c}{T_c}\right|^{1-\alpha}\ , \quad \alpha < 0$$





Polyakov Loop Susceptibility

- not a response function, no direct relation to critical behavior



hotQCD preliminary

Conclusions

- LGT calculations start to produce quantitative predictions on QCD thermodynamics that provide input to the interpretation of heavy ion experiments

  EoS, Tc, transport coefficients, spectral functions,
  phase boundary, charge fluctuations,.....

- How sensitive is the QCD transition with physical quark masses to universal properties at the chiral phase transition?
- How does the phase transition vary with chemical potentials?

- Use staggered fermion action with reduced taste violation (HISQ) with physical quark masses close to the cont. limit
- Use chiral fermion formulations (DWF) in thermodynamic calculations