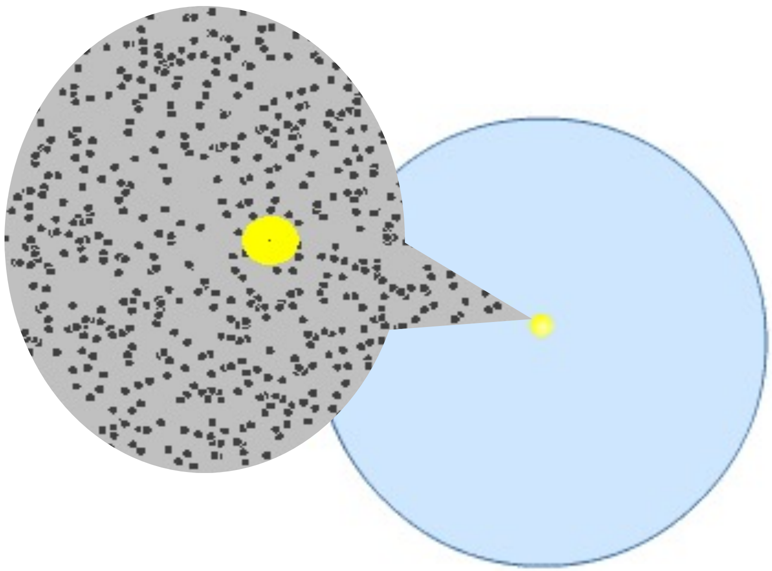
Indirect detection of gravitons through quantum decoherence

Sugumi Kanno (Kyushu University)

Based on
S. Kanno, J. Soda (Kobe), J. Tokuda (Kobe)
PRD103 (2021) 4, 044017
PRD104 (2021) 8, 083516

Our idea

Brownian motion



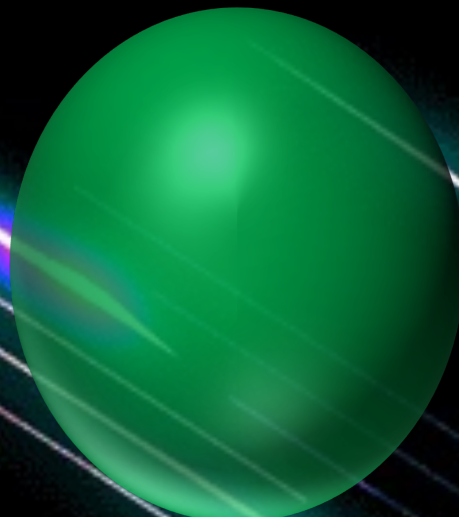
Water molecules was discovered indirectly through the Brownian motion of a minute particle

We want to discover gravitons indirectly through something

Consider a macroscopic object

What if the object is surrounded by gravitons?

Gravitons ← Water molecules



Macroscopic object ← Minute particle

The same setup as the Brownian motion

⬇

Gravitons should create noise in the object

We may be able to prove the existence of gravitons indirectly through noise

What to calculate?

Derive a Langevin equation
(stocastic differential equation)
of this system and identify gravitons

Action for gravitational fields

Metric of tensor perturbations

$$ds^2 = \left[-dt^2 + \left(\delta_{ij} + h_{ij}\right) dx^i dx^j\right]$$

satisfy $h_{ij}{}^{,j} = h^i{}_i = 0$

Action

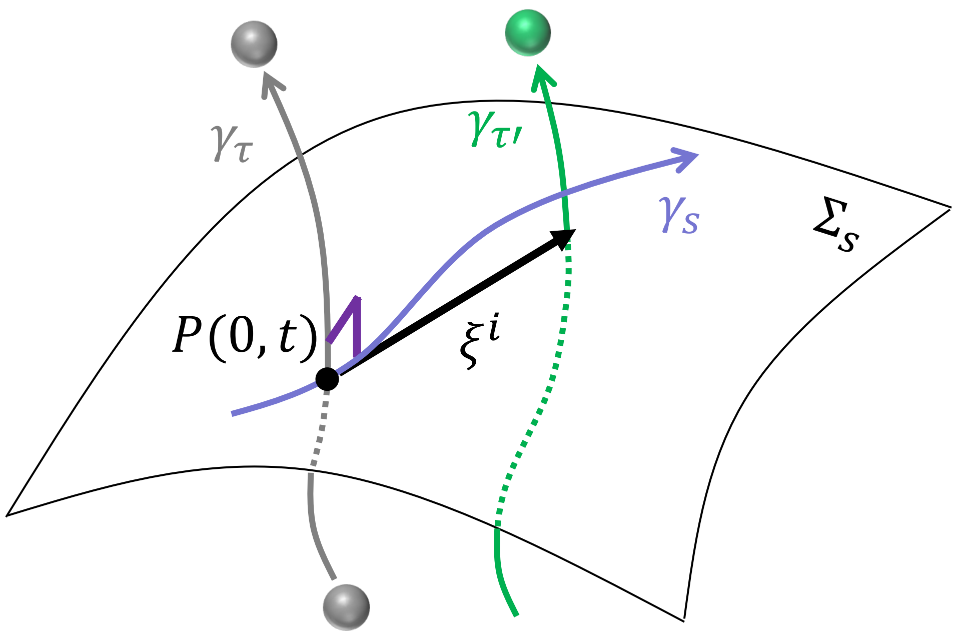
$$S = \frac{M_{\rm pl}^2}{2} \int d^4x \sqrt{-g}\, R \qquad , M_{\rm pl}^2 = \frac{1}{8\pi G}$$

$$= \frac{M_{\rm pl}^2}{4} \int dt\, d^3x \left[\frac{1}{2}\dot{h}_{ij}\dot{h}_{ij} - \frac{1}{2} h_{ij,k}\, h_{ij,k}\right] \qquad \cdot = \partial_t$$

Action for an object : Fermi Normal Coordinate

An object does not feel gravity ➡ *Geodesic deviation*

Local inertial coordinates that are adapted to the geodesic



Action of an object up to the 2nd order of ξ^i is

Gauge invariant ⋯▸ TT gauge

$$S = \int dt \left[\frac{m}{2} \left(\dot{\xi}^i\right)^2 - \frac{m}{2} R_{0i0j}(0,t) \xi^i \xi^j \right]$$

interaction between gravity and an object

In TT gauge

$$R_{0i0j} = -\frac{1}{2} \ddot{h}_{ij}$$

The total action for $h_{\boldsymbol{k}}^{A}$ and an object & quantization

Total Action in Fourier mode

Mass of an object

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,R - m\int d\tau \qquad , M_{\rm pl}^2 = \frac{1}{8\pi G}$$

$$= \int dt \sum_{\mathbf{k}} \sum_{A=+,\times} \left[\frac{1}{2}\,\dot{h}_{\mathbf{k}}^{A}\,\dot{h}_{\mathbf{k}}^{*A} - \frac{1}{2}k^2 h_{\mathbf{k}}^{A}\,h_{\mathbf{k}}^{*A}\right]$$

$$+\int dt\left[\frac{m}{2}\left(\dot{\xi}^i\right)^2 + \frac{1}{2}\frac{m}{M_{\rm pl}}\frac{1}{\sqrt{V}}\sum_{\mathbf{k},A}\left[\,e_{ij}^{A}(\mathbf{k})\,\ddot{h}_{\mathbf{k}}^{*A}\,\xi^i\xi^j\,\right]\right]$$

qubic interaction

Quantize the $h_{\mathbf{k}}^{A}$ and the ξ^i (in the interaction picture)

We promote the free metric field $h_{\mathbf{k}}^{A}(t)$ to the operator $\hat{h}_{\mathbf{k}}^{A}(t)$

Positive freq. mode in the Minkowski space

The Minkowski vacuum

$$\hat{h}_{\mathbf{k}}^{A}(t) = \hat{a}_{\mathbf{k}}^{A}v_k(t) + \hat{a}_{-\mathbf{k}}^{A\dagger}\,v_k^*(t) \qquad \hat{a}_{\mathbf{k}}^{A}|0\rangle = 0 \qquad \left[\hat{a}_{\mathbf{k}}^{A}, \hat{a}_{\mathbf{p}}^{B\dagger}\right] = \delta^{AB}\delta_{\mathbf{kp}}$$

Since $\xi^i(t)$ is just a position, we promote $\hat{\xi}^i(t)$ to the operator as well

EOM for the total system

EOMs

$$\begin{cases} \ddot{\hat{h}}_{\mathbf{k}}^{*A} + k^2 \hat{h}_{\mathbf{k}}^{A} = \frac{1}{2}\frac{m}{M_{\mathrm{pl}}\sqrt{V}}\, e_{ij}^{A}(\mathbf{k})\, \frac{d^2}{dt^2}\left(\hat{\xi}^i \hat{\xi}^j\right) \\ \ddot{\hat{\xi}}^i = \frac{m}{M_{\mathrm{pl}}\sqrt{V}} \sum_{\mathbf{k},A} \left[\, e_{ij}^{A}(\mathbf{k})\, \ddot{\hat{h}}_{\mathbf{k}}^{A}\, \hat{\xi}^j \,\right] \end{cases}$$

⬅ Solvable

The solution of the $\hat{h}_{k}^{A}$ is obtained by starndard Green's funciton technique

initial position — initial velocity

$$\hat{h}_{\mathbf{k}}^{A} = \delta\hat{h}_{\mathbf{k}}^{A}(0)\cos kt + \delta\dot{\hat{h}}_{\mathbf{k}}^{A}(0)\,\frac{\sin kt}{k}$$

quantum fluctuation (gravitons)

$$+\frac{1}{2}\frac{m}{M_{\mathrm{pl}}\sqrt{V}}\, e_{ij}^{A}(\mathbf{k}) \int_0^t dt' \frac{\sin k(t-t')}{k} \frac{d^2}{dt'^2}\left(\hat{\xi}^i(t')\,\hat{\xi}^j(t')\right)$$

quantum fluctuation (object)

Plug this solution into the EOM for $\hat{\xi}^i$ and get EOM for $\hat{\xi}^i$ (Langevin equation)

Langevin equation of geodesic deviation

Kanno, Soda & Tokuda (2020)
Parikh, Wilczek & Zahariade (2020)

Langevin equation

$$\ddot{\hat{\xi}}^i + \omega^2 \hat{\xi}^j(t) + \frac{m}{40\pi M_{\rm pl}^2}\left(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{k\ell}\right)\hat{\xi}^i \frac{d^5}{dt^5}\left(\hat{\xi}^k \hat{\xi}^\ell\right)$$

frictional force (radiation reaction force)

$$= -\hat{N}_{ij}(t)\hat{\xi}^j(t)$$

random force ⋯ not determined by an initial condition of $\hat{\xi}^i$

where $\hat{N}_{ij}(t)$ consists of the initial condition of gravitons (noise)

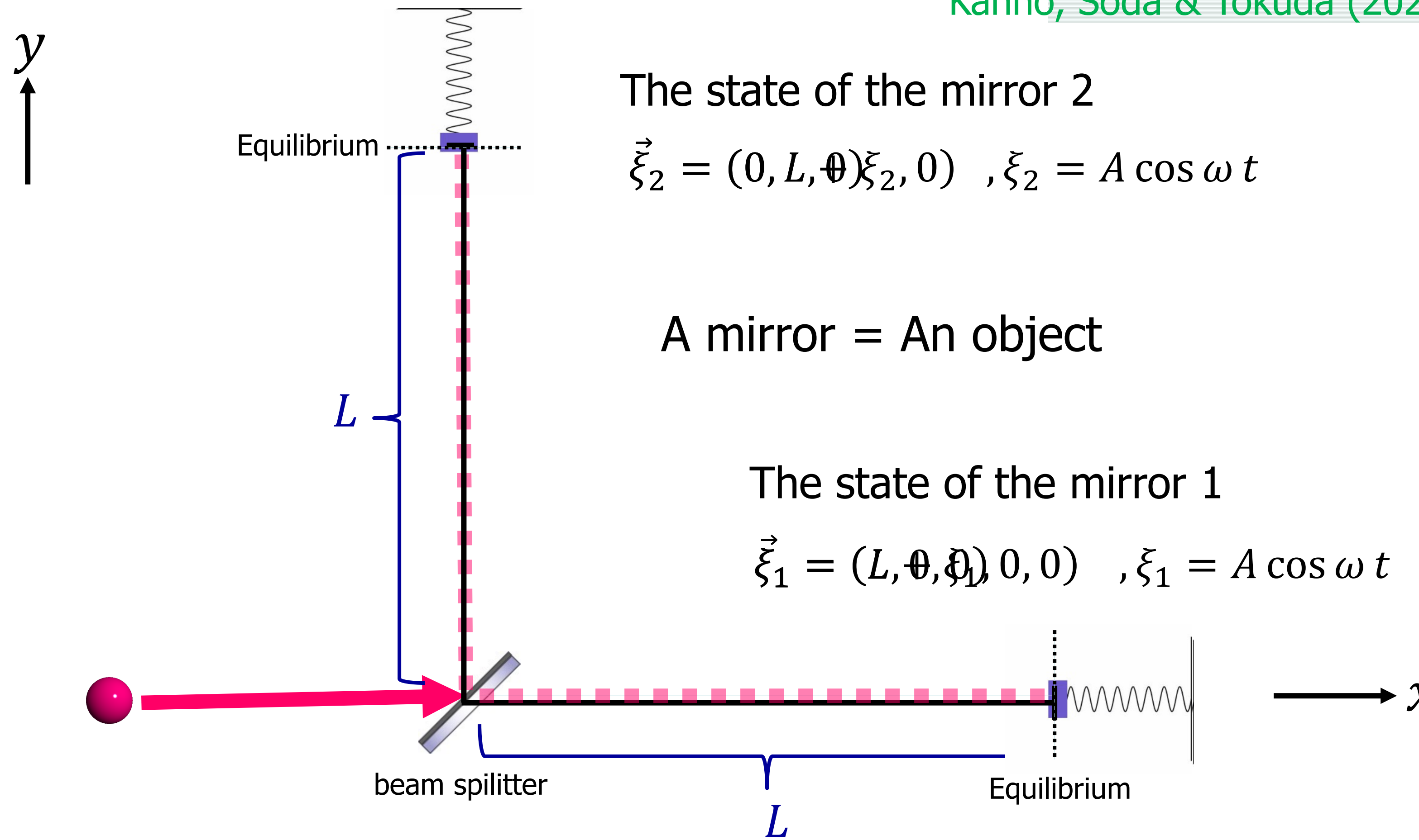
$$\hat{N}_{ij}(t) = \frac{1}{M_{\rm pl}\sqrt{V}} \sum_{\boldsymbol{k},A} k^2 e_{ij}^A(\boldsymbol{k}) \left\{ \delta \hat{h}_{\boldsymbol{k}}^A(0)\cos kt + \delta \dot{\hat{h}}_{\boldsymbol{k}}^A(0) \frac{\sin kt}{k} \right\}$$

The noise is identified as quantum fluctuations of gravitons

We consider a setup for measuring the decoherence time due to this noise of gravitons.

Setups

Kanno, Soda & Tokuda (2021)



The state of the mirror 2

$\vec{\xi}_2 = (0, L, 0)$ / $\vec{\xi}_2 = (0, L + \xi_2, 0)$, $\xi_2 = A\cos\omega t$

A mirror = An object

The state of the mirror 1

$\vec{\xi}_1 = (L, 0, 0)$ / $\vec{\xi}_1 = (L + \xi_1, 0, 0)$, $\xi_1 = A\cos\omega t$

A photon is in a superposition state of being in both the upper and lower arms until it is detected by the oscillation of either mirrors

The state of both mirrors is maximally entangled state

$$\frac{1}{\sqrt{2}}|\vec{\xi}_1\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|\vec{\xi}_2\rangle$$

, $|0\rangle \equiv$ ground state

The initial state

The initial entangled mirror state

If we consider the environmental gravitons,

$$|\psi(t_\mathrm{i})\rangle = \left(\frac{1}{\sqrt{2}}|\vec{\xi}_1\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|\vec{\xi}_2\rangle\right)|h\rangle$$

initial time

initial state of gavitons

The density operator of the total system is

Now we focus on the system of mirrors, the reduced density operator is

$$\rho(t_\mathrm{i}) = |\psi(t_\mathrm{i})\rangle\langle\psi(t_\mathrm{i})|$$

$$\rho_m(t_\mathrm{i}) = \mathrm{Tr}_h|\psi(t_\mathrm{i})\rangle\langle\psi(t_\mathrm{i})|$$

$$= \sum_i \langle i|\left(\frac{1}{\sqrt{2}}|\vec{\xi}_1\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|\vec{\xi}_2\rangle\right)|h\rangle\langle h|\left(\frac{1}{\sqrt{2}}\langle\vec{\xi}_1|\langle 0| + \frac{1}{\sqrt{2}}\langle 0|\langle\vec{\xi}_2|\right)|i\rangle\langle i|h\rangle$$

$$= \rho_{11}(t_\mathrm{i}) + \rho_{22}(t_\mathrm{i}) + \rho_{12}(t_\mathrm{i}) + \rho_{21}(t_\mathrm{i})$$

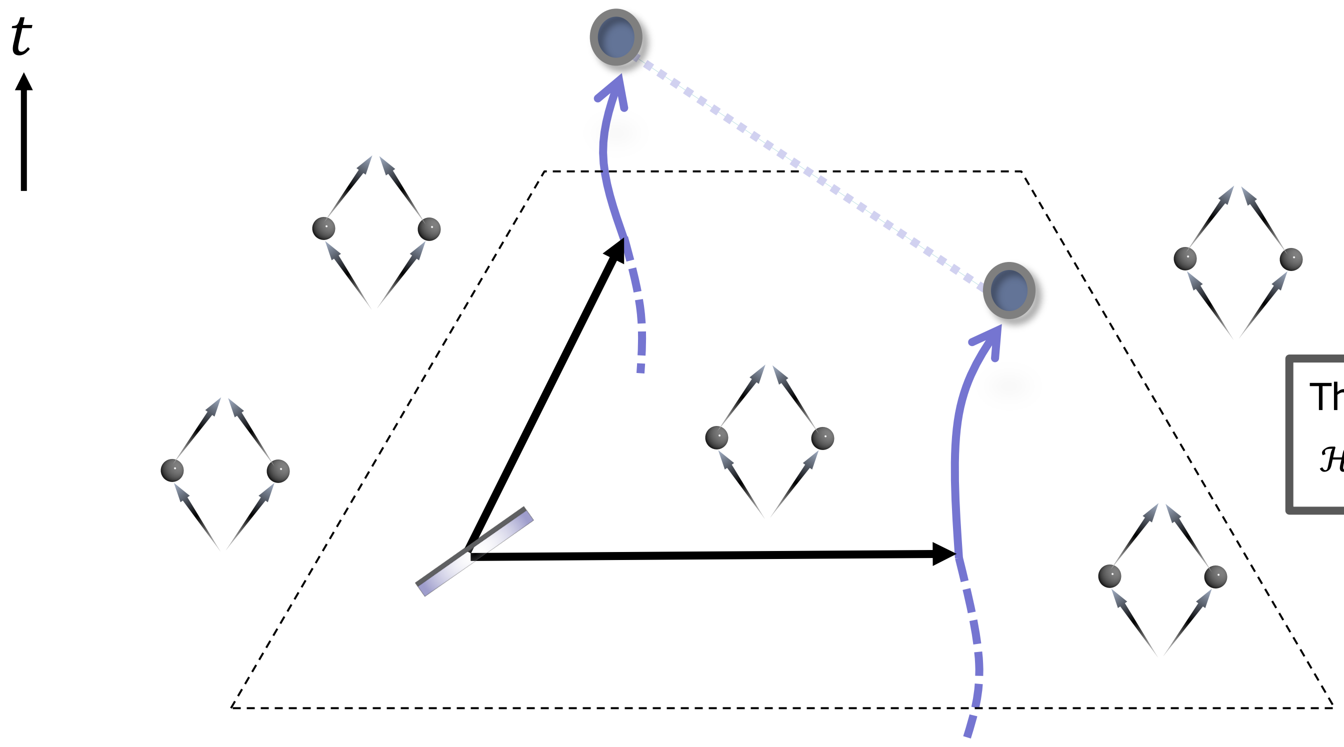
e.g. $\rho_{11} \equiv \frac{1}{2}|\vec{\xi}_1\rangle|0\rangle\langle\vec{\xi}_1|\langle 0|$

off-diagonal elements: "quantum interference term"

The interference term appears due to the initial entangled state of the mirrors

Time evolution of the total system (schematic pic.)

The total system evolves in time according to the Liouvill-von Neumann eq



The noise of gravitons

$\hat{N}_{ij}$

The interaction Hamiltonian

$\mathcal{H}_{\rm int}(t,x) \sim \ddot{h}_{ij}(t,0)\xi^i\xi^j$

The state of gravitons changes as a result of the interaction with each mirror

$$|\psi(t_{\rm f})\rangle = \frac{1}{\sqrt{2}}|\vec{\xi}_1\rangle\,|0\rangle\,|h;\vec{\xi}_1\rangle + \frac{1}{\sqrt{2}}|0\rangle|\vec{\xi}_2\rangle\,|h;\vec{\xi}_2\rangle$$

system gravitons

final time

The system of the mirrors gets entangled with enviromnental gravitons

After time evolution

"The total system (mirrors+gravitons) gets entangled as a result of the interaction"

↓ From the point of view of the mirrors

"The decoherece of the mirrors occurs"

The final state of the mirror after solving the Liouvill-von Neumann eq is expressed as

Φ : influence functional

$$\rho_m(t_\mathrm{f}) = \mathrm{Tr}_h|\psi(t_\mathrm{f})\rangle\langle\psi(t_\mathrm{f})| = \rho_{11}(t_\mathrm{i}) + \rho_{22}(t_\mathrm{i}) + \exp(i\Phi)\rho_{12}(t_\mathrm{i}) + \exp(-i\Phi^*)\rho_{21}(t_\mathrm{i})$$

↓ imaginary part Φ, Φ^* ↓

$\exp(-\mathrm{Im}\Phi)$ $\exp(\mathrm{Im}\Phi^*)$

The environmental gravitons affect the interference terms through $\mathrm{Im}\Phi$

Since the imaginary part of Φ supress the interference term, it is refered to as the *decoherence functional*

$\Gamma = \mathrm{Im}\Phi \sim 1$ $\cdots$ decoherence is effective

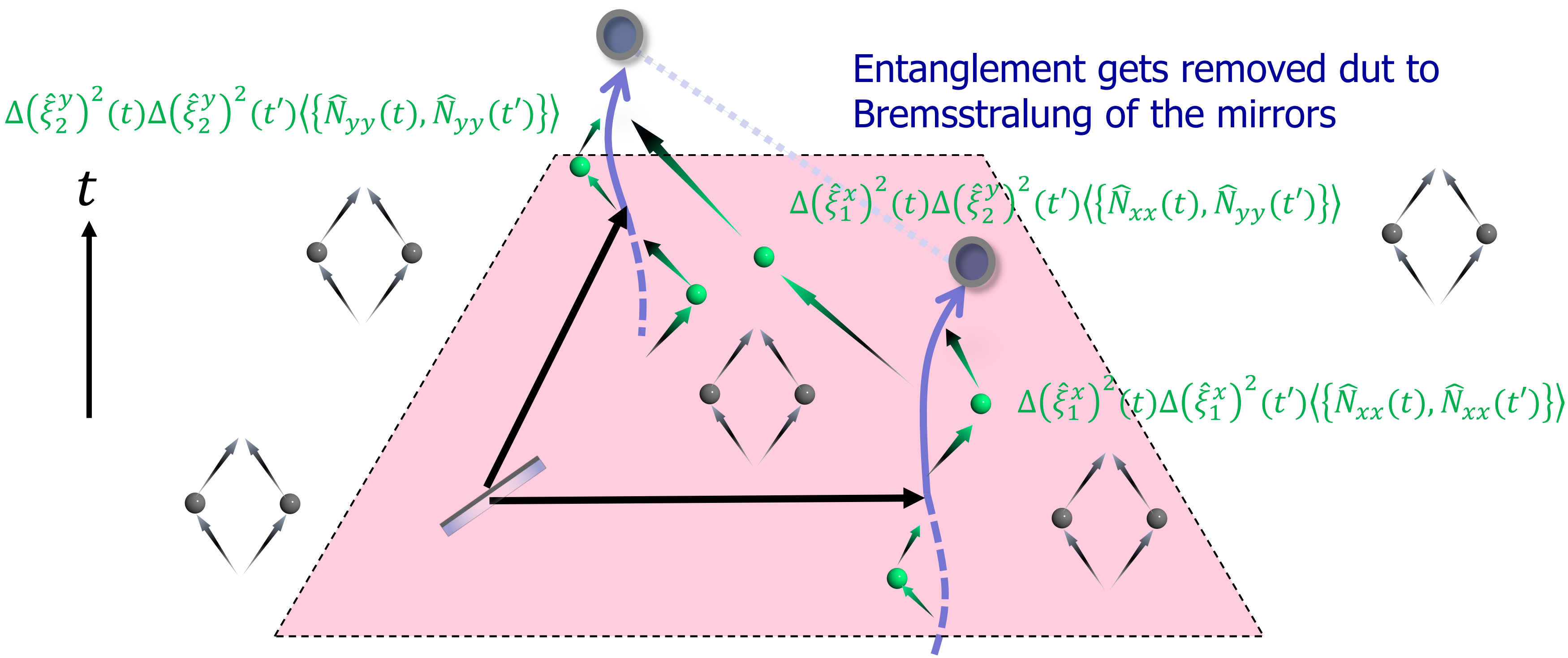
If Γ is calculated, we can read off the decoherence time caused by gravitons
→ indirect detection of gravitons

How decoherece happens?

The decoherence functional

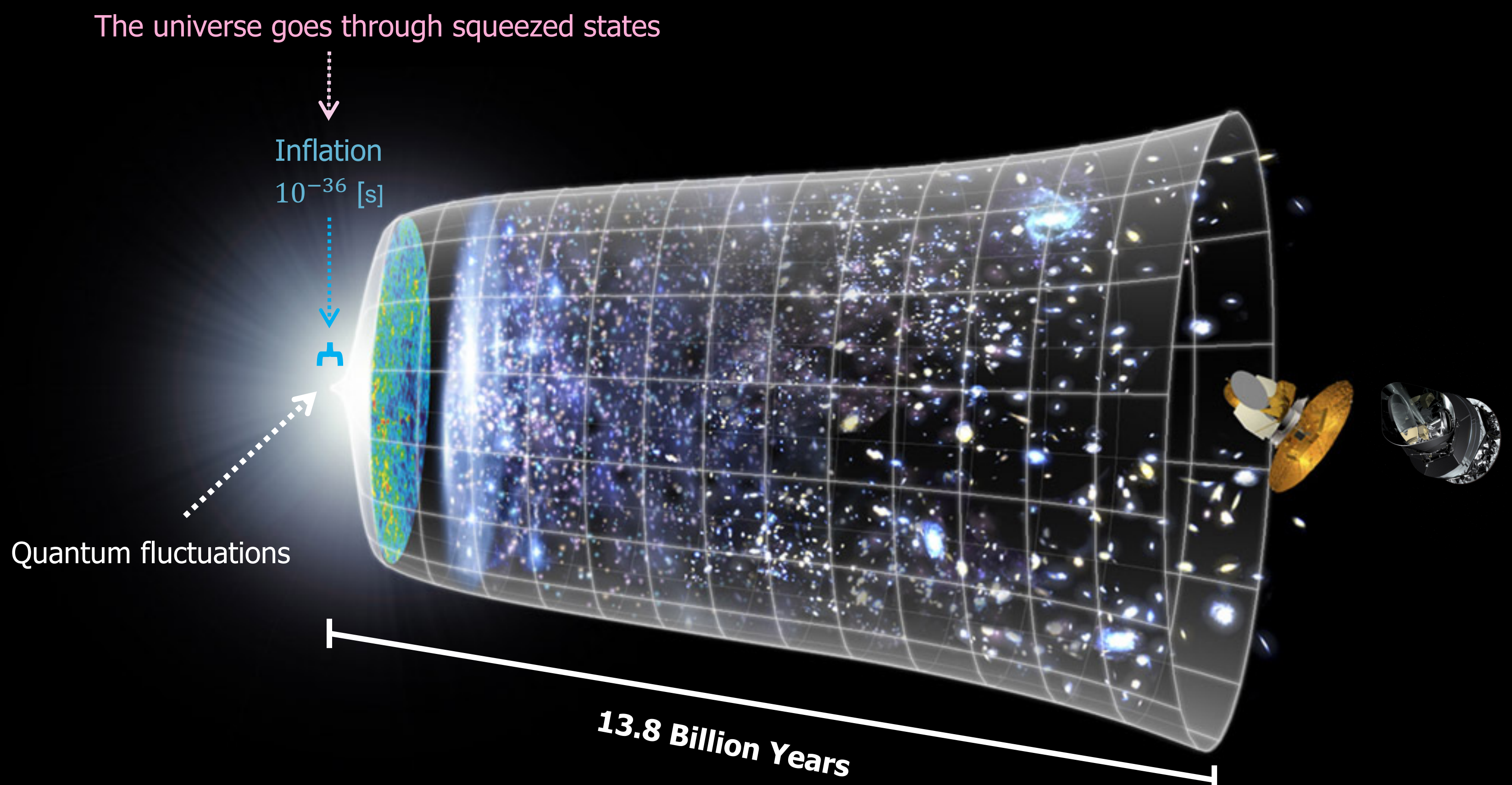
separation of a superposition of the mirrors — Correlation of graviton ⬅ in squeezed states

$$\Gamma = \frac{m^2}{8}\int_0^{t_{\mathrm{f}}} dt\, \Delta\left(\xi^i \xi^j\right)(t) \int_0^{t_{\mathrm{f}}} dt' \Delta\left(\xi^k \xi^\ell\right)(t') \left\langle \left\{ \hat{N}_{ij}(t), \hat{N}_{k\ell}(t') \right\} \right\rangle \sim 1$$



Inflationary universe predicts squeezed states

Our current state should be squeezed if the universe went through inflation in the past



Sqeezed states?

In an expanding universe, the vacuum that seems to be the most natural is different in each era

The relation between the vacuum during inflation and radiation dominated era

$$|0\rangle_{\mathrm{I}} = \prod_{\mathbf{k}} N_k \exp\left(e^{-i(\theta_k+\varphi_k)} \tanh r_k \, \hat{b}^\dagger_{\mathbf{k}} \hat{b}^\dagger_{-\mathbf{k}}\right) |0_{\mathbf{k}}\rangle_{\mathrm{R}} \otimes |0_{-\mathbf{k}}\rangle_{\mathrm{R}}$$

normalization

Squeezing parameters

Inflation

The squeeze operator

Radiation dominated era

The vacuum during inflation looks like a two-mode squeezed state of the modes $\mathbf{k}$ and $-\mathbf{k}$ of a gravitational field from the point of view of the radiation dominated era

If we expand the squeeze operator in Taylor series

$$|0\rangle_{\mathrm{I}} \sim |0_{\mathbf{k}}\rangle_{\mathrm{R}} |0_{-\mathbf{k}}\rangle_{\mathrm{R}} + e^{-i(\theta_k+\varphi_k)} \tanh r_k \, |1_{\mathbf{k}}\rangle_{\mathrm{R}} |1_{-\mathbf{k}}\rangle_{\mathrm{R}} + \cdots + e^{-in(\theta_k+\varphi_k)} \tanh^n r_k \, |n_{\mathbf{k}}\rangle_{\mathrm{R}} |n_{-\mathbf{k}}\rangle_{\mathrm{R}}$$

The squeezed state is an entangled state between the modes $\mathbf{k}$ and $-\mathbf{k}$ of gravitons

A huge number of entangled gravitons are filled in the squeezed states

In the highly squeezing limit $r_k \to \infty$, the vacuum $|0\rangle_{\mathrm{I}}$ looks like a maximally entangled state

Decoherence time

Large quantum objects: two of LIGO’s mirrors, each weighing 40 kg. (Courtesy: Caltech/MIT/LIGO Lab)

Other possible sources of decoherence?

Kanno, Soda & Tokuda (2021)

Common scattering-induced decoherence: thermal photons & air molecules

Decoherence due to thermal photons < Decoherence due to air molecules

The decoherence rate

Mass of an individual air molecule $\sim 0.5\times 10^{-25}$ kg

$$\Lambda = \frac{8}{3\hbar^2} n\sqrt{2\pi M}\, R^2 (k_B T)^{\frac{3}{2}} A^2$$

R: radius of a mirror ~ 0.17 meter (LIGO)

Number density of air molecules $\sim 10^{12}$ per 1 m^3 for ultrahigh vacuuum 10^{-10} Pa

The decoherence time $= \Lambda^{-1} \approx 1200 \left(\frac{R}{0.17\ \mathrm{meter}}\right)^{-2} \left(\frac{T}{10\ \mathrm{K}}\right)^{-\frac{2}{3}}$ s $\quad \gg 20$ s

Decoherence induced by gravitons occurs faster than that by air molecules

Also, coherent component of gravitons does not contribute to the decoherence rate
Kanno, Soda & Tokuda (2020)

Our setup is not disturbed by common scattering-induced decoherence

The squeeze state contains huge # of gravitons, much larger than the # of air molecules

Summary

We performed an experimental setup for detecting gravitons indirectly by observing the decoherence time of the entanglement between the macroscopic mirrors of an equal-arm interferometer.

We estimated that the decoherence time induced by the noise of gravitons in squeezed states stemming from inflation is approximately 20s for 40km long arms and 40kg mirrors.

If the decoherence time in laboratory agreed with our prediction, it implies discovery of gravitons.