Axion/Hidden-Photon Dark Matter

Conversion into

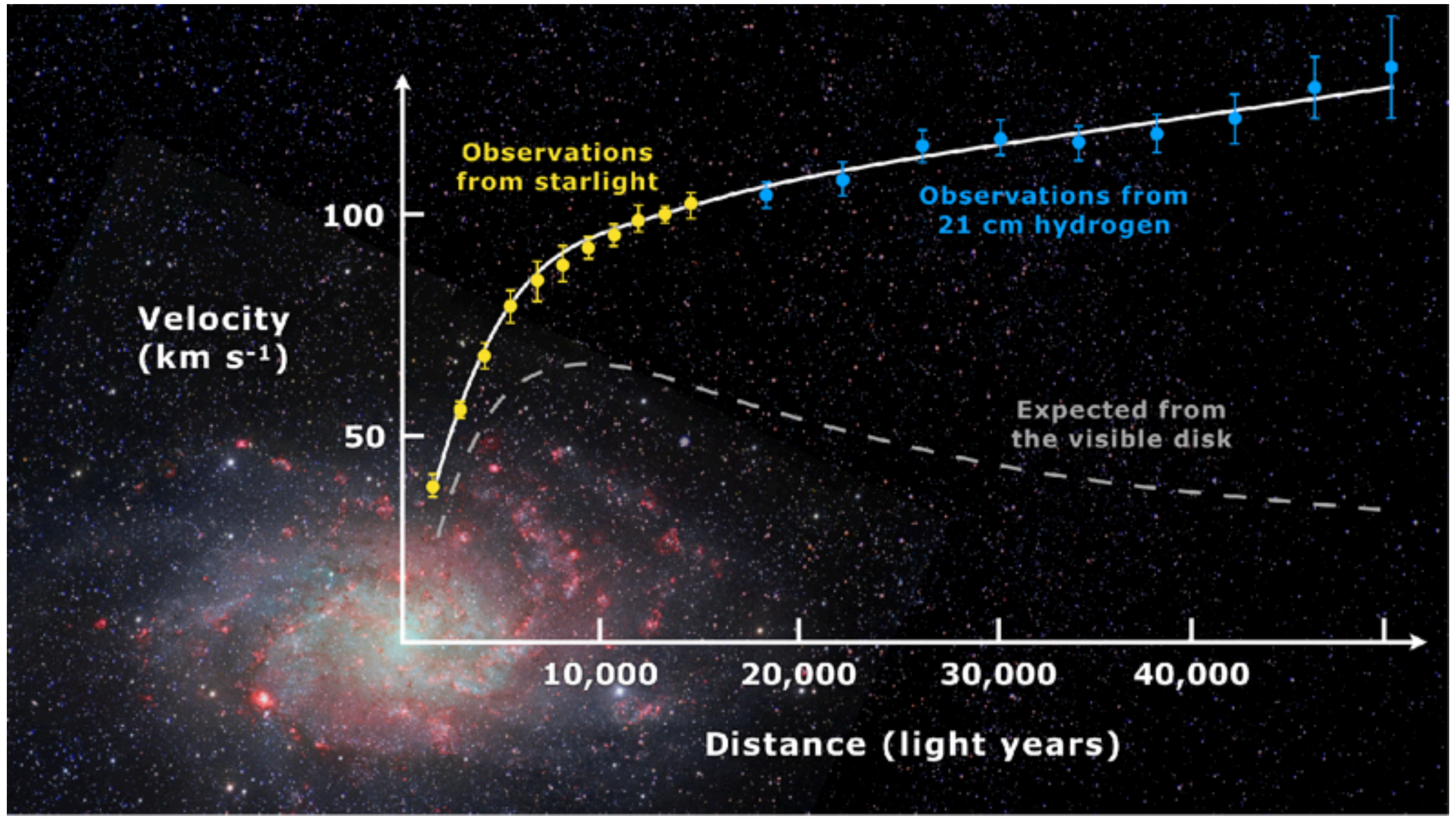
Condensed Matter Axion

~ searching for $\mathcal{O}(1)\,\mathrm{meV}$ axion(ALPs), hidden photon DM

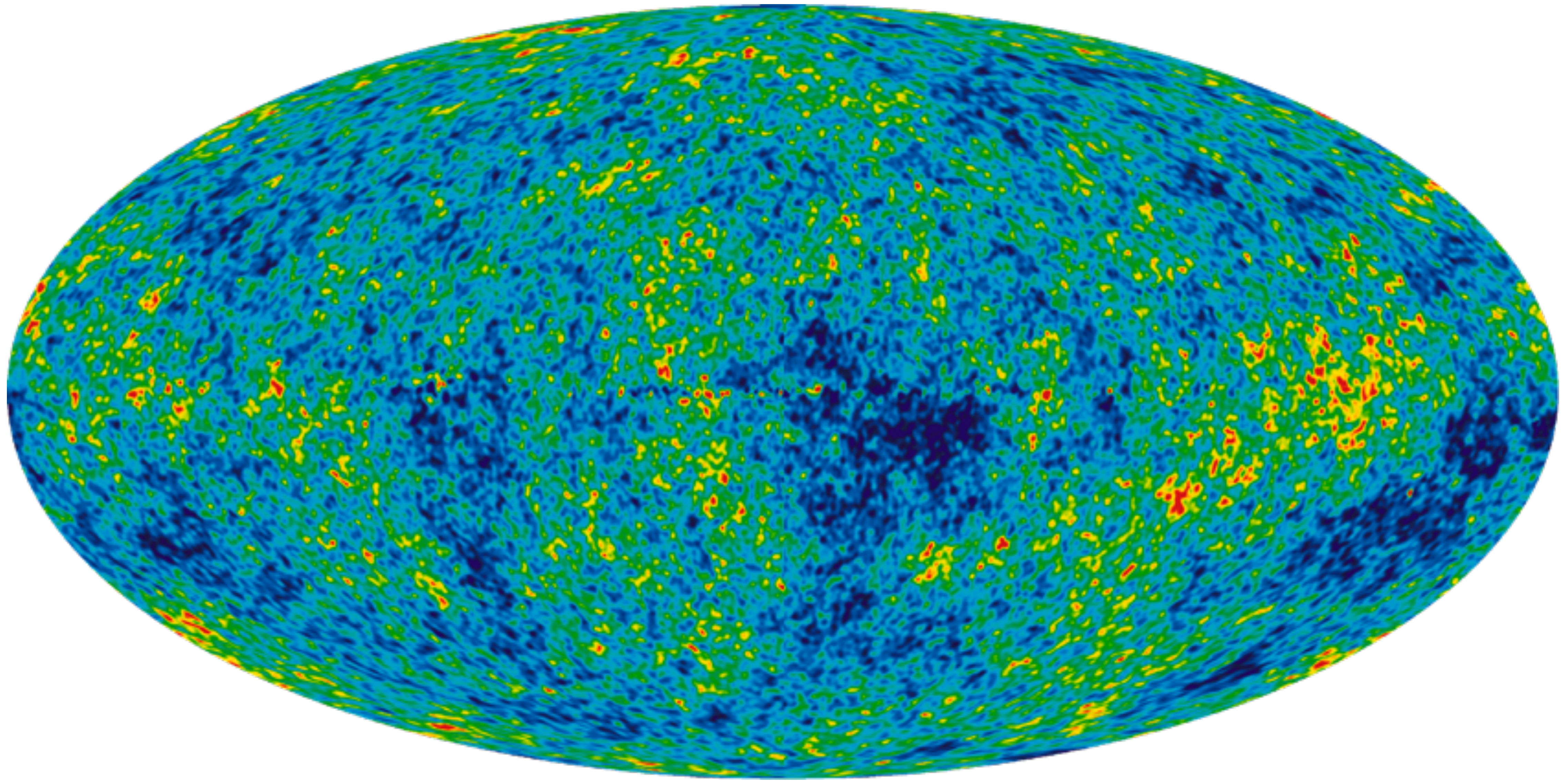
S. Chigusa, K. Nakayama, T. Moroi, arXiv: 2102.06179

2022/2/8 So Chigusa (LBNL/UC Berkeley) @ KEK IPNS-IMSS-QUP Joint workshop

Dark Matter as a hint of new physics



Wikipedia "Galaxy rotation curve", E. Corbelli, P. Salucci (2000)



Wikipedia "Cosmic microwave background", 9 years of WMAP data

"Known"

✓ DM existence, abundance
✓ Has gravitational interaction

"Unkown"

✓ DM mass
✓ Non-gravitational interactions

Various DM models

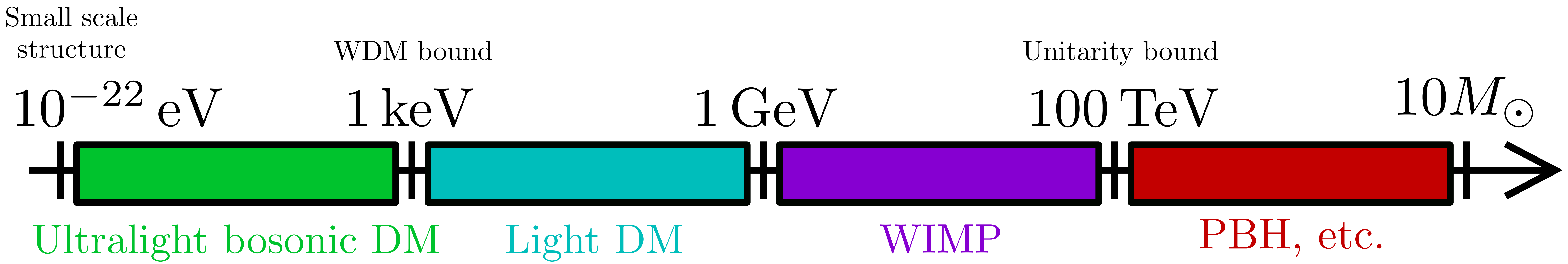
✓ Common features

- Abundance: $\rho_\chi \sim 0.3\,\mathrm{GeV/cm^3}$ around us
- Velocity: $v_\chi \sim 10^{-3}c$

✓ Model dependent features

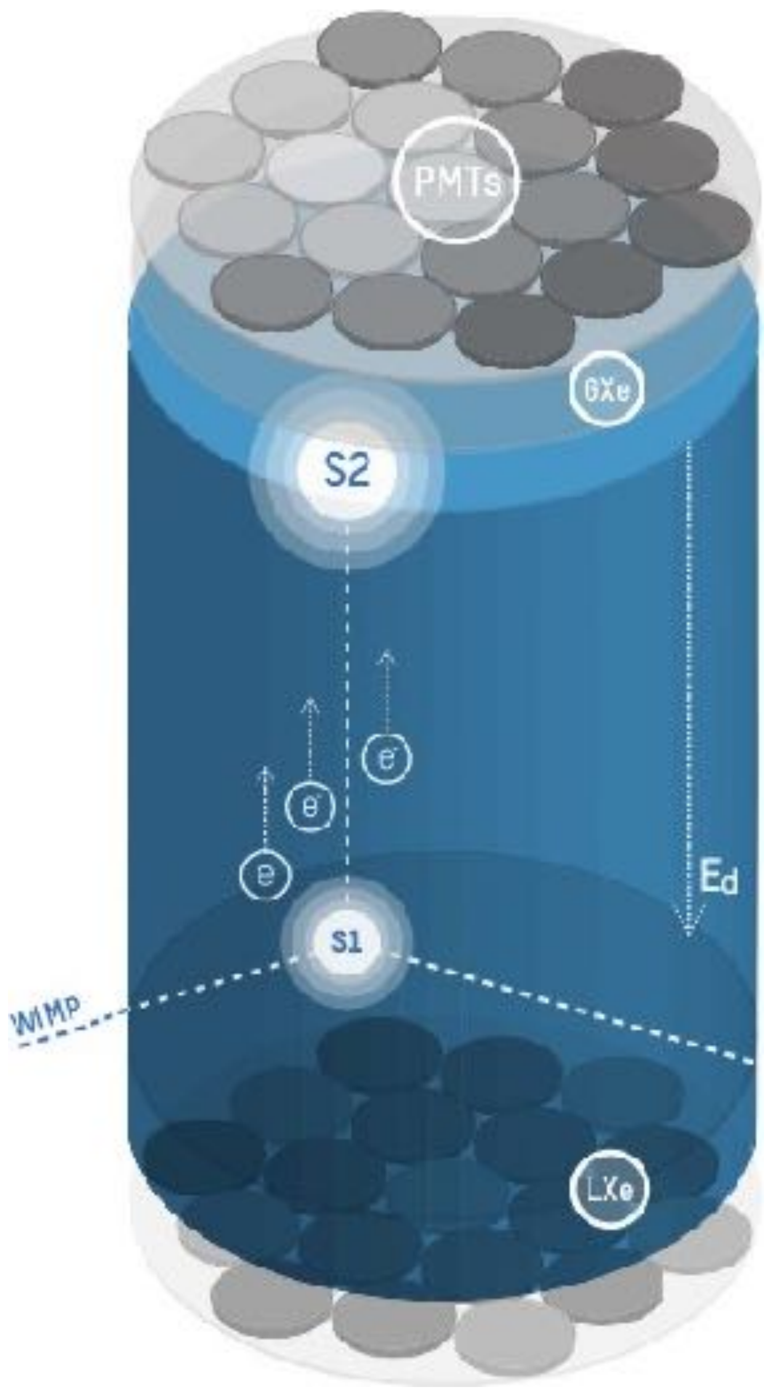
- Broad mass window
- Interaction with visible particles

✓ How to detect DM candidates?



Direct detection of DM

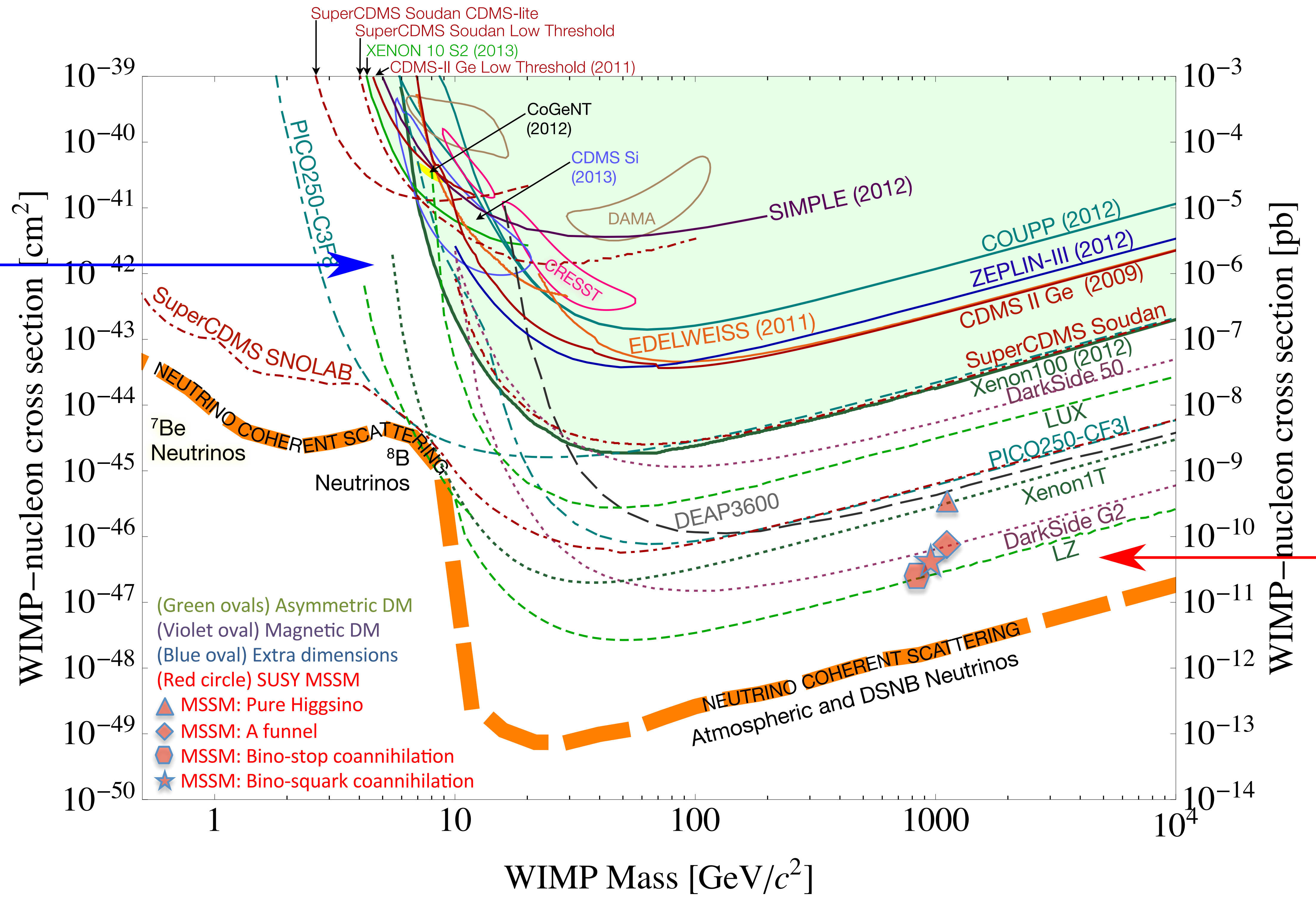
✓ Example: DM-nucleus scattering



DM

Wikipedia “XENON experiment”

$E_R^{\rm thr} \sim 10\,{\rm keV}$

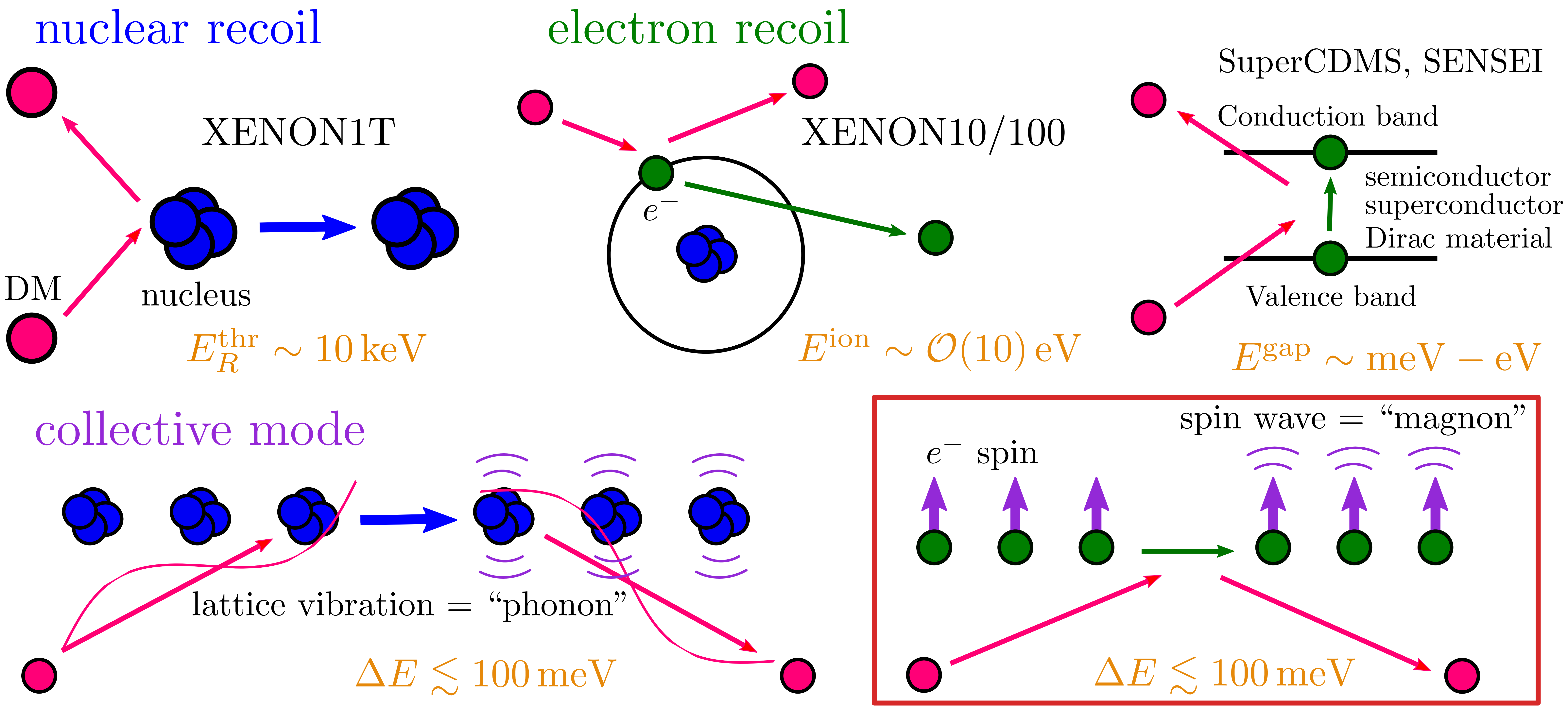


$n_\chi \propto m_\chi^{-1}$

CF1 Snowmass report, 1310.8327

✓ Recently more focus on lighter region

Target (quasi-)particle for direct detection



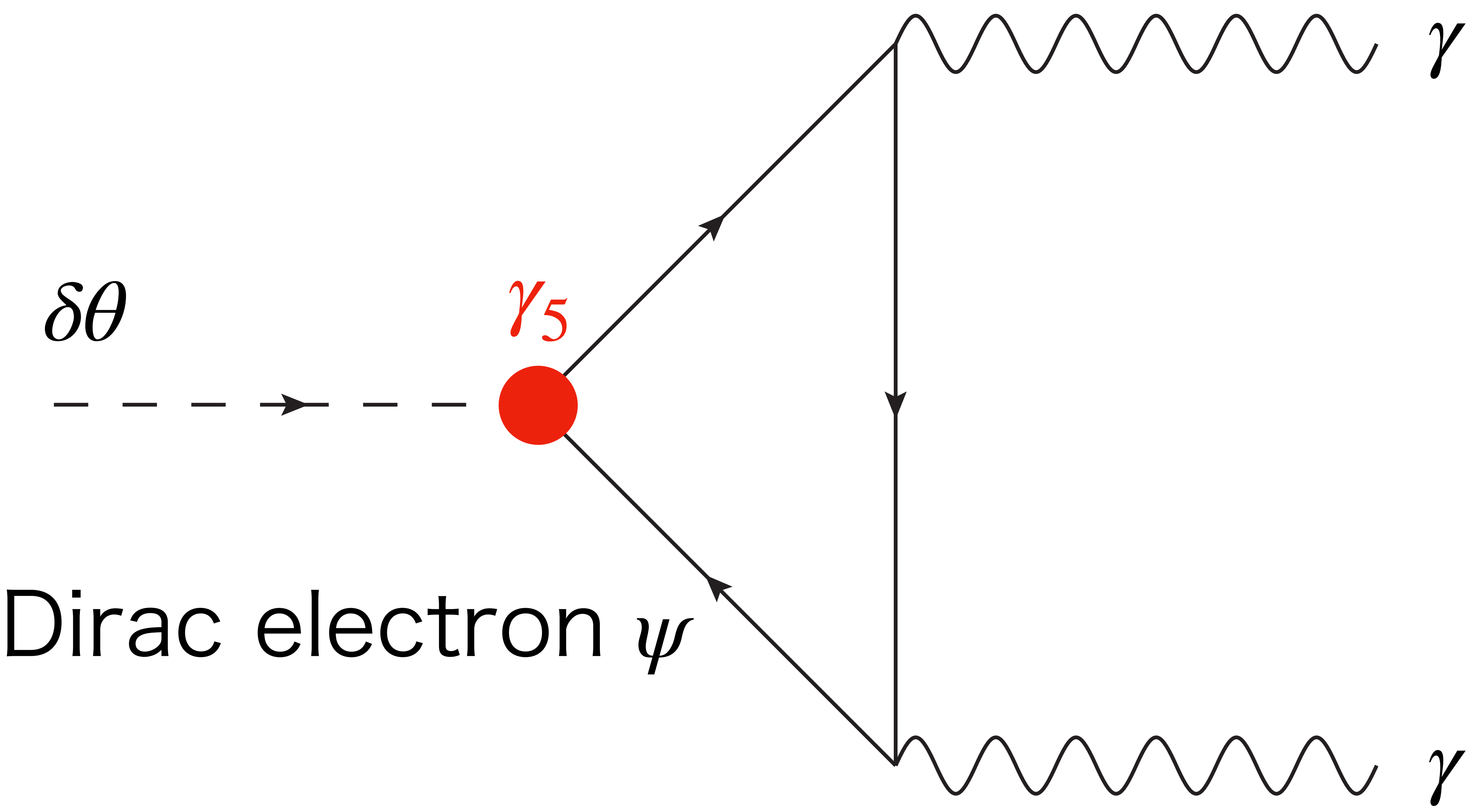
✓ Collective modes are important for light DM ($m_\chi \lesssim {\rm MeV}$)

Today I focus on "axion quasi-particle"

Introduction to axion excitation

Summary: What is "axion"?

✓ "Axion" is fluctuation of spins $\delta\theta$ inside magnetic materials



$\propto \delta\theta\, F^{\mu\nu}\tilde{F}_{\mu\nu}$

"axion" is named after this interaction

R. Li, J. Wang, X. Qi, S. Zhang Nature Physics 6, 284–288 (2010)

✓ Example: Anti-ferromagnetic topological insulator

Summary: Fu-Kane-Mele-Hubbard model

$$H = -\sum_{\langle i,j\rangle,\sigma} t_{ij}\, c^{\dagger}_{i\sigma} c_{j\sigma}$$

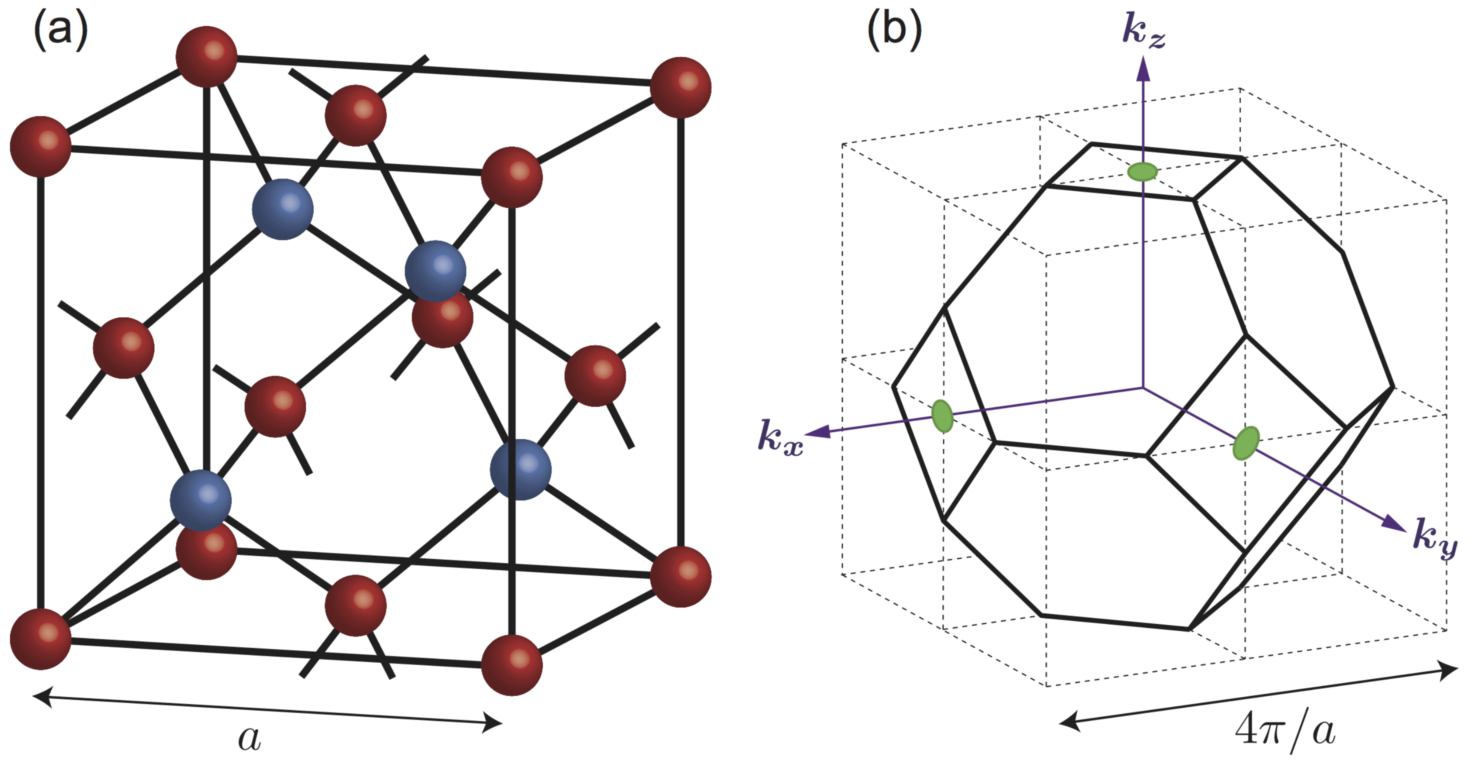
$$+ i\lambda \sum_{<<i,j>>} c_i^{\dagger} \vec{\sigma} \cdot (\vec{d}^{\,1}_{ij} \times \vec{d}^{\,2}_{ij}) c_j$$

$$+ U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Fu-Kane-Mele model
(Model of topological insulator)

Hubbard interaction
- Anti-ferromagnetic ordering
- Spin-electron interaction

Fu-Kane-Mele model



lattice structure Brillouin zone

A. Sekine, K. Nomura J. Phys. Soc. Jpn. 83, 104709 (2014)

✓ Hamiltonian (Effective model of outermost electrons)

$$H = -\sum_{<i,j>} \sum_{\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + i\lambda \sum_{<<i,j>>} c^{\dagger}_{i} \vec{\sigma} \cdot (\vec{d}^{\,1}_{ij} \times \vec{d}^{\,2}_{ij}) c_j$$

Spin-orbit interaction

- $c_{i\sigma}$: electron annihilation operator @ site i w/ spin σ

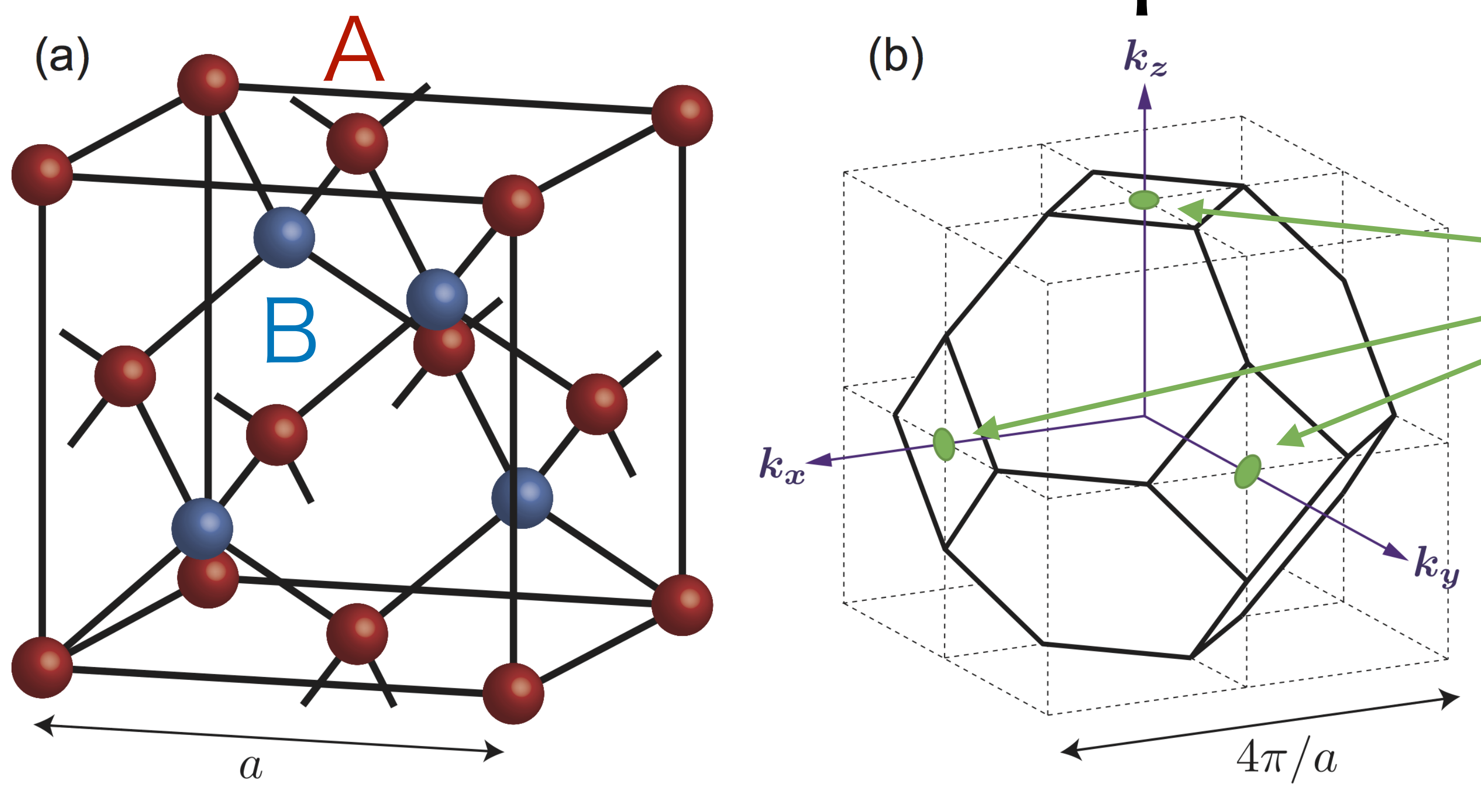
✓ Distorted diamond lattice:

- $t_{ij} = t + \delta t$ for some specific direction
- $t_{ij} = t$ for others

Band structure of FKM model

✓ e^- annihilation operators in momentum space

- $c_{\vec{k}\uparrow,A}, c_{\vec{k}\downarrow,A}, c_{\vec{k}\uparrow,B}, c_{\vec{k}\downarrow,B}$
- $c_{\vec{k}} = (c_{\vec{k}\uparrow,A}\ c_{\vec{k}\downarrow,A}\ c_{\vec{k}\uparrow,B}\ c_{\vec{k}\downarrow,B})^T$



3 Dirac points

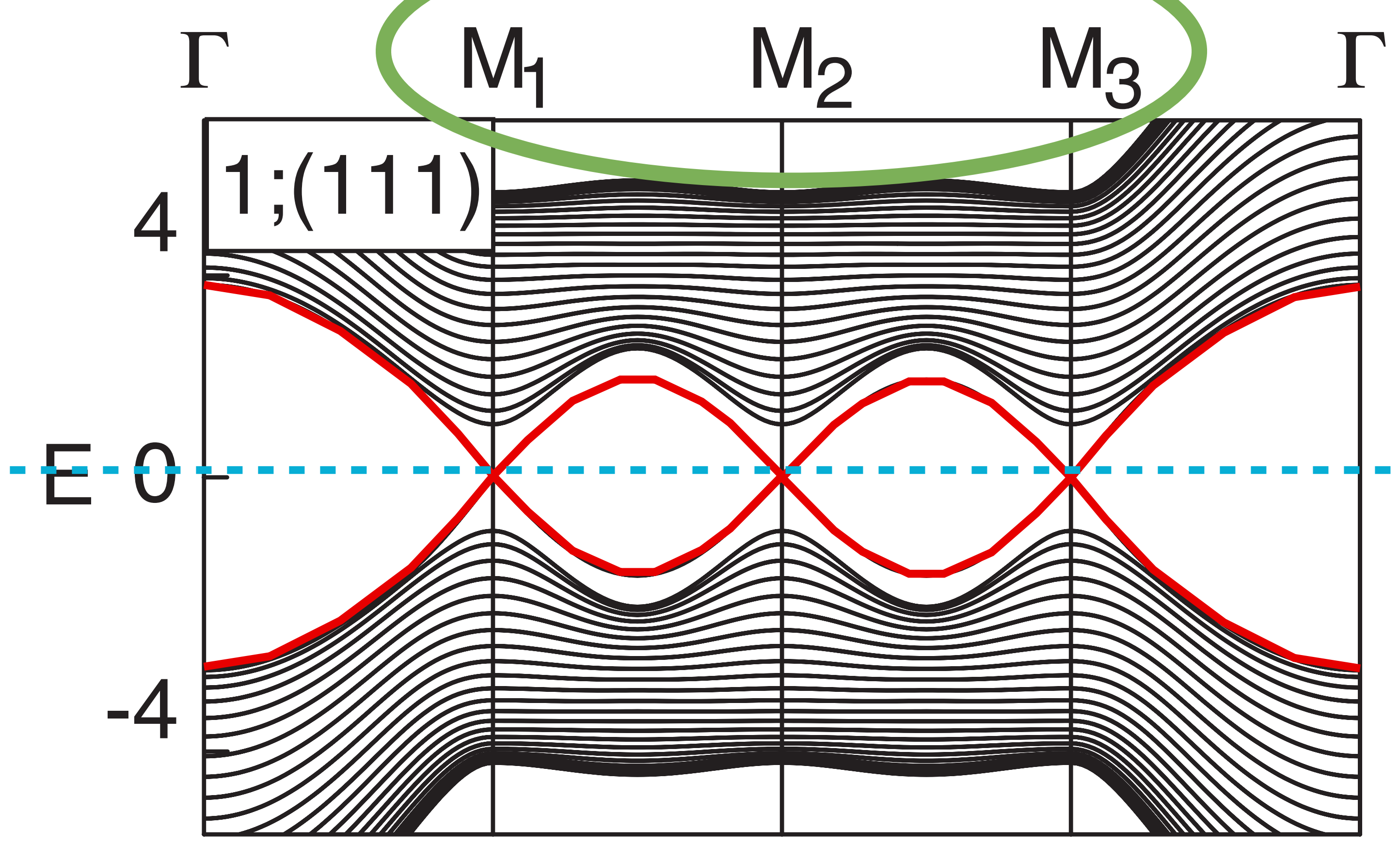
L. Fu, C. L. Kane, E. J. Mele, PRL 98, 106803 (2007)

✓ 4-band model

- $H = \sum_{\vec{k}} c^\dagger_{\vec{k}} \mathscr{H}_{\vec{k}} c_{\vec{k}}$
- $\mathscr{H}_{\vec{k}} = \sum_{\mu=1}^{5} R_\mu(\vec{k})\alpha_\mu$

$$\alpha_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix}, \quad \alpha_4 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \alpha_5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\{\alpha_\mu, \alpha_\nu\} = 2\delta_{\mu\nu}$$



Symmetry enhanced points
small gap $\sim \delta t$

✓ Energy eigenvalues $E_\pm(\vec{k}) = \pm\sqrt{\sum_\mu \left(R_\mu(\vec{k})\right)^2}$

Hubbard interaction and magnetism

✓ Coulomb interaction makes it difficult to fill 2 e^-s in an orbital

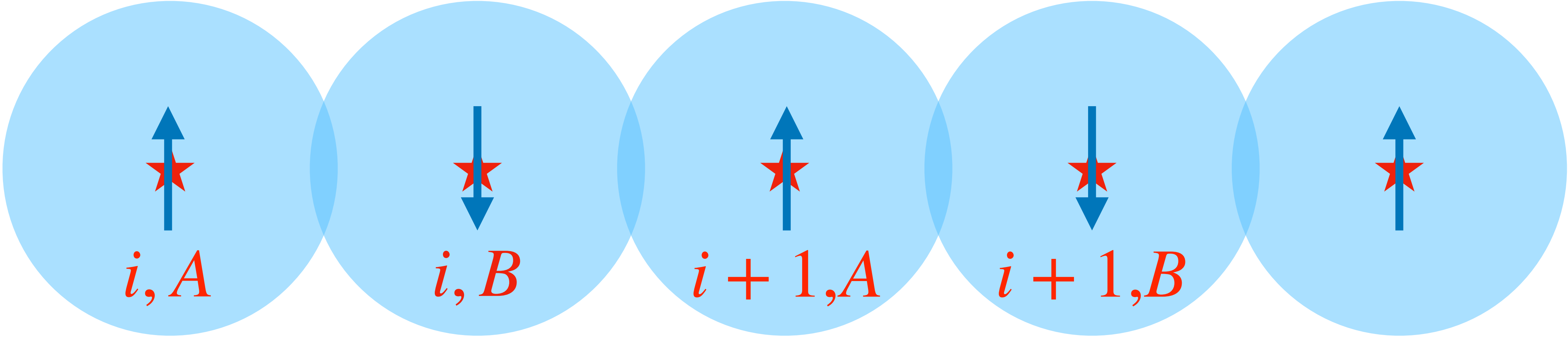
$$H = -t \sum_{<i,j>} \sum_{\sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \qquad n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$$

Hubbard interaction

✓ Large U limit, treating t as perturbation

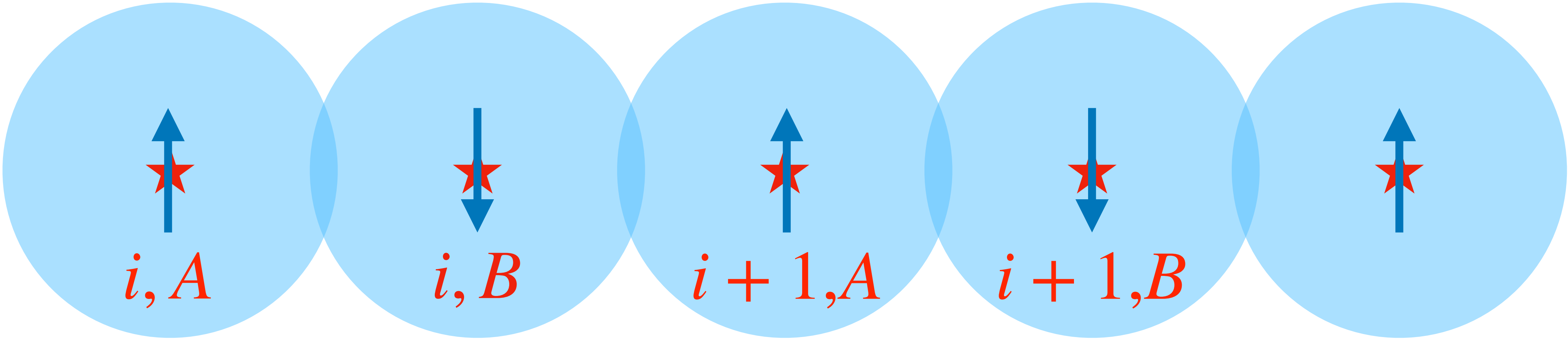
$$H_{\text{eff}} \sim H_t \frac{1}{H_U} H_t = \frac{t^2}{U} \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$$

✓ $t^2/U > 0 \rightarrow$ Anti-ferromagnetic ordering



Spin - electron interaction

✓ Order parameter $\left\langle \vec{S}_{i,A} \right\rangle = -\left\langle \vec{S}_{i,B} \right\rangle \equiv \vec{m}$



✓ Mean-field approximation on $H_U = U \sum_i n_{i\uparrow} n_{i\downarrow}$

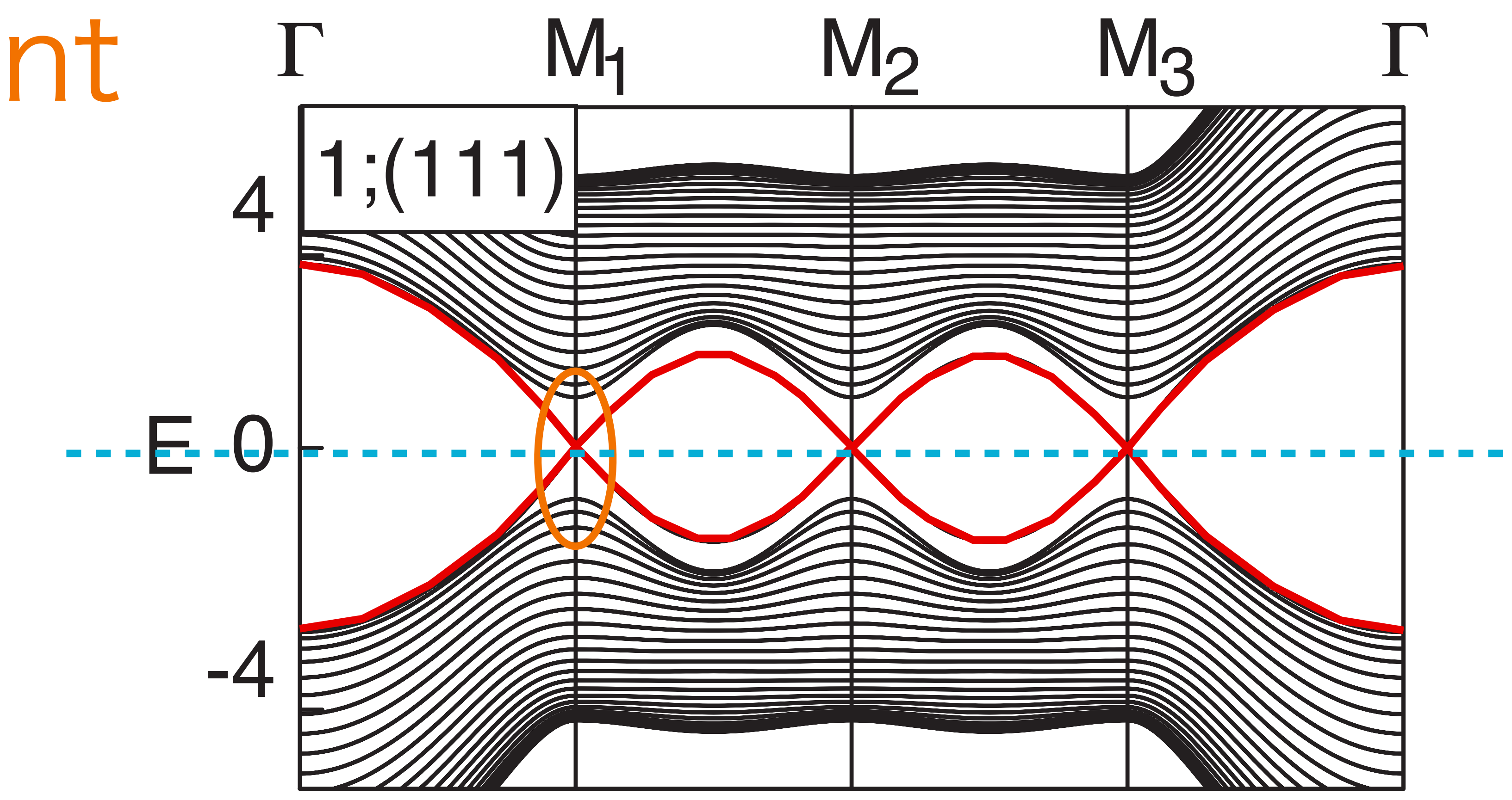
$$H_U \simeq U \sum_i (\underbrace{< n_{i\uparrow} >}_{1/2 + m_z} n_{i\downarrow} + \underbrace{< n_{i\downarrow} >}_{1/2 - m_z} n_{i\uparrow} - < n_{i\uparrow} > < n_{i\downarrow} >$$

$$- \underbrace{< c^\dagger_{i\uparrow} c_{i\downarrow} >}_{m_x + i m_y} c^\dagger_{i\downarrow} c_{i\uparrow} - \underbrace{< c^\dagger_{i\downarrow} c_{i\uparrow} >}_{m_x - i m_y} c^\dagger_{i\uparrow} c_{i\downarrow} + < c^\dagger_{i\uparrow} c_{i\downarrow} > < c^\dagger_{i\downarrow} c_{i\uparrow} >)$$

✓ Nothing but interaction between $\vec{m}$ and e^-s

$E \simeq \mathcal{O}(\delta t)$ phenomenology

✓ Linearlized Hamiltonian around a Dirac point



- $H(\vec{M}_1 + \vec{q}) \simeq c^\dagger_{\vec{M}_1+\vec{q}} \mathcal{H}'_{\vec{q}} c_{\vec{M}_1+\vec{q}}$
- $\mathcal{H}'_{\vec{q}} = \sum_i q'_i \alpha_i + \delta t \alpha_4 + U m_x \alpha_5$

w/ $q'_1 = atq_1,\ q'_2 = 2a\lambda q_2,\ q'_3 = 2a\lambda q_3$ & $\{\alpha_\mu, \alpha_\nu\} = 2\delta_{\mu\nu}$

✓ Equivalent effective action

$$S = \int d^4x \sum_{r=1,2,3} \overline{\psi}_r \left[i\gamma^\mu(\partial_\mu - ieA_\mu) - \delta t - i\gamma_5 U m_r \right] \psi_r$$

✓ Properties of 3 Dirac electrons

- mimics free relativistic particles with mass δt (at linear approximation)
- axionic (i.e., via γ_5) interaction with magnetization

$E \ll \delta t$ phenomenology

✓ Integrate out Dirac electrons with mass δt

✓ Fujikawa's method (chiral rotation)

$$S_\theta = \frac{\alpha_e}{4\pi}\int d^4x\, \theta F_{\mu\nu}\widetilde{F}^{\mu\nu} \qquad \theta \equiv \pi + \sum_r \theta_r = \pi + \sum_r \tan^{-1}\left(\frac{Um_r}{\delta t}\right)$$

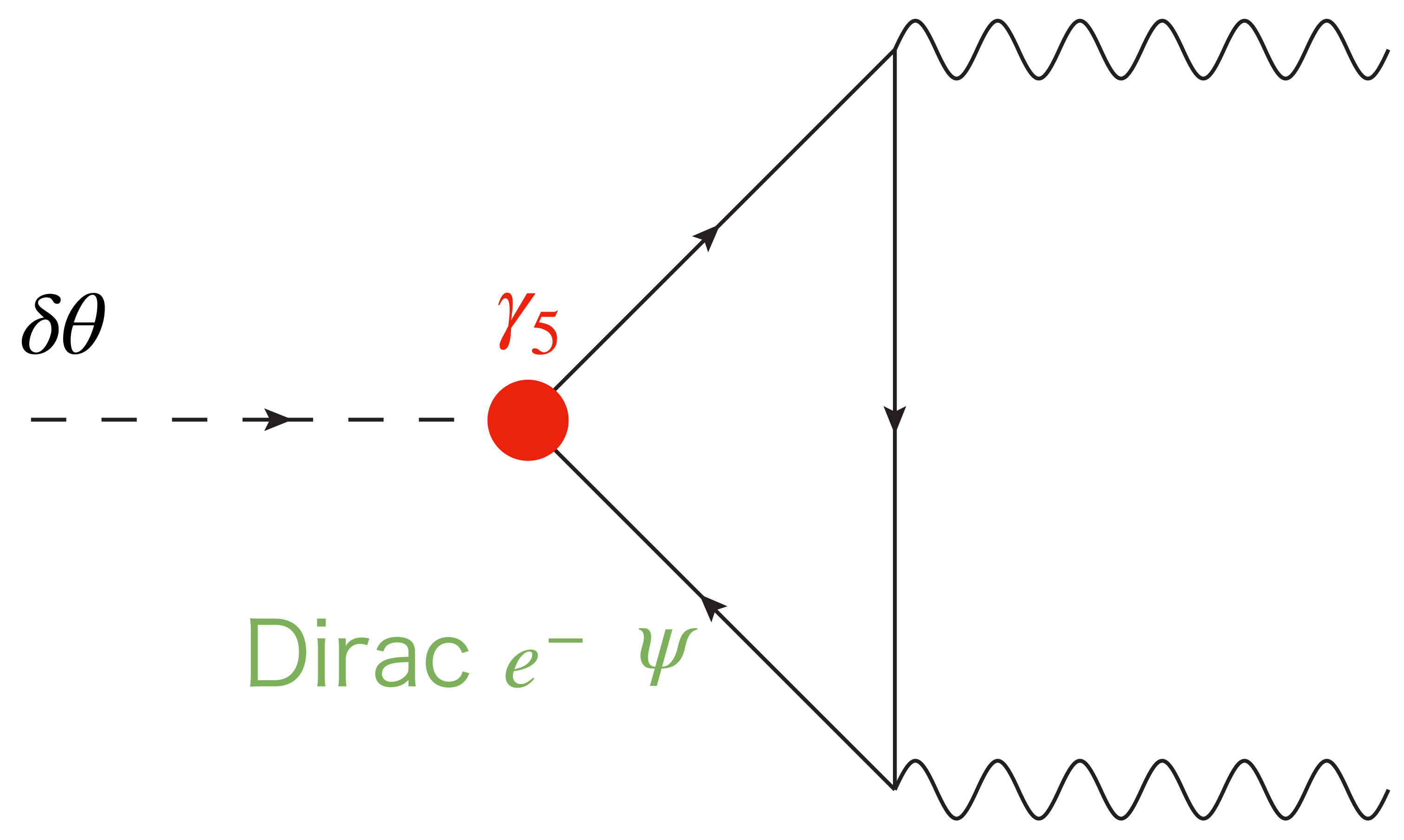
A. Sekine, K. Nomura J. Phys. Soc. Jpn. 83, 104709 (2014)

✓ Same applies for dynamical spin fluctuations

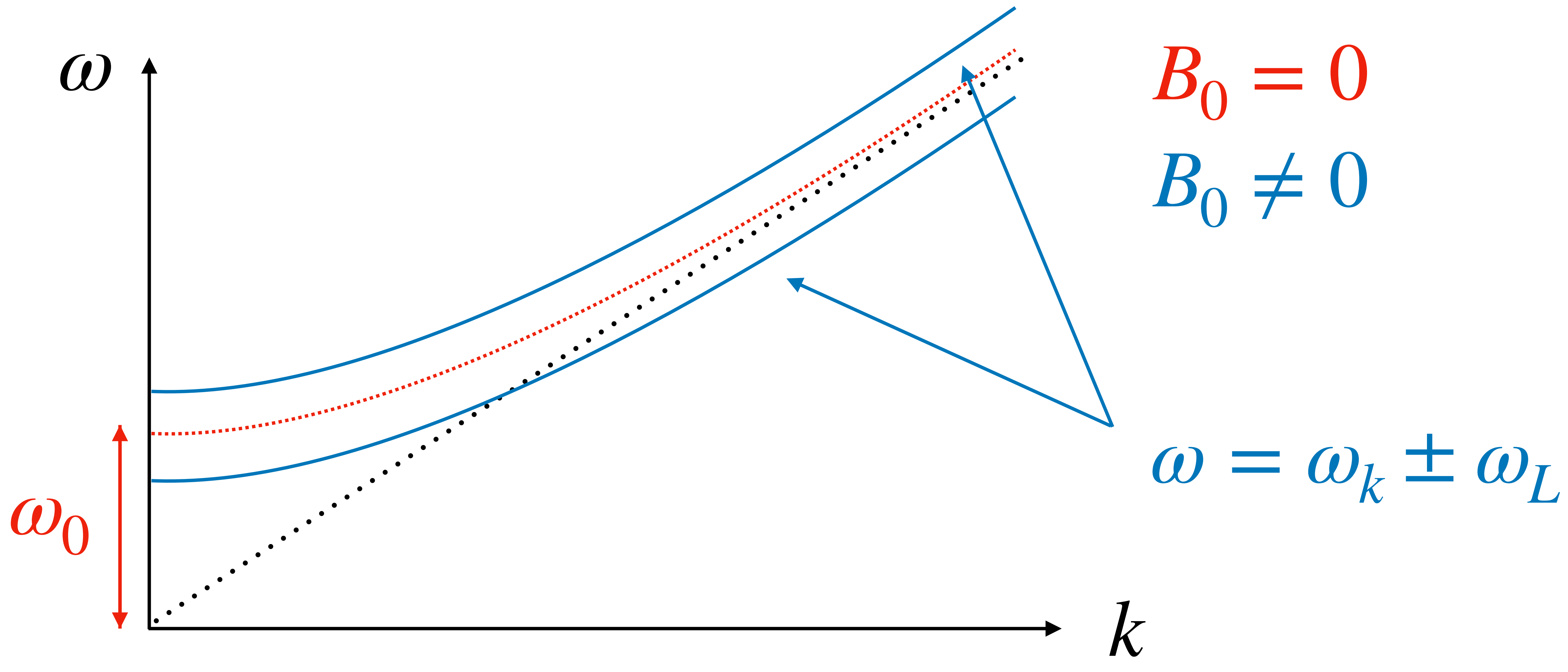
- $\vec{m}(t,\vec{x}) = \vec{m} + \delta\vec{m}(t,\vec{x})$

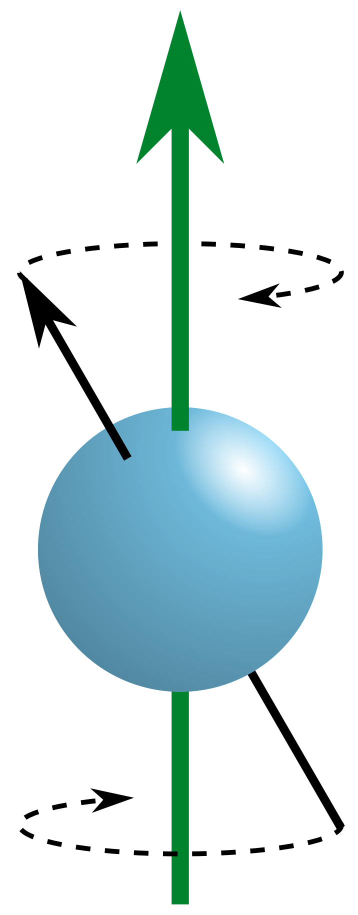
$$\delta\theta(t,\vec{x}) = \sum_r \frac{U/\delta t}{1 + U^2 m_r^2/\delta t^2}\delta m_r(t,\vec{x})$$

with $U/\delta t \sim \mathcal{O}(1)$



Properties of spin fluctuations





Wikipedia "Larmor Precession"

✓ 2 "magnon" modes with spin $\pm 1/2$

- Common intrinsic gap ω_0
- External $\vec{B}_0$ induces mass splitting $\omega_L = g\mu_B B_0$

✓ Dynamical axion in FKMH is linear combination of 2 magnon modes

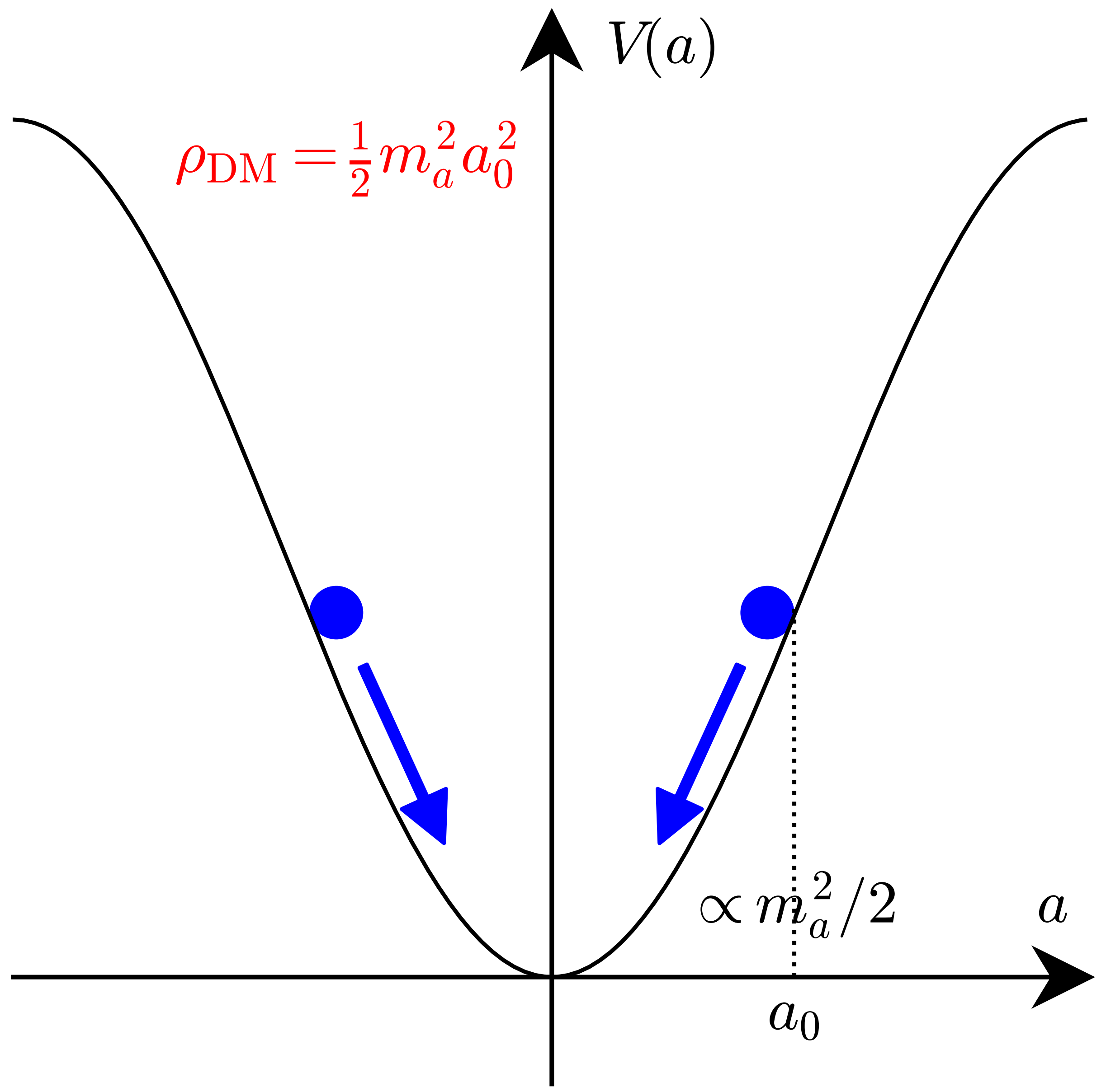
Application to DM direct detection

Axion / Hidden-photon DM

✓ Energy stored as coherent oscillation

$$\rho_{\rm DM} = \frac{1}{2} m_a^2 a_0^2$$

- mimics non-relativistic matter $\rho_{\rm DM} \propto (\text{scale factor})^{-3}$
- light bosons can be DM candidates



✓ Effectively works as oscillating EM fields

- Axion DM $\mathscr{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$

  ➡ $\vec{E}_a = - g_{a\gamma\gamma} a B_0$ under background $\vec{B}_0$

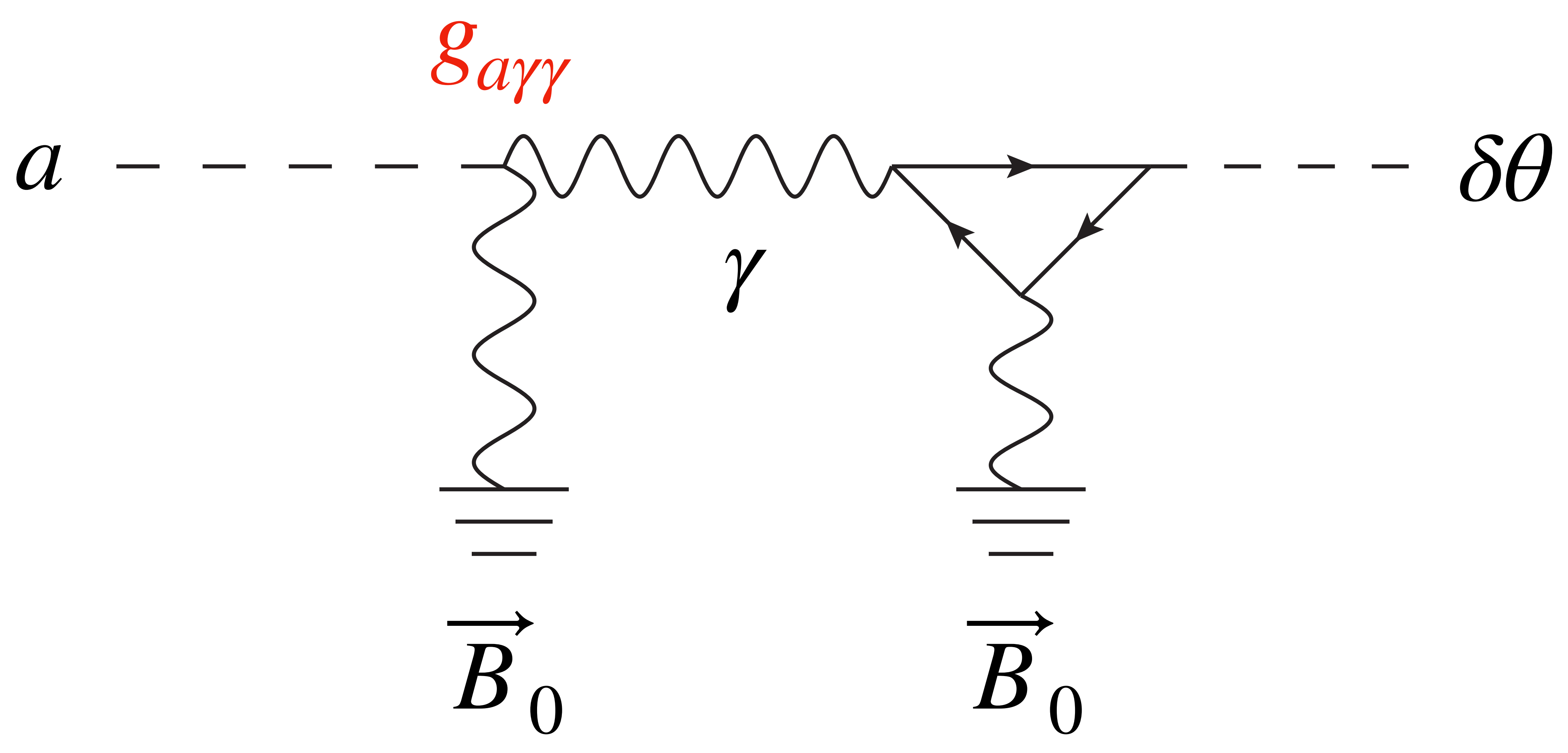
- Hidden-photon DM $\mathscr{L}_{Hee} = - \epsilon_H e H_\mu \bar{\psi} \gamma^\mu \psi$

  ➡ $\vec{E}_H \propto - \epsilon_H \partial_t \vec{H}$

DM direct detection w/ cond-mat axion

✓ Axion (ALPs) DM

$$\mathscr{L} = g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$



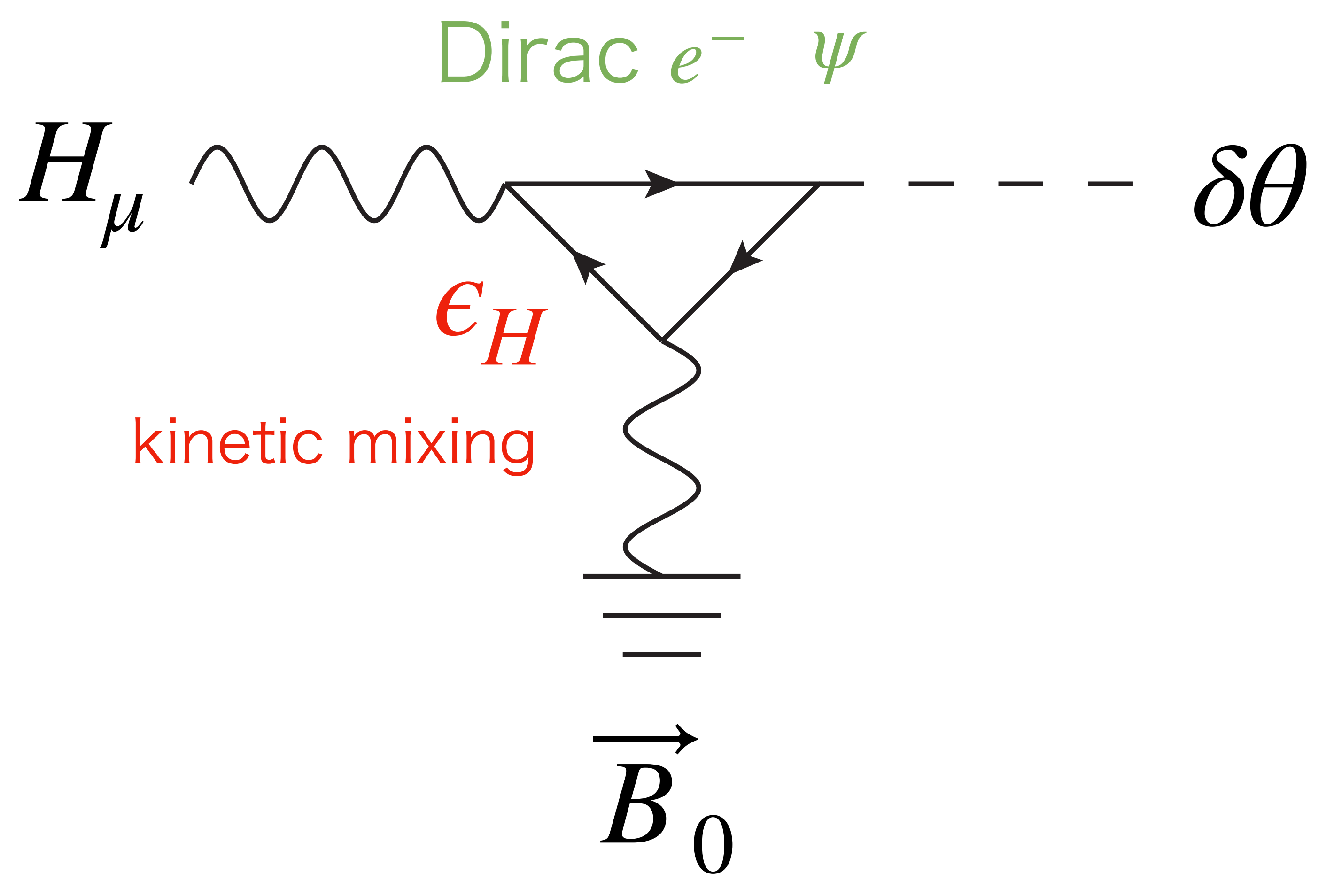
D. J.E. Marsh, K. C. Fong, E. W. Lentz, L. Šmejkal, M. N. Ali, PRL 123, 121601 (2019)

J. Schütte-Engel, D. J. E. Marsh, A. J. Millar, A. Sekine, et al. [2102.05366]

S. Chigusa, T. Moroi, K. Nakayama [2102.06179]

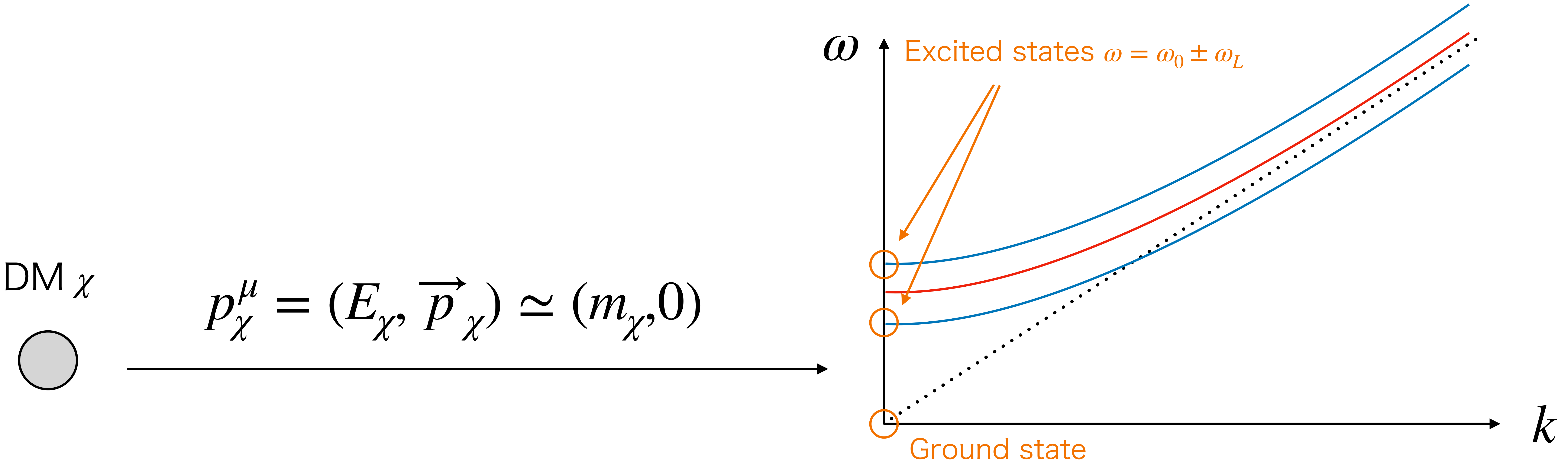
✓ Hidden-photon DM

$$\mathscr{L} = -\epsilon_H e H_\mu \bar{\psi} \gamma^\mu \psi,$$



S. Chigusa, T. Moroi, K. Nakayama [2102.06179]

Kinematics of DM-axion conversion



✓ DM de-Broglie wave length > material size

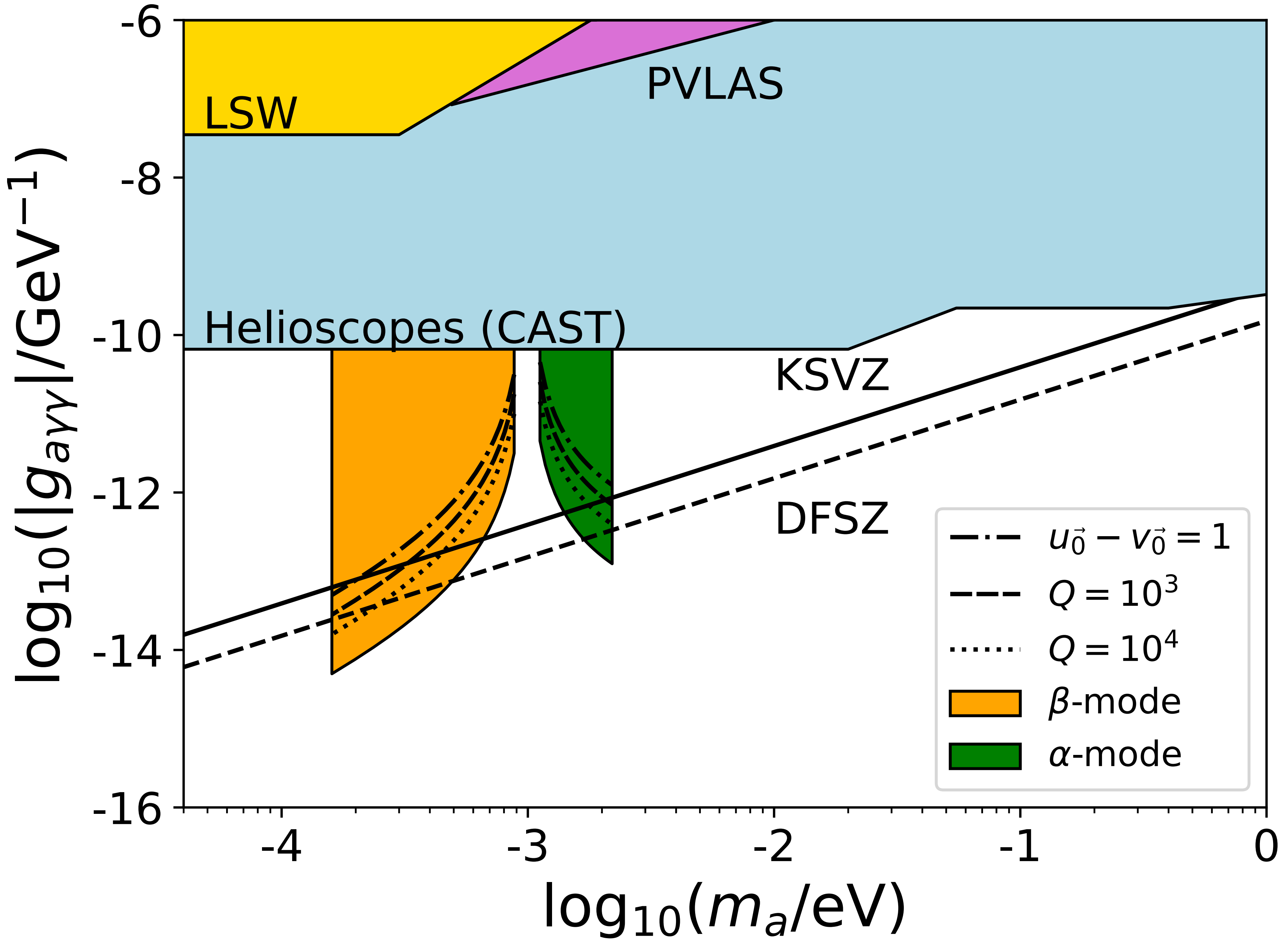
- only $k = 0$ mode of axion is relevant

✓ $m_{\chi} \simeq \omega_0 \pm \omega_L$ leads to resonant conversion

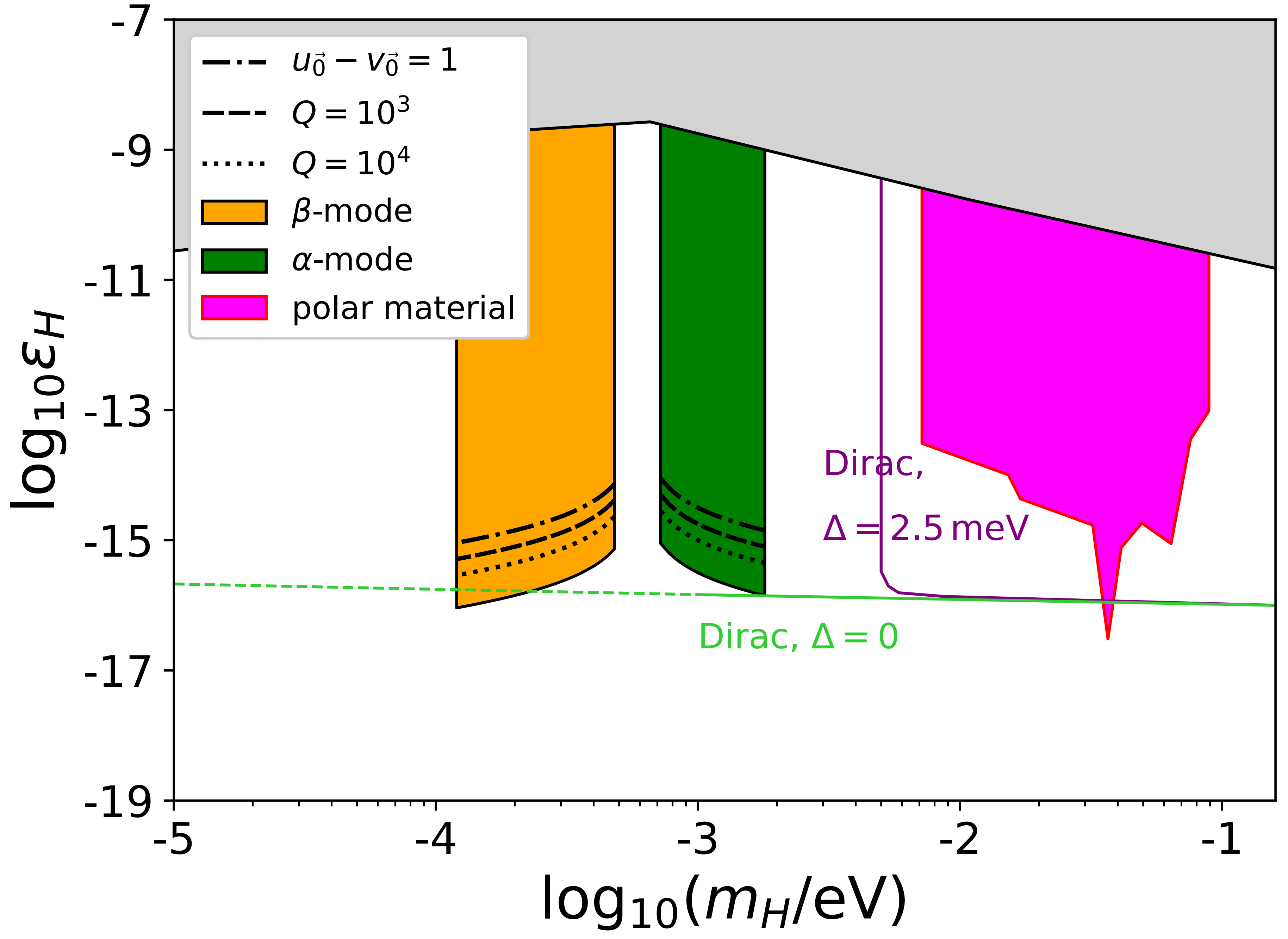
- scan $\vec{B}_0$ to search for broad mass range

Sensitivities on DM candidates

✓ Axion (ALPs) DM



✓ Hidden photon DM



✓ $B_0 = 1\,\mathrm{T} \sim 10\,\mathrm{T}$, $100\,\mathrm{s}$ obs. for each point, $V = (10\,\mathrm{cm})^3$ & $T = 1\,\mathrm{yr}$ & $Q = 10^6$

✓ Typical values for material properties (mass range depend on them)

Comparison with magnon

	axion	magnon → Soda-san's talk
excitation energy	$\sim \mathcal{O}(10)\,\mathrm{meV}$ depend on anisotropy controlled by $\vec{B_0}$	$\sim \mathcal{O}(10)\,\mathrm{meV}$ depend on anisotropy controlled by $\vec{B_0}$
DM coupling probed	$a - \gamma$ $H - e$	$a - e \quad (a - N)$ $H - e$
target materials	anti-ferro topo insulator, etc FKMH, $(Bi_{1-x}Fe_x)_2Se_3$, $Mn_2Bi_2Te_5$	magnetic material YIG, $NiPS_3$
references	D. Marsh+ [1807.08810] J. Schütte-Engel+ [2102.05366] S. Chigusa+ [2102.07910]	R. Barbieri+ [1606.02201] S. Chigusa+ [2001.10666] T. Ikeda+ [2102.08764] A. Mitridate+ [2005.10256]

Conclusion

✓ Possibility of using axion excitation to search for DM

✓ Formulation and primitive estimation of sensitivity

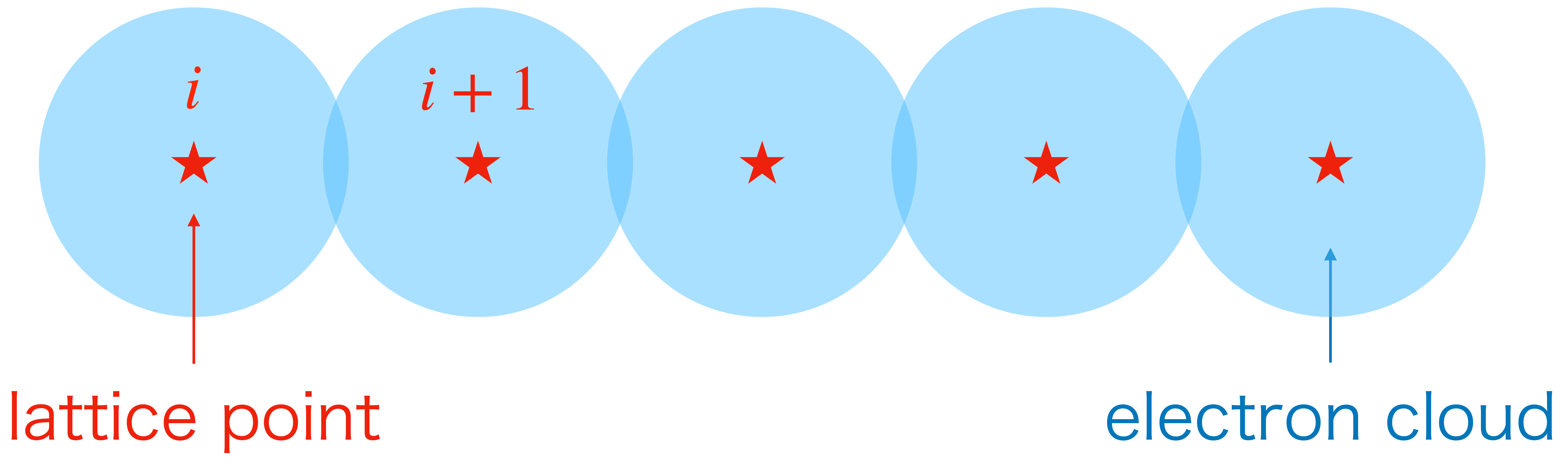
- Direct detection of $\mathcal{O}(1)\,\mathrm{meV}$ bosonic DM

✓ For detection of signal, quantum sensors will play important roles

backup slides

Tight-binding model

✓ Consider electrons tightly bound to lattice points



✓ Each nucleus has electron orbital with $E = \epsilon$: $\langle i | H | i \rangle = \epsilon$

✓ Small overlap of nearest orbitals : $\langle i | H | i+1 \rangle = -t \quad (|t| \ll \epsilon)$

$$H = \epsilon \sum_i c_i^\dagger c_i \boxed{-t \sum_{<i,j>} c_i^\dagger c_j}$$

Hopping term

$< i, j >$: nearest neighbors

Coulomb repulsion (Hubbard int.)

✓ Coulomb interaction makes it difficult to fill 2 e^{-}s in an orbital

$$H = -t \sum_{<i,j>} \sum_{\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} \qquad n_{i\sigma} = c^{\dagger}_{i\sigma} c_{i\sigma}$$

Hubbard interaction

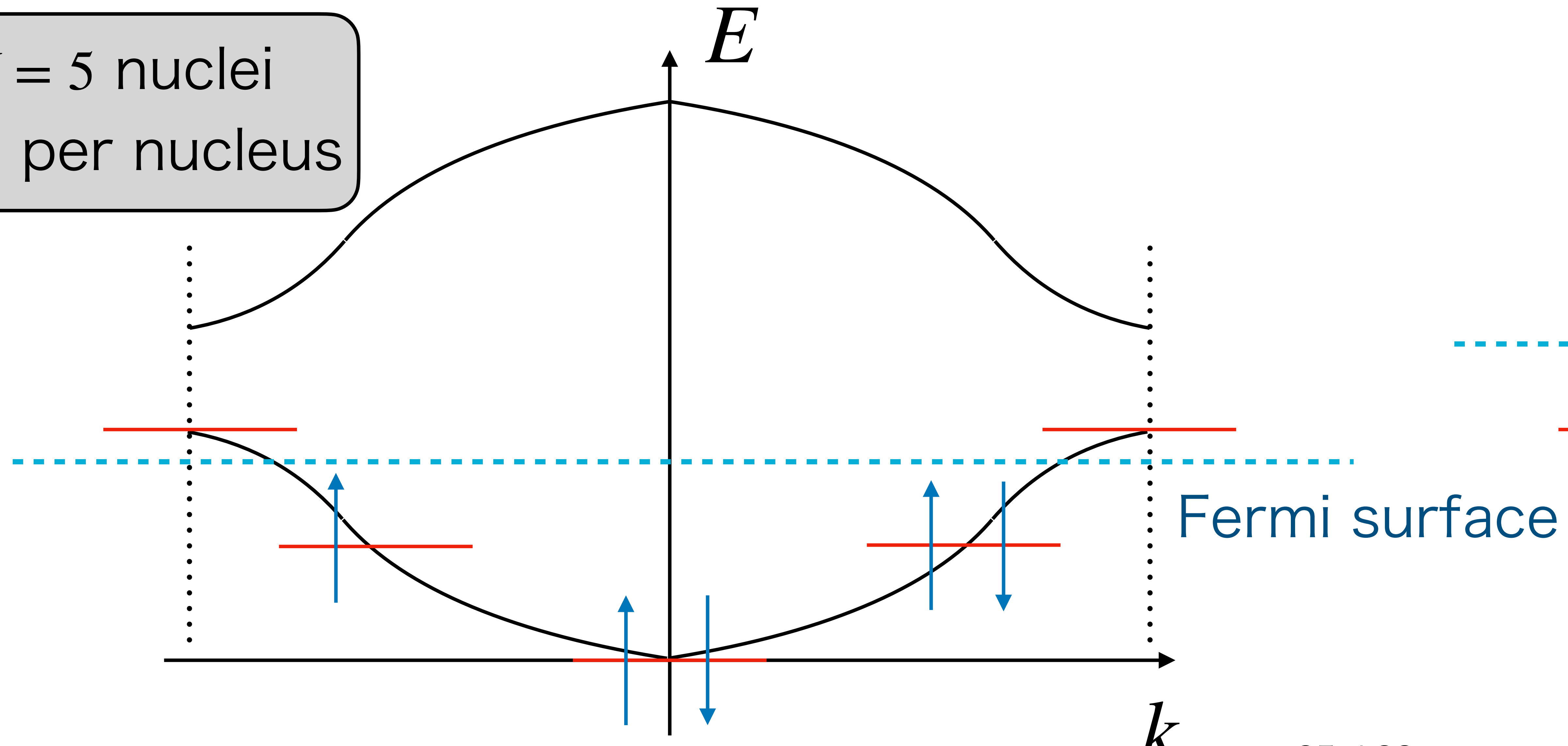
$\sigma \in \{\uparrow, \downarrow\}$: electron spin

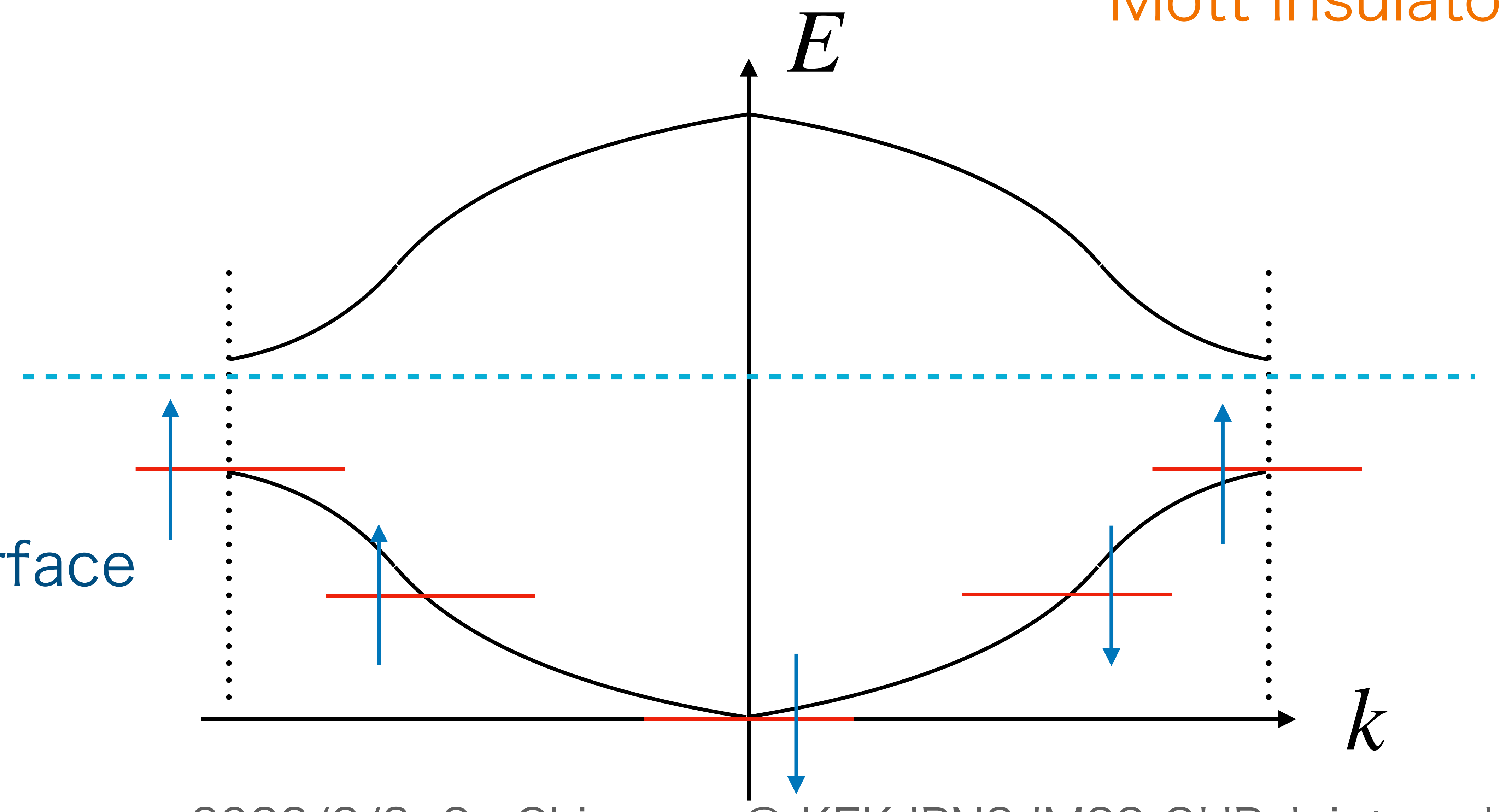
✓ Band theory of solid states tells us
odd # of e^{-}s per nucleus = metal

However,
✓ large U may make it insulator

Mott insulator

$N = 5$ nuclei
1 e^{-} per nucleus





Spin-orbit interaction → Topo. insulator

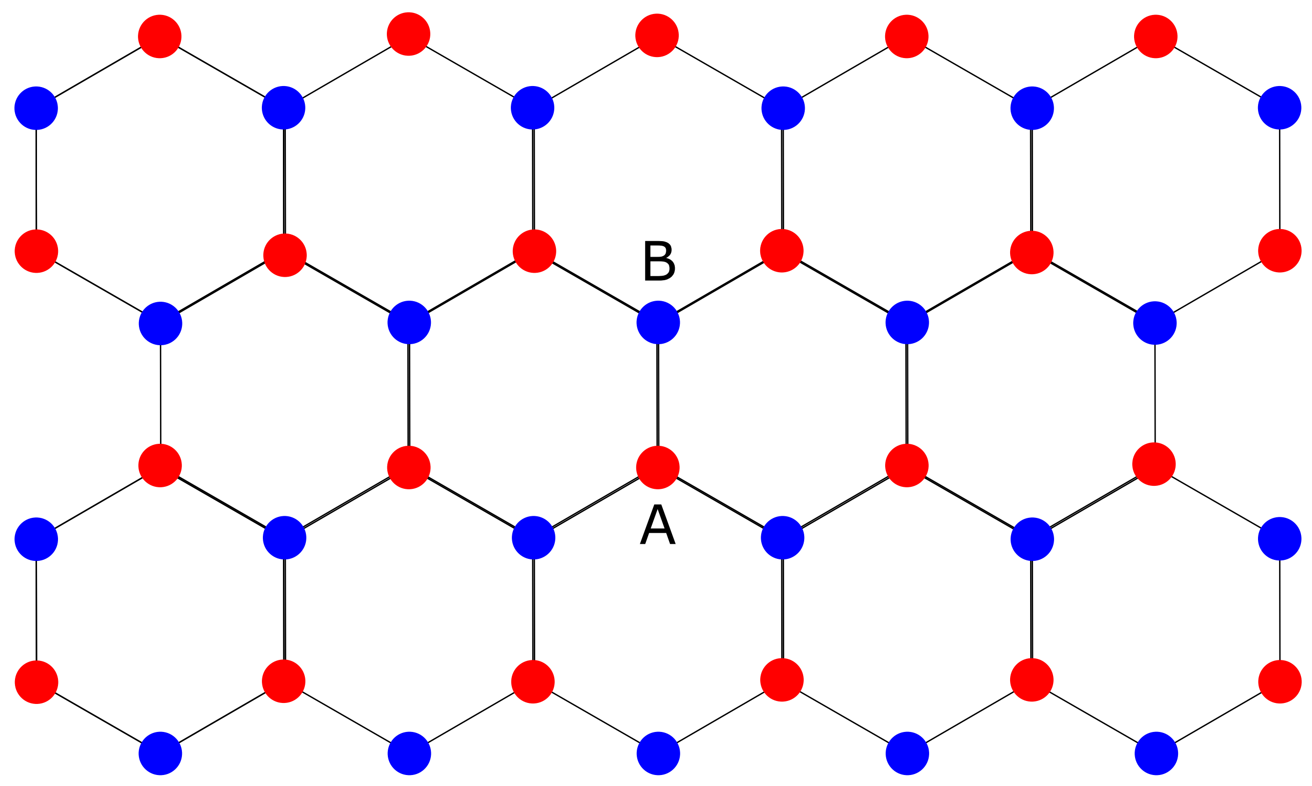
$$H = -t \sum_{<i,j>} \sum_{\sigma} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{<<i,j>>} c^\dagger_i \vec{\sigma} \cdot (\vec{d}^{\,1}_{ij} \times \vec{d}^{\,2}_{ij}) c_j$$

✓ SO interaction plays important role in topological insulators

Kane-Mele model

C. L. Kane, E. J. Mele PRL 95 (2005) 226081 & PRL 95 (2005) 146802

✓ First example of (time-reversal symmetric) topo. insulator



2D Honeycomb lattice

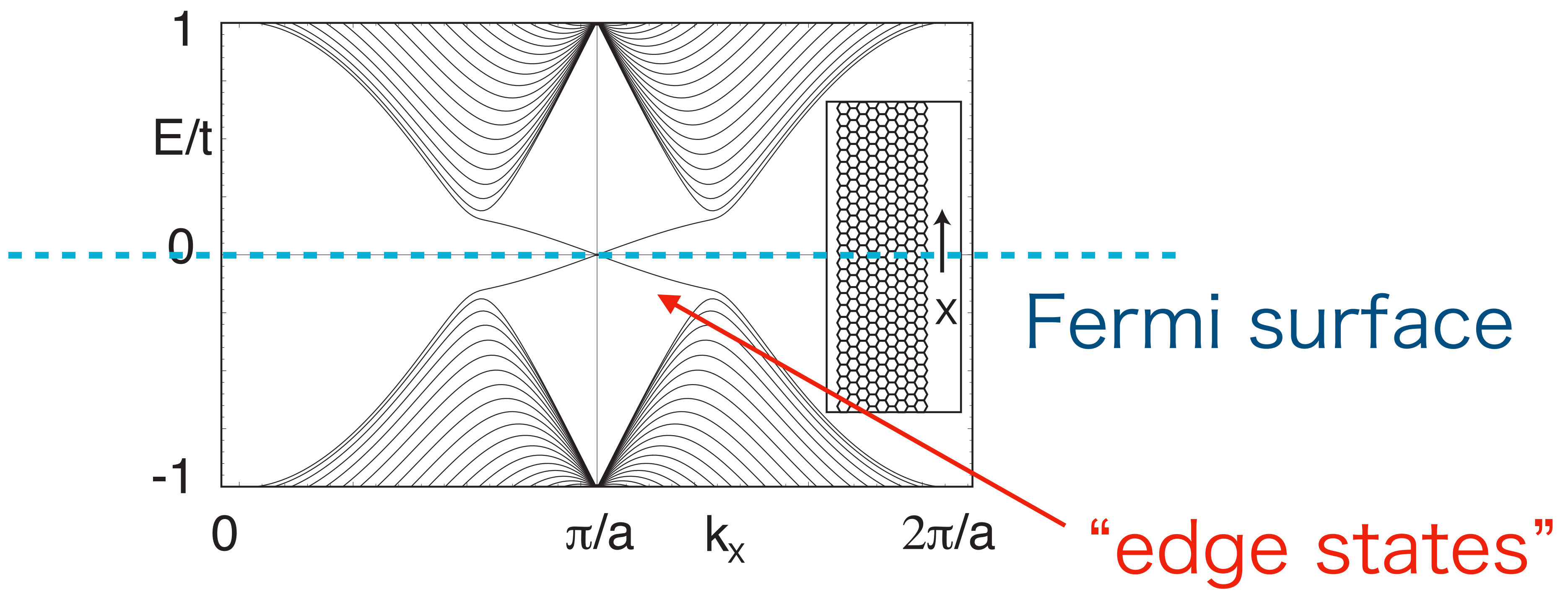


FIG. 1: (a) One dimensional energy bands for a strip of graphene (shown in inset) modeled by (7) with $t_2/t = .03$. The bands crossing the gap are spin filtered edge states.

Relationship w/ topology

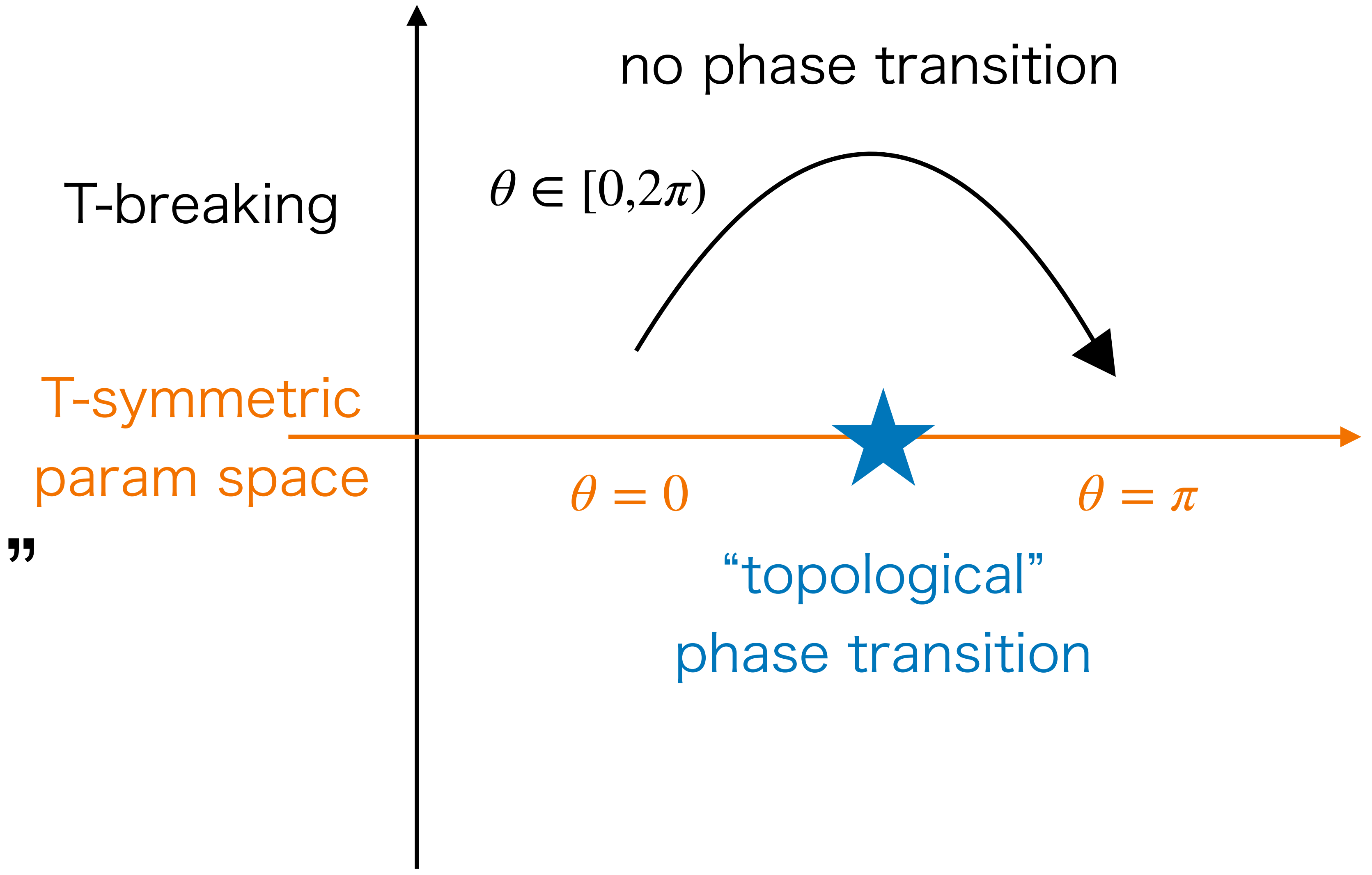
✓ Normal / Topological insulators have different topologies = No continuous deformation

- Topological invariant θ is evaluated w/ berry connection

$$\mathscr{A}_i^{\alpha\beta} = -i\langle u_k^\alpha | \frac{\partial}{\partial k_i} | u_k^\beta \rangle \qquad \theta \equiv \frac{1}{4\pi}\int_{\mathrm{BZ}} d^3k\, \epsilon^{ijk}\, \mathrm{Tr}\left[\mathscr{A}_i \partial_j \mathscr{A}_k + i\frac{2}{3}\mathscr{A}_i \mathscr{A}_j \mathscr{A}_k\right]$$

Berry connection Bloch states↔energy eigenstates Brillouin zone

- Time reversal symmetry forces θ to be one of below
  - $\theta = 0$ (normal insulator)
  - $\theta = \pi$ (topological insulator)
- SPT phase
  = "symmetry protected topological phase"



θ is axion term

✓ Topological EM response

$$S = \frac{\alpha}{4\pi}\int dt d^3x\, \theta\, \underline{F_{\mu\nu}\widetilde{F}^{\mu\nu}} \quad ; \quad \widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$= 4\vec{E}\cdot\vec{B}$

- θ is "static" axion term
- $\vec{B}$ induces electric polarization $\vec{P} \propto \theta\vec{B}$
- $\vec{E}$ induces magnetization $\vec{M} \propto \theta\vec{E}$

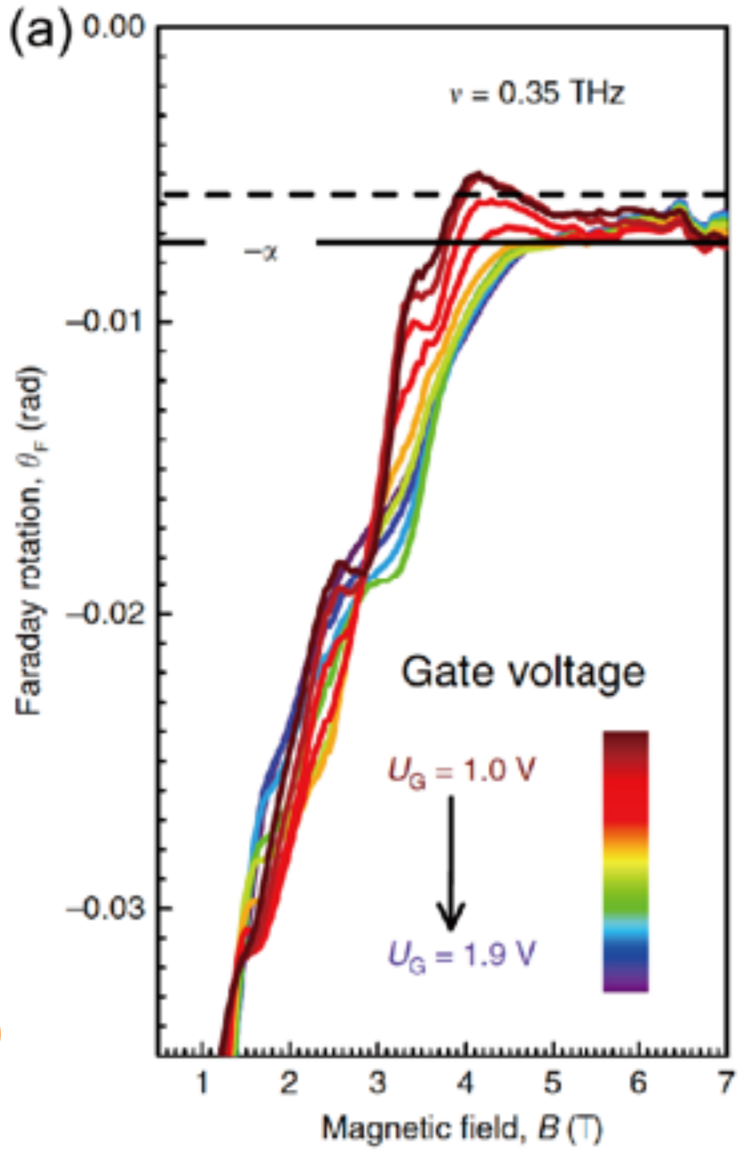
✓ Rich phenomenology like

- Faraday rotation

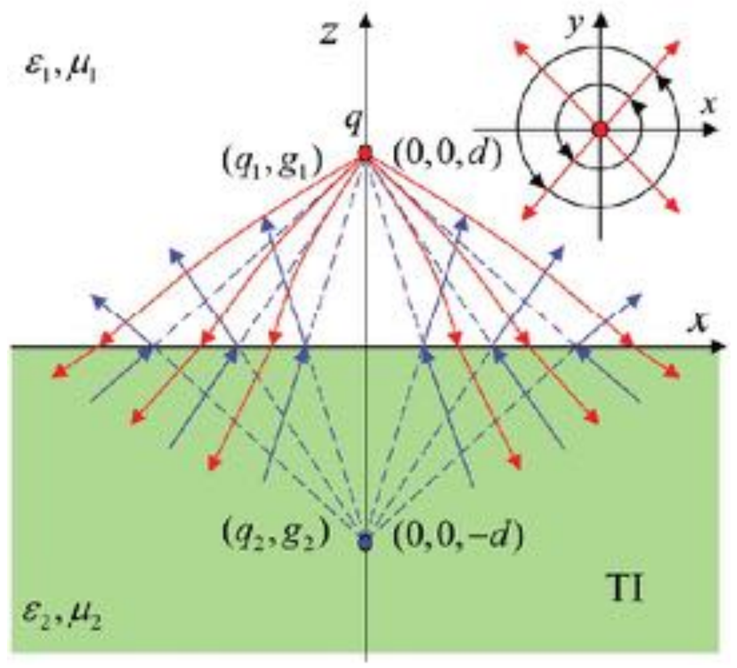
rotation of polarization plane of linearly polarized photon

cf. cosmological birefringence

V. Dziom+ Nat. Commun. 8, 15197 (2017)



- Image monopole effect



X. Qi+ Science 323, 1184 (2009)

Emergence of magnetic fields as if "image monopole" exists

Sketch: emergence of dynamical axion

✓ So, what's next?

- Topological Insulator + Hubbard interaction
- Anti-ferromagnetic ordering spontaneously breaks T-symmetry
  - $\langle \vec{S} \rangle \neq 0$ gives additional contribution and $\theta \neq \pi$
- Spin fluctuation becomes dynamical axion $\delta\theta$
- Topological EM interaction

$$\mathscr{L}_{\rm int} = \frac{\alpha}{\pi} \delta\theta \vec{E} \cdot \vec{B}$$

connection to chiral anomaly

$$H = -t \sum_{<i,j>} \sum_{\sigma} c^{\dagger}_{i\sigma} c_{j\sigma}$$ Tight-binding

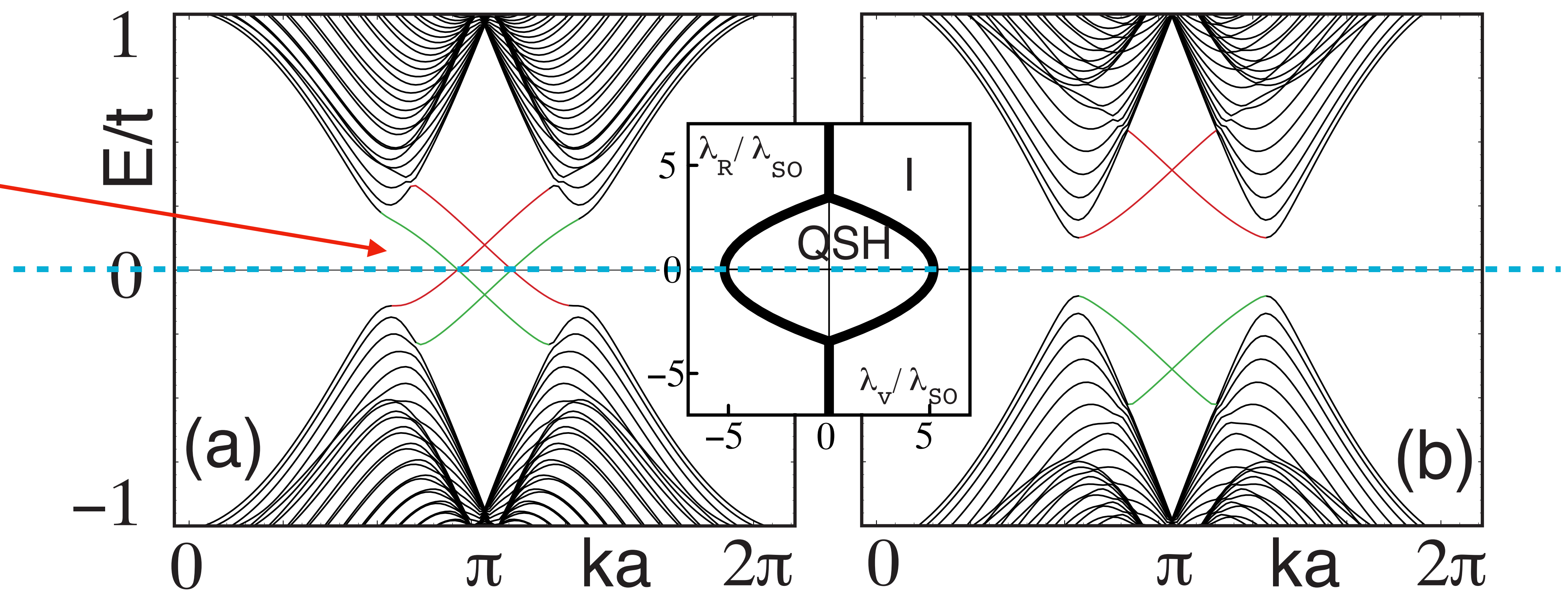
$$+ i\lambda \sum_{<<i,j>>} c^{\dagger}_{i} \vec{\sigma} \cdot (\vec{d}^{\,1}_{ij} \times \vec{d}^{\,2}_{ij}) c_j$$ Topological insulator

$$+ U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$ Anti-ferromagnet

Topological insulator

✓ Insulator bulk + metal surface   cf) bulk-edge correspondence

- Edge states localized on surface leads to conductivity



Topo. insulator phase   Normal insulator phase

✓ Hot Topics in cond-mat & significant developments after Kane-Mele

- Bernevig-Hughes-Zhang (BHZ) model
  B. A. Bernevig, T. L. Hughes, S. Zhang, Science, 314, 1757 (2006)
- HgTe/CdTe quantum well structure explained by BHZ model
  First experimental observation   M. König, et al. Science, Sciencexpress 318, 766 (2007)
- 3 dim. topological insulators : Fu-Kane-Mele model、$Bi_{1-x}Sb_x$
  L. Fu, C. L. Kane, E. J. Mele, PRL 98, 106803 (2007)   D. Hsieh, Nature 452, 970 (2008)

Quantization of spin fluctuations

✓ Quantized spin waves = magnon

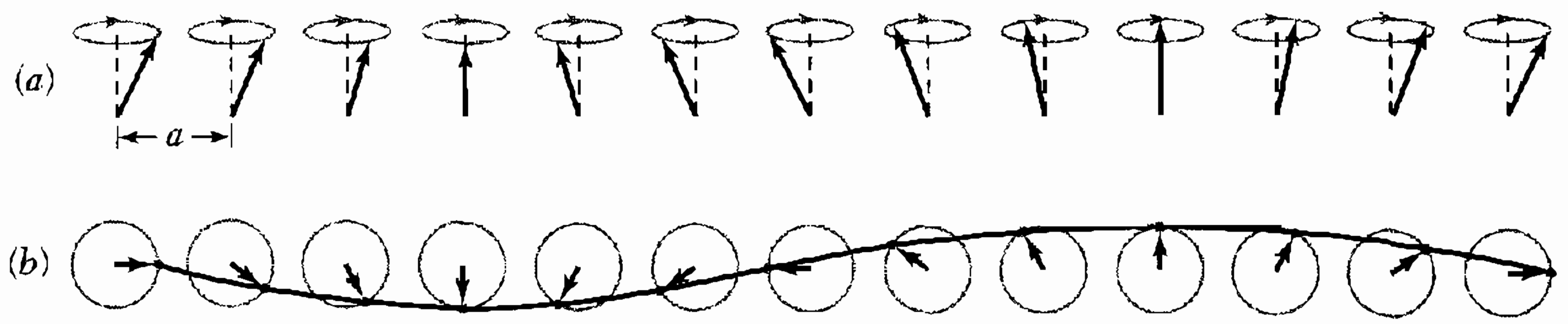


Figure 9 A spin wave on a line of spins. (a) The spins viewed in perspective. (b) Spins viewed from above, showing one wavelength. The wave is drawn through the ends of the spin vectors.

C. Kittel "Introduction to Solid State Physics [8th ed]"

✓ Anti-ferromagnetic Holstein-Primakoff transf.

$$S_\ell^+ = \sqrt{2s - a_\ell^\dagger a_\ell}\, a_\ell \qquad S_{\ell'}^+ = b_{\ell'}^\dagger \sqrt{2s - b_{\ell'}^\dagger b_{\ell'}}$$

$$S_\ell^- = a_\ell^\dagger \sqrt{2s - a_\ell^\dagger a_\ell} \qquad S_{\ell'}^- = \sqrt{2s - b_{\ell'}^\dagger b_{\ell'}}\, b_{\ell'}$$

$$S_\ell^z = s - a_\ell^\dagger a_\ell \qquad S_{\ell'}^z = -s + b_{\ell'}^\dagger b_{\ell'}$$

sublattice A — sublattice B

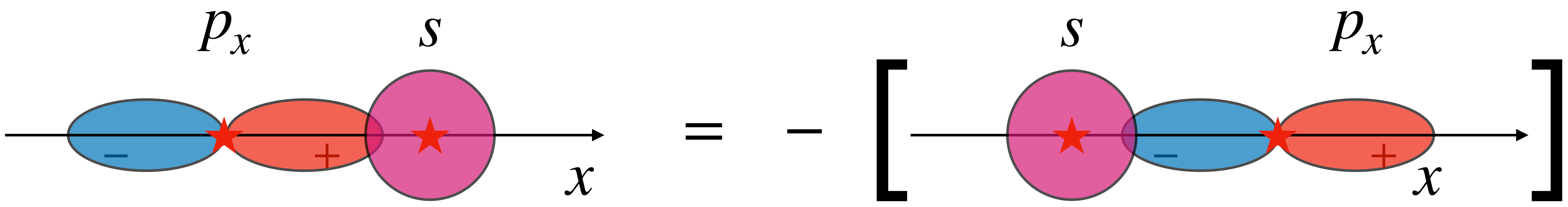
Relates bosonic / spin operators consistently

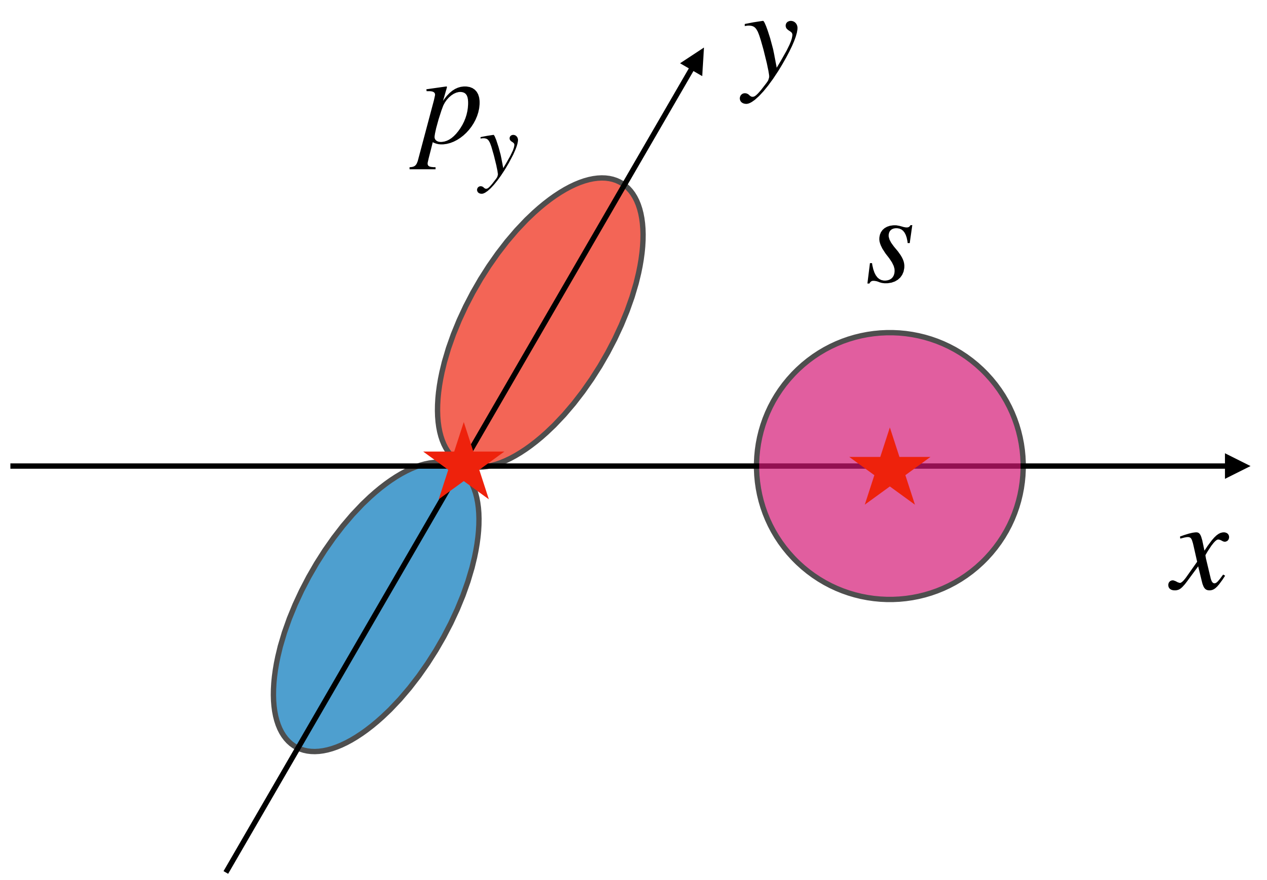
$$\left[a_\ell, a_m^\dagger\right] = \delta_{\ell m} \quad \Rightarrow \quad \left[S_\ell^i, S_m^j\right] = i\epsilon^{ijk} S_\ell^k \delta_{\ell m}$$

$$\left[b_{\ell'}, b_{m'}^\dagger\right] = \delta_{\ell' m'} \quad \Rightarrow \quad \left[S_{\ell'}^i, S_{m'}^j\right] = i\epsilon^{ijk} S_{\ell'}^k \delta_{\ell' m'}$$

Evaluation of hopping terms

✓ Shape and properties of e^- orbitals are important





$= 0$

Slater-Koster relations

TABLE I. Energy integrals for crystal in terms of two-center integrals.

$E_{s,s}$	$(ss\sigma)$
$E_{s,x}$	$l(sp\sigma)$
$E_{x,x}$	$l^2(pp\sigma)+(1-l^2)(pp\pi)$
$E_{x,y}$	$lm(pp\sigma)-lm(pp\pi)$
$E_{x,z}$	$ln(pp\sigma)-ln(pp\pi)$
$E_{s,xy}$	$\sqrt{3}lm(sd\sigma)$
E_{s,x^2-y^2}	$\frac{1}{2}\sqrt{3}(l^2-m^2)(sd\sigma)$
$E_{s,3z^2-r^2}$	$[n^2-\frac{1}{2}(l^2+m^2)](sd\sigma)$
$E_{x,xy}$	$\sqrt{3}l^2m(pd\sigma)+m(1-2l^2)(pd\pi)$
$E_{x,yz}$	$\sqrt{3}lmn(pd\sigma)-2lmn(pd\pi)$
$E_{x,zx}$	$\sqrt{3}l^2n(pd\sigma)+n(1-2l^2)(pd\pi)$

etc.

$$\vec{r}_j - \vec{r}_i = (l, m, n)$$

J. C. Slater, G. F. Koster (1954)

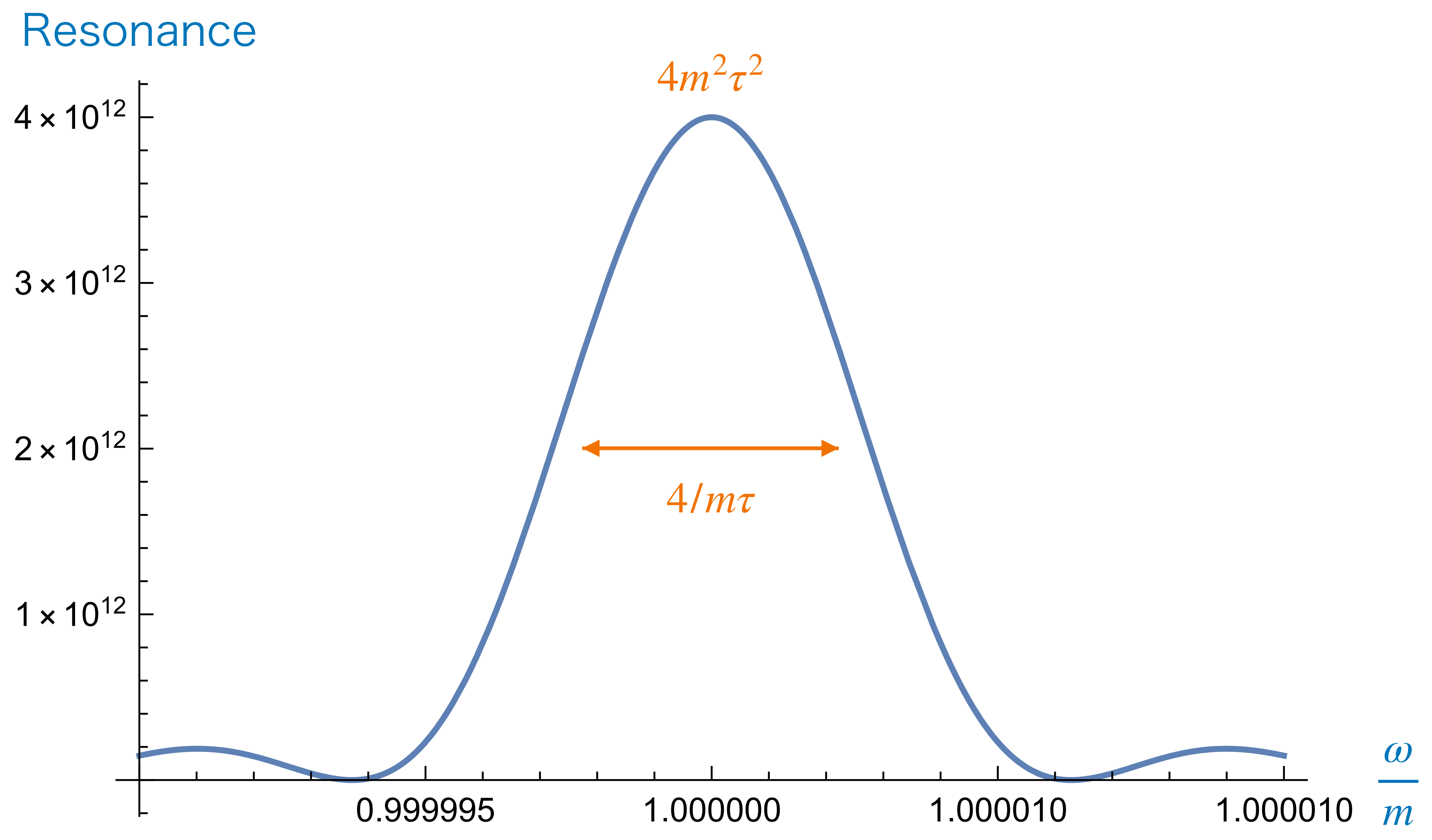
✓ Direction-dependent hopping

- contains information of shape of lattice

Resonance effect

✓ Resonance enhances rate

- long coherence time $\tau \leftrightarrow$ narrower & higher peak
- $\delta(\omega/m - 1)$ @ $\tau \to \infty$



✓ Upper limit on τ determined by

- cond-mat axion lifetime
- DM coherence time $\tau_\chi \sim 1/m_\chi v_\chi^2$