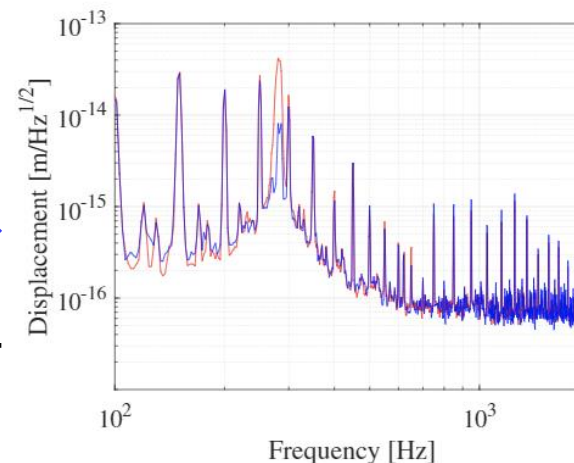
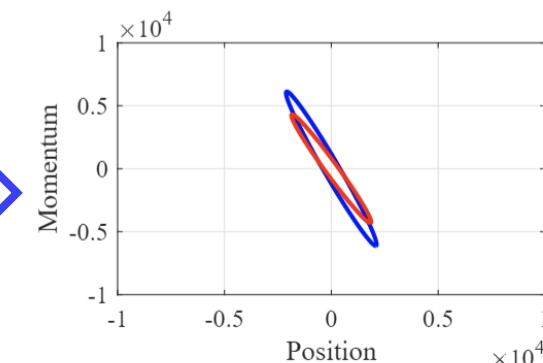


Continuous meas.



Quantum filter

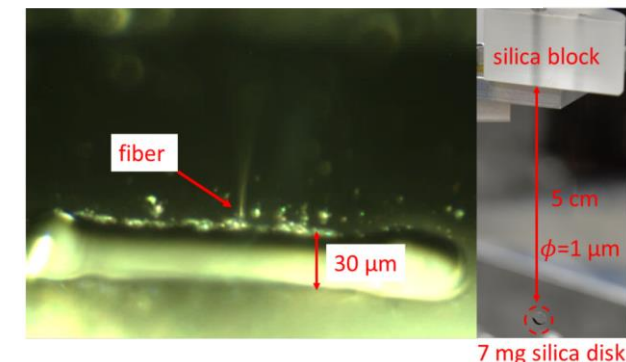
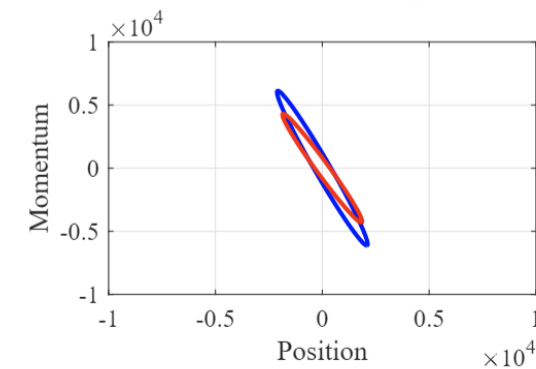
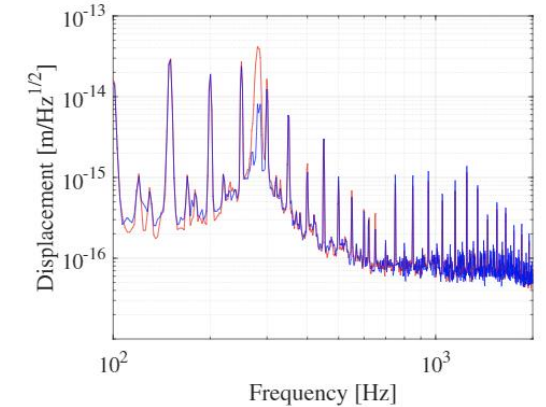
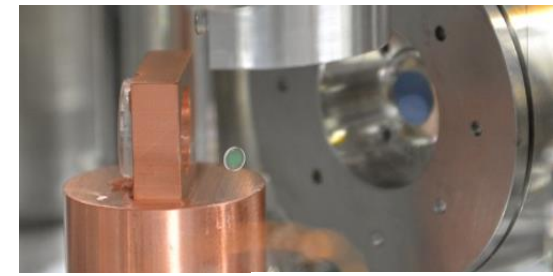


Demonstration of conditional mechanical squeezing of a massive pendulum

Gakushuin University
Nobuyuki Matsumoto
2022/2/10 @KEK

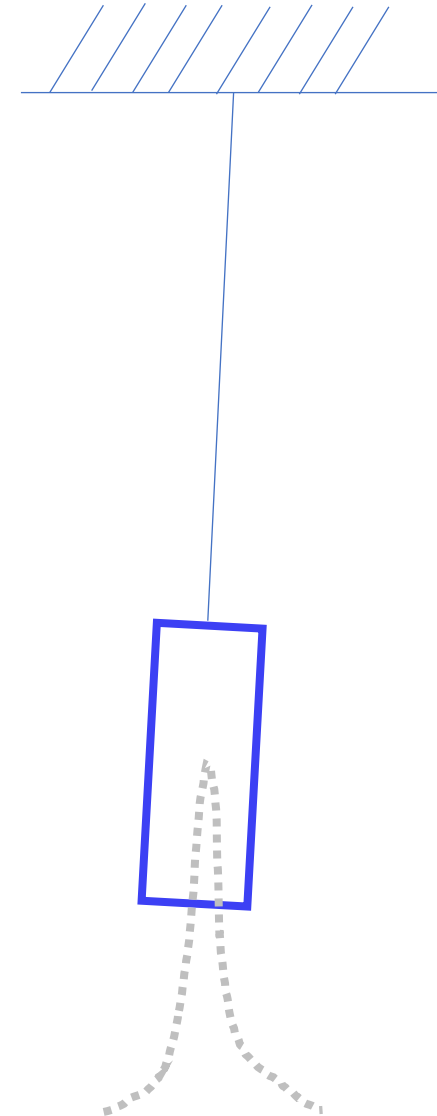
Contents

- Massive objects and thermal decoherence
- Linear continuous measurement
(related article: PRL 122, 071101 (2019))
- Conditional mechanical position squeezing
(related article: arXiv:2008.10848)
- Low-loss pendulum towards quantum squeezing
(related article: PRL 124, 221102 (2020))
- Summary



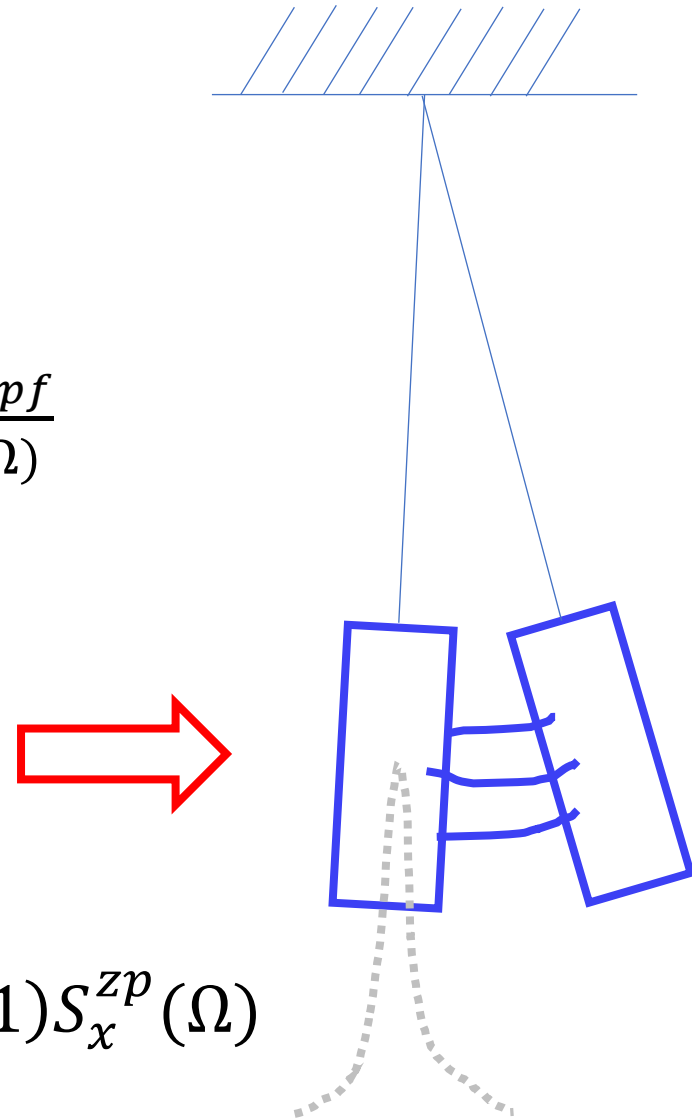
Massive objects and thermal decoherence

- Mass m
- Frequency Ω
- Dissipation γ
- zero point fluctuation spectrum: $S_x^{zp}(\Omega) = \frac{8x_{zpf}^2}{\gamma(\Omega)}$
- $x_{zpf} = \sqrt{\frac{\hbar}{2m\Omega}} = 3 \times 10^{-18} \text{ m} \left(\frac{1 \text{ kg}}{m}\right)^{0.5} \left(\frac{1 \text{ Hz}}{\frac{\Omega}{2\pi}}\right)^{0.5}$



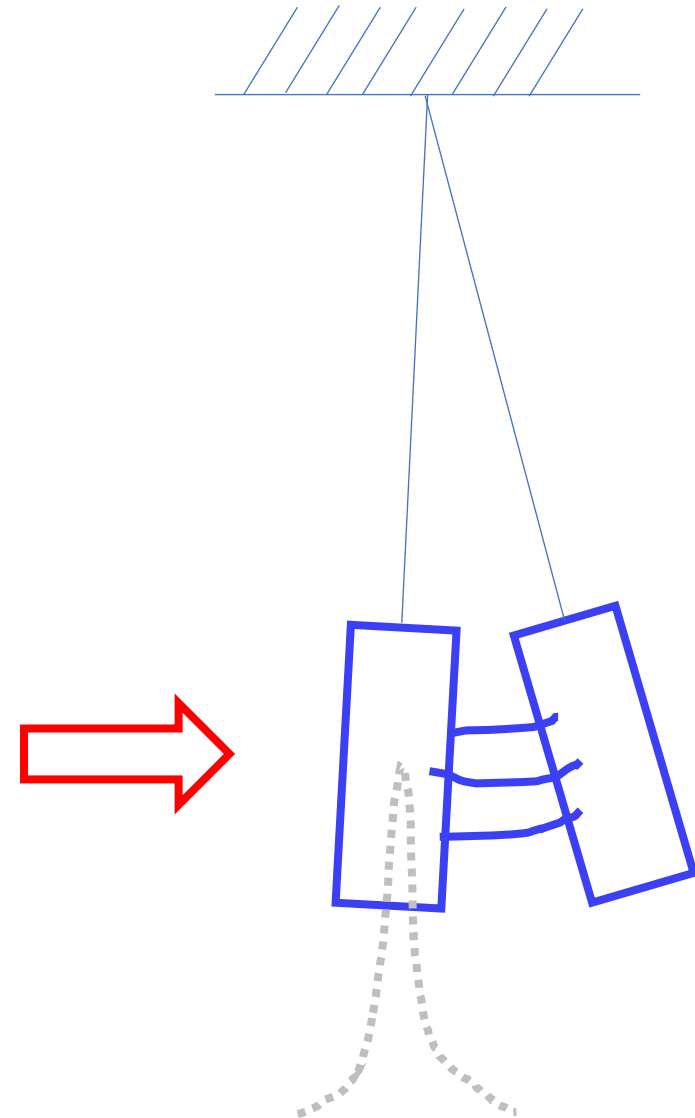
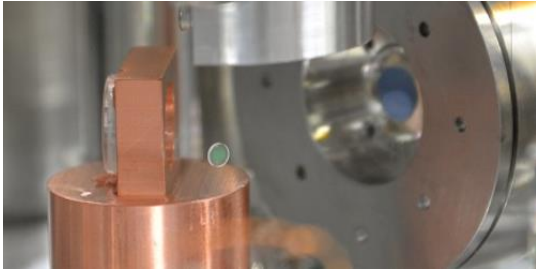
Massive objects and thermal decoherence

- Mass m
- Frequency Ω
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- zero point fluctuation spectrum: $S_x^{zp}(\Omega) = \frac{8x_{zpf}^2}{\gamma(\Omega)}$
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- Thermal decoherence
→ Quantum + Brown motion: $S_{xx}(\Omega) = (2n_{th} + 1)S_x^{zp}(\Omega)$



Thermal phonon

- $n_{th} \simeq \frac{k_B T}{\hbar \Omega} \sim 2 \times 10^{10} \left(\frac{T}{300 \text{ K}} \right) \left(\frac{300 \text{ Hz}}{\frac{\Omega}{2\pi}} \right)$



Measurement-based control

- Feedback control

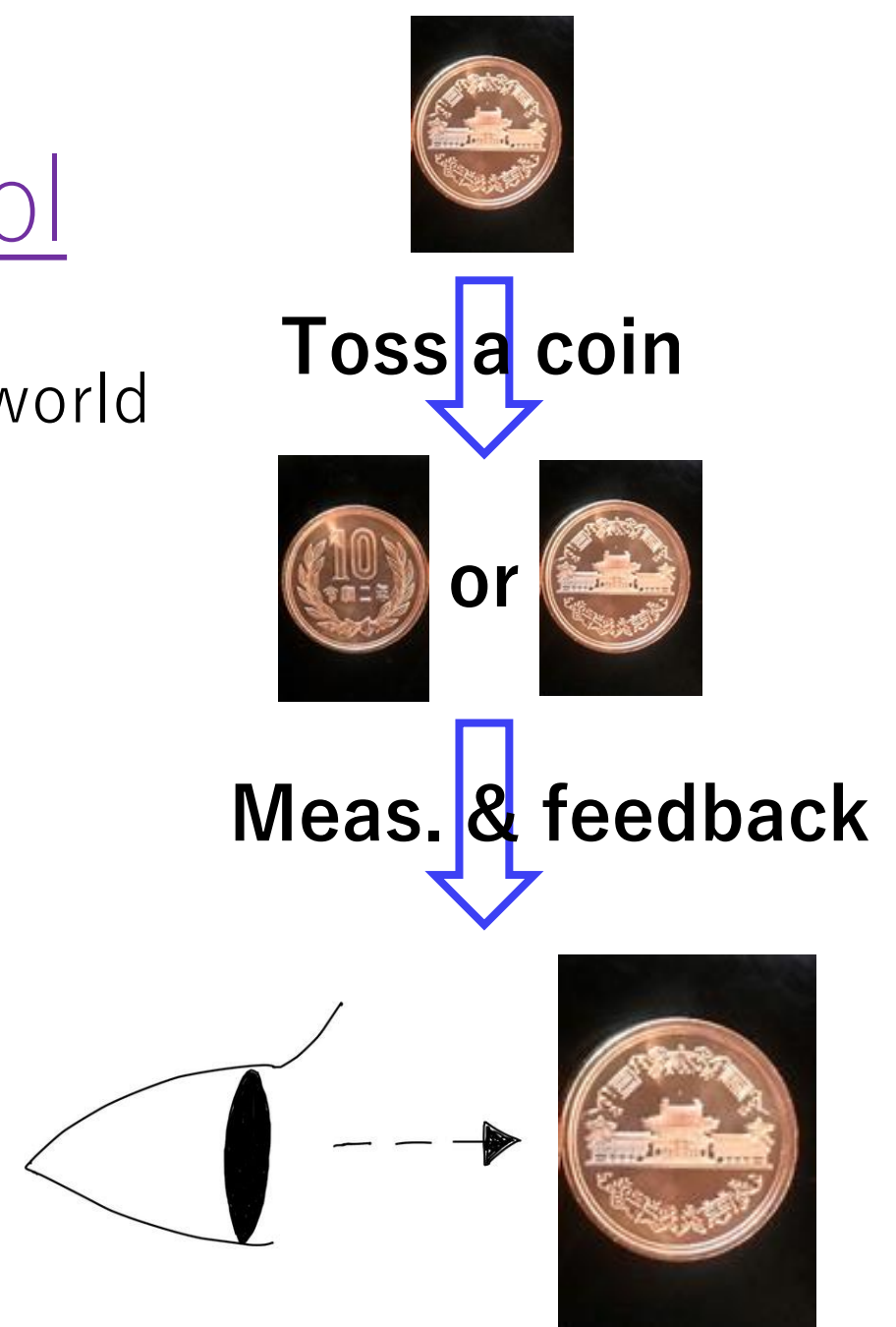
Control algorithm implemented in real physical world

e.g. Coin probability $P(k) = \frac{1}{2}, k = heads, tails$

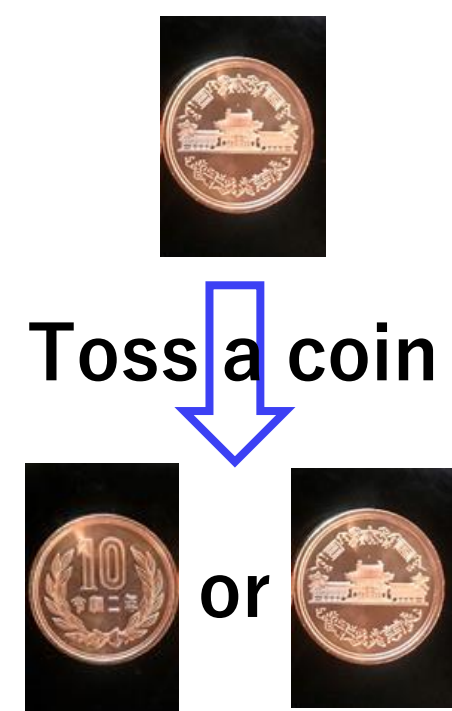


Measurement-based control

- Feedback control
Control algorithm implemented in real physical world
e.g. Coin probability $P(k) = \frac{1}{2}, k = heads, tails$
Coin is back, then **actuators** turn it over
 $P(heads) = 1$



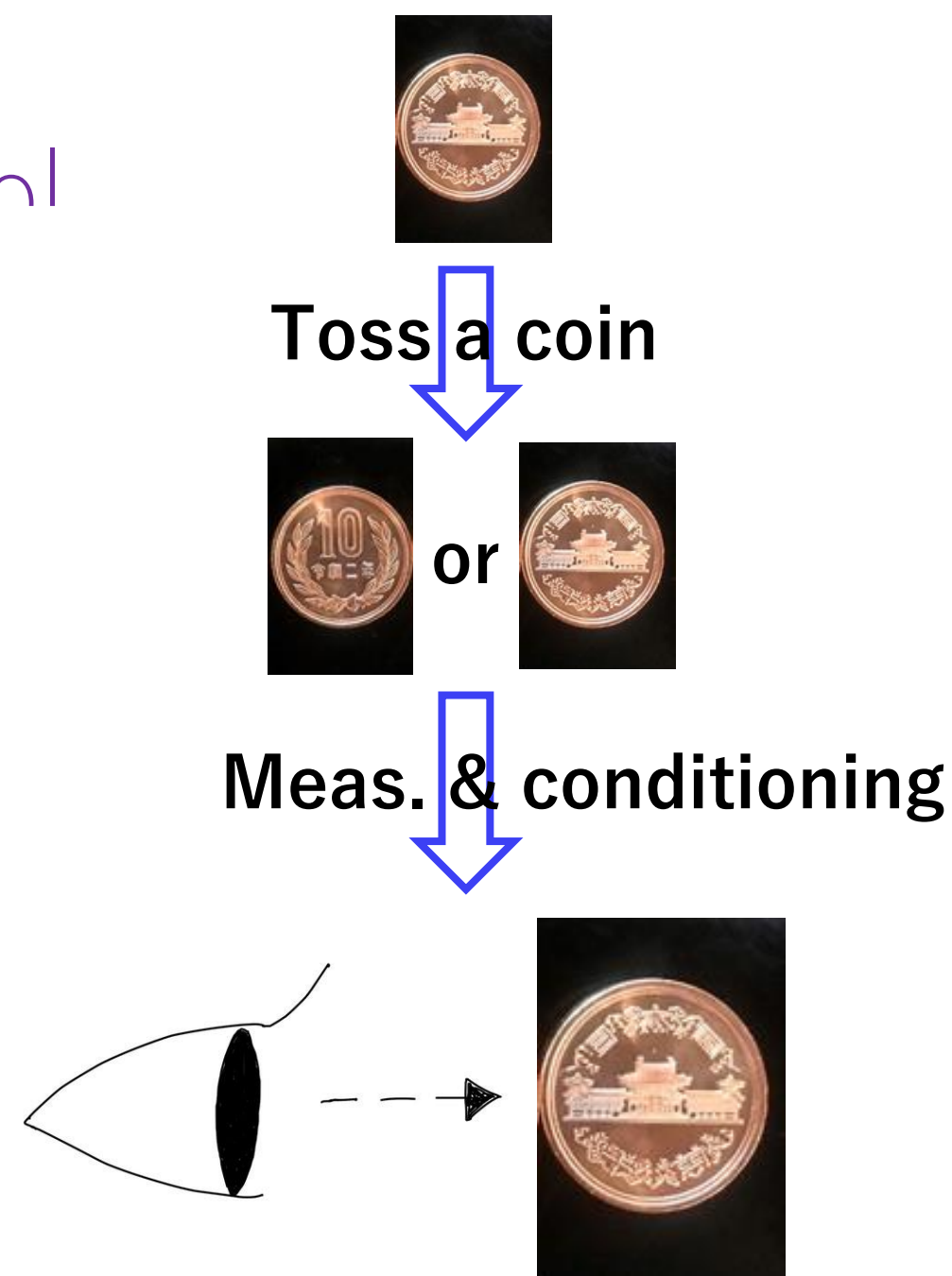
Measurement-based control



- Conditioning
Control algorithm without actuator
→ create a conditional state
e.g. Coin probability $P(k) = \frac{1}{2}, k = heads, tails$

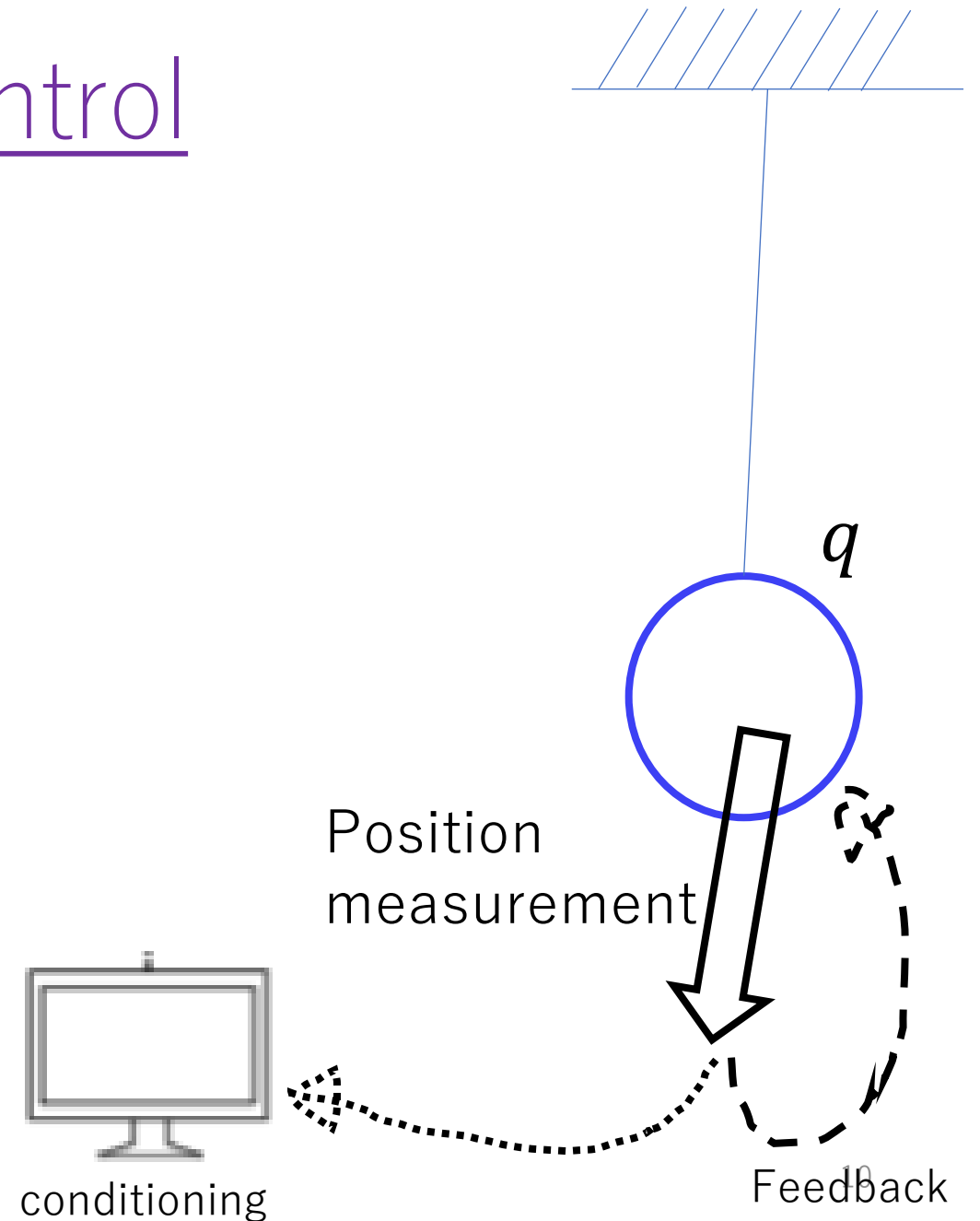
Measurement-based control

- Conditioning
Control algorithm without actuator
→ create a conditional state
e.g. Coin probability $P(k) = \frac{1}{2}, k = \text{heads}, \text{tails}$
Someone tells you it is front, then
 $P(\text{heads}|\text{heads}) = 1$



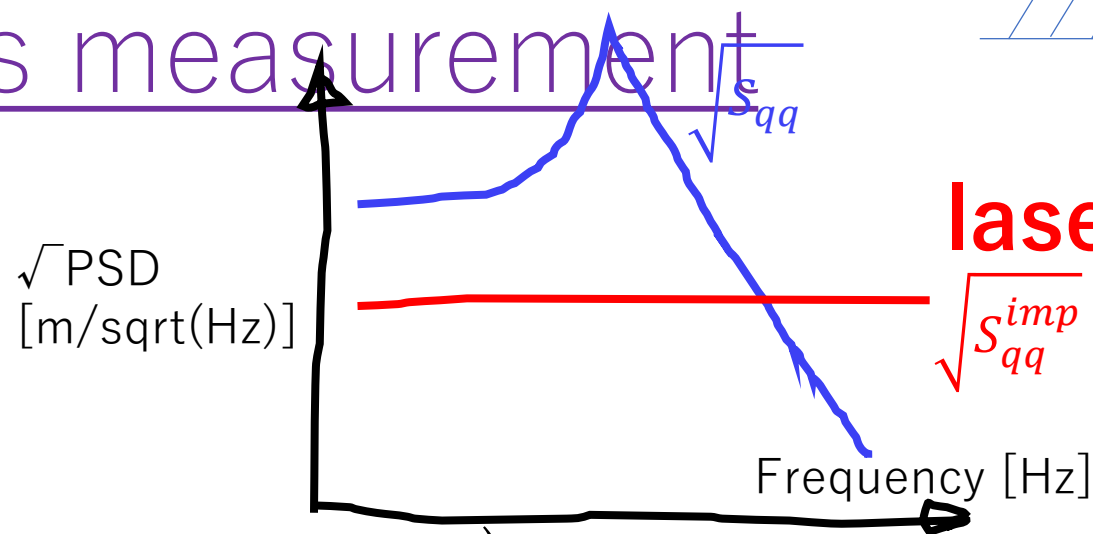
Measurement-based control

- Continuous position measurement
→ Obtain velocity by differentiating
- Reduce velocity by FB or conditioning
→ reduce mode temperature
→ cooling
- We use the both of control



Linear continuous measurement

- $X(t) = Aq(t) + \zeta(t)$
- X : optical power
- q : position
- A : sensitivity coefficient
- ζ : sensing noise (read out noise, impression noise)



laser noise

$$\sqrt{S_{qq}^{imp}}$$

Frequency [Hz]

q

- Displacement-referred noise level: $S_{qq}^{imp} = \frac{S_{\zeta\zeta}}{A^2}$
- Sensitivity (ZPF-referred noise level): $n_{imp} = \frac{S_{qq}^{imp}}{2S_{qq}^{zpf}} \propto \gamma$

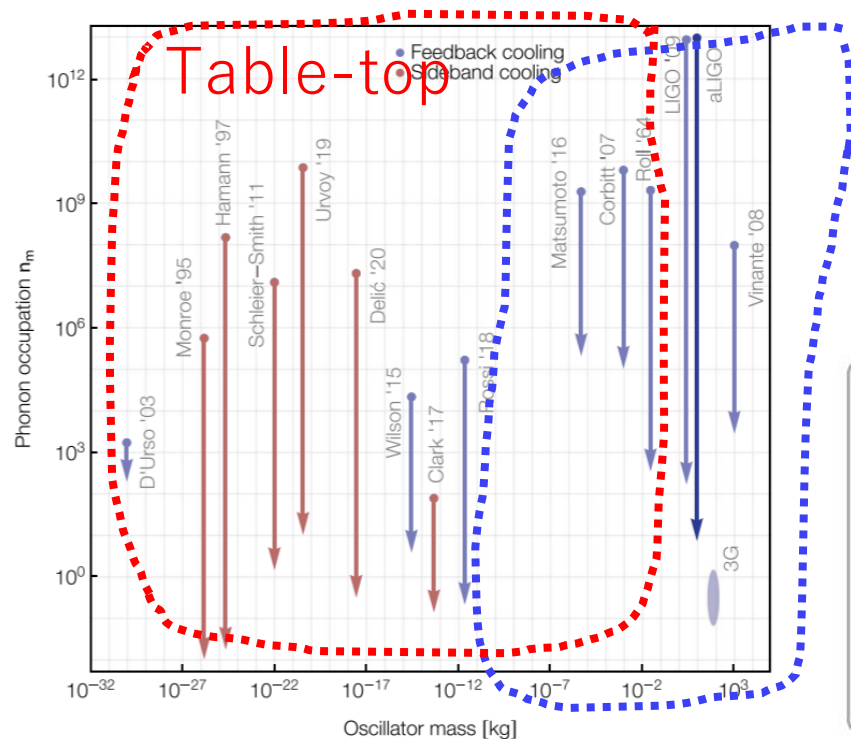
- Requirement: $n_{imp} \sim \frac{1}{2n_{th}} \sim 2 \times 10^{-11} \left(\frac{300 \text{ K}}{T} \right) \left(\frac{\frac{\Omega}{2\pi}}{300 \text{ Hz}} \right)$

laser

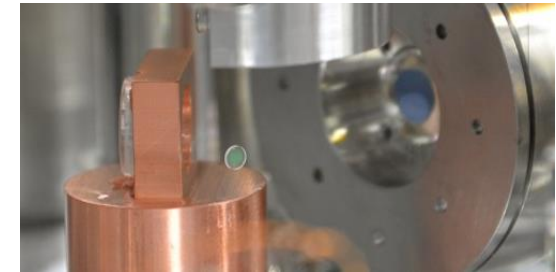
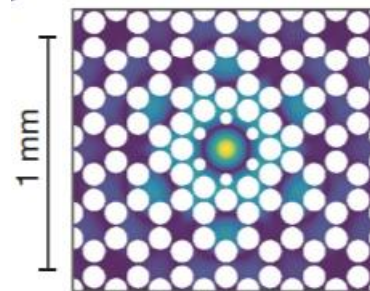
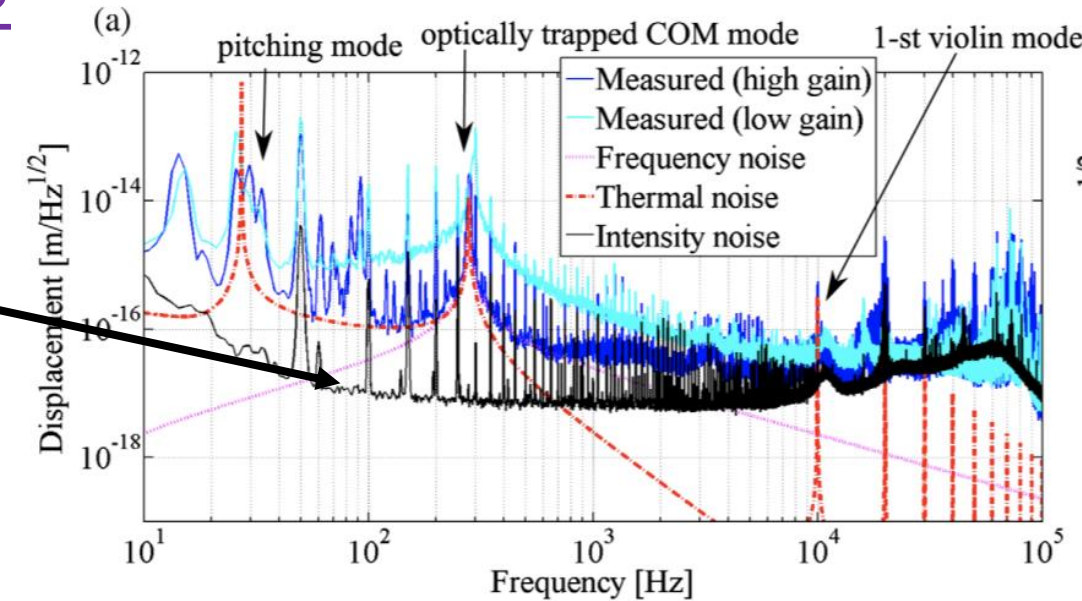
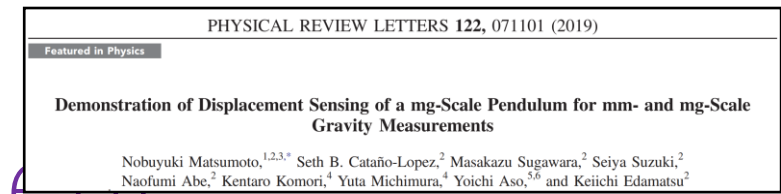
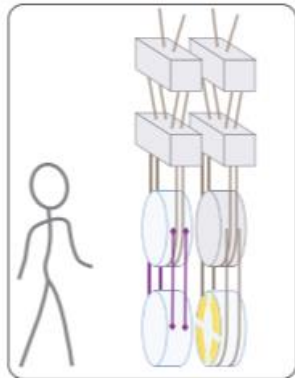
$$X(t) = Aq(t) + \zeta(t)$$

Result1: continuous measurement

- $\sqrt{S_{qq}^{imp}} \simeq 10^{-17} \frac{\text{m}}{\sqrt{\text{Hz}}}$
- $n_{imp} \simeq 3 \times 10^{-8}$
- Requirement: $n_{imp} \sim 2 \times 10^{-11}$



Unexplored region

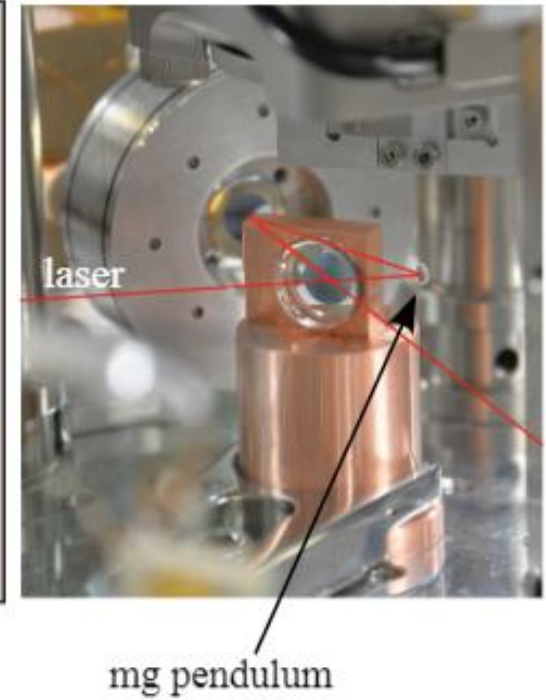
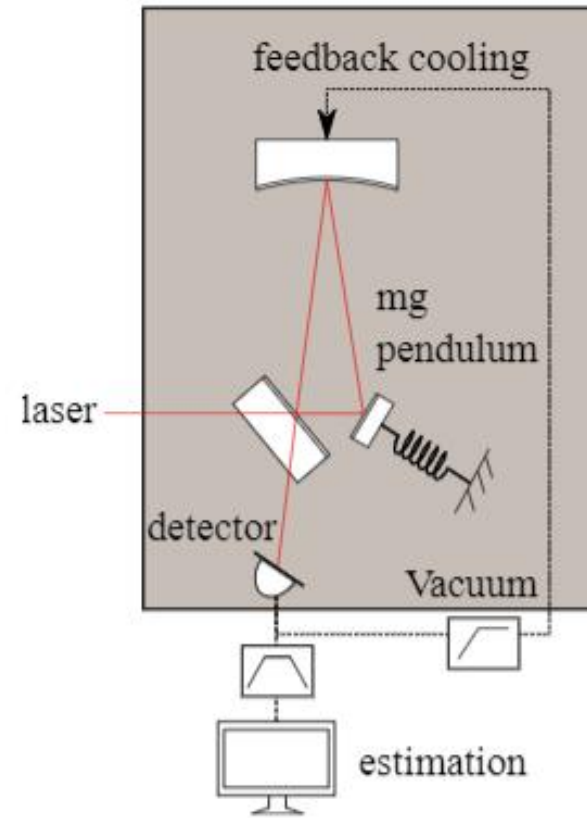


$n_{imp} \simeq 10^{-7}$, Nature 563, 53–58 (2018).

$n_{imp} \simeq 3.5 \times 10^{-13}$, Science 372, 1333–1336 (2021)

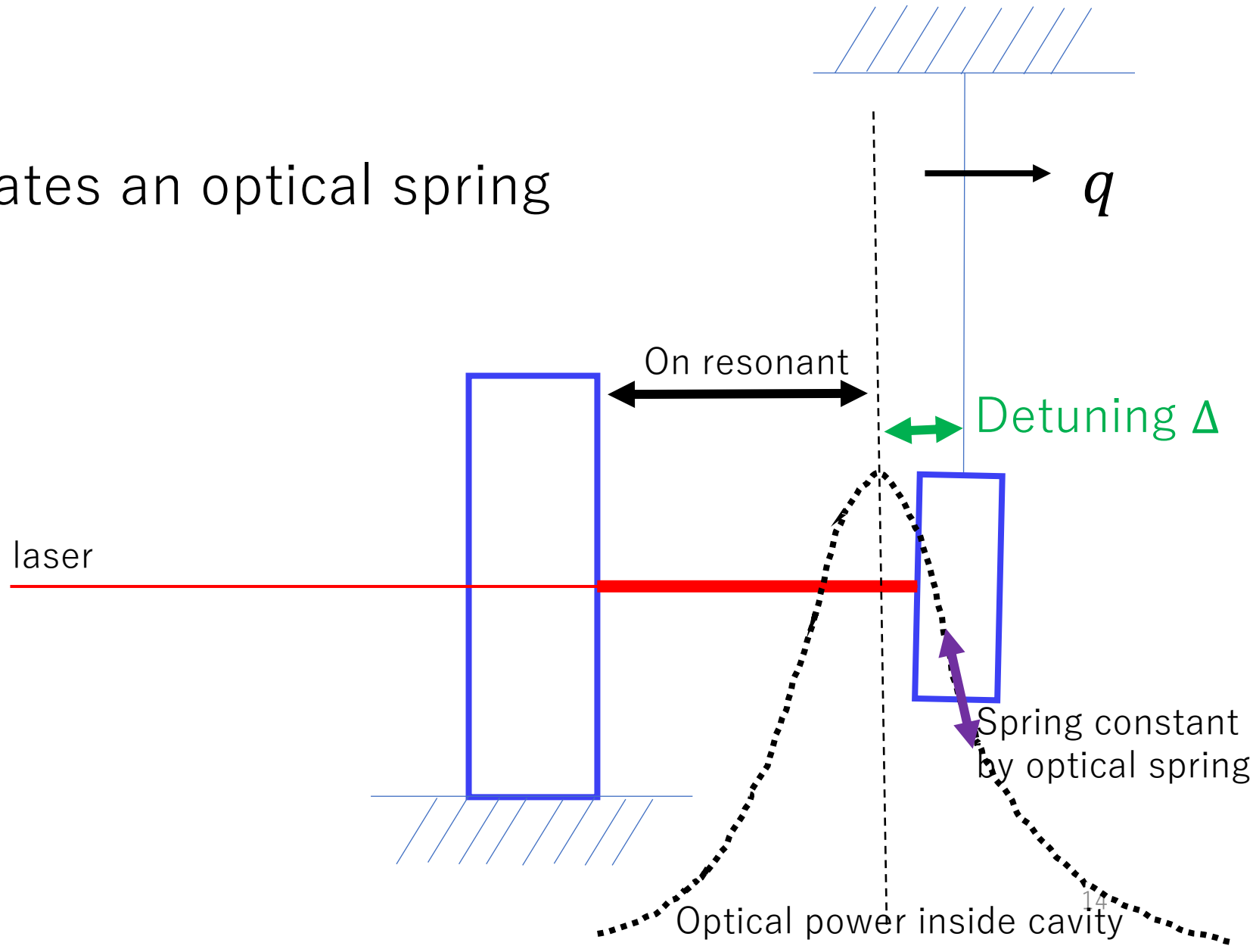
Our system

- Hamiltonian
- $H = \frac{\hbar\Omega}{4}(q^2 + p^2) - \frac{\hbar\Delta}{4}(x^2 + y^2) + \hbar gxq$
- Equation of motion
- $\dot{q} = \omega_m p$
- $\dot{p} = -\omega_m q - \gamma_m p + \text{noise terms}$
- $X = Aq + \text{noise terms}$ (X : traveling wave, x : cavity mode)
- Optical spring increases resonance: $\Omega \rightarrow \omega_m$
- Feedback cooling decreases mode temperature



Optical spring

- Detuned cavity creates an optical spring



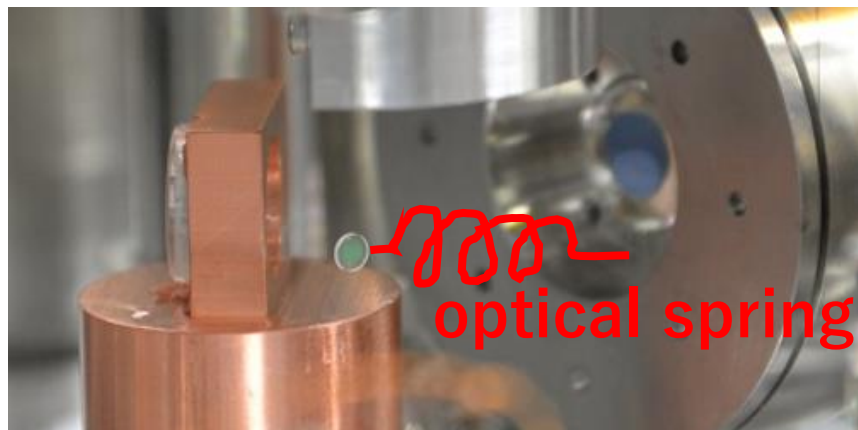
Optical spring

- Cavity with a pendulum
→ **optically trapped pendulum**

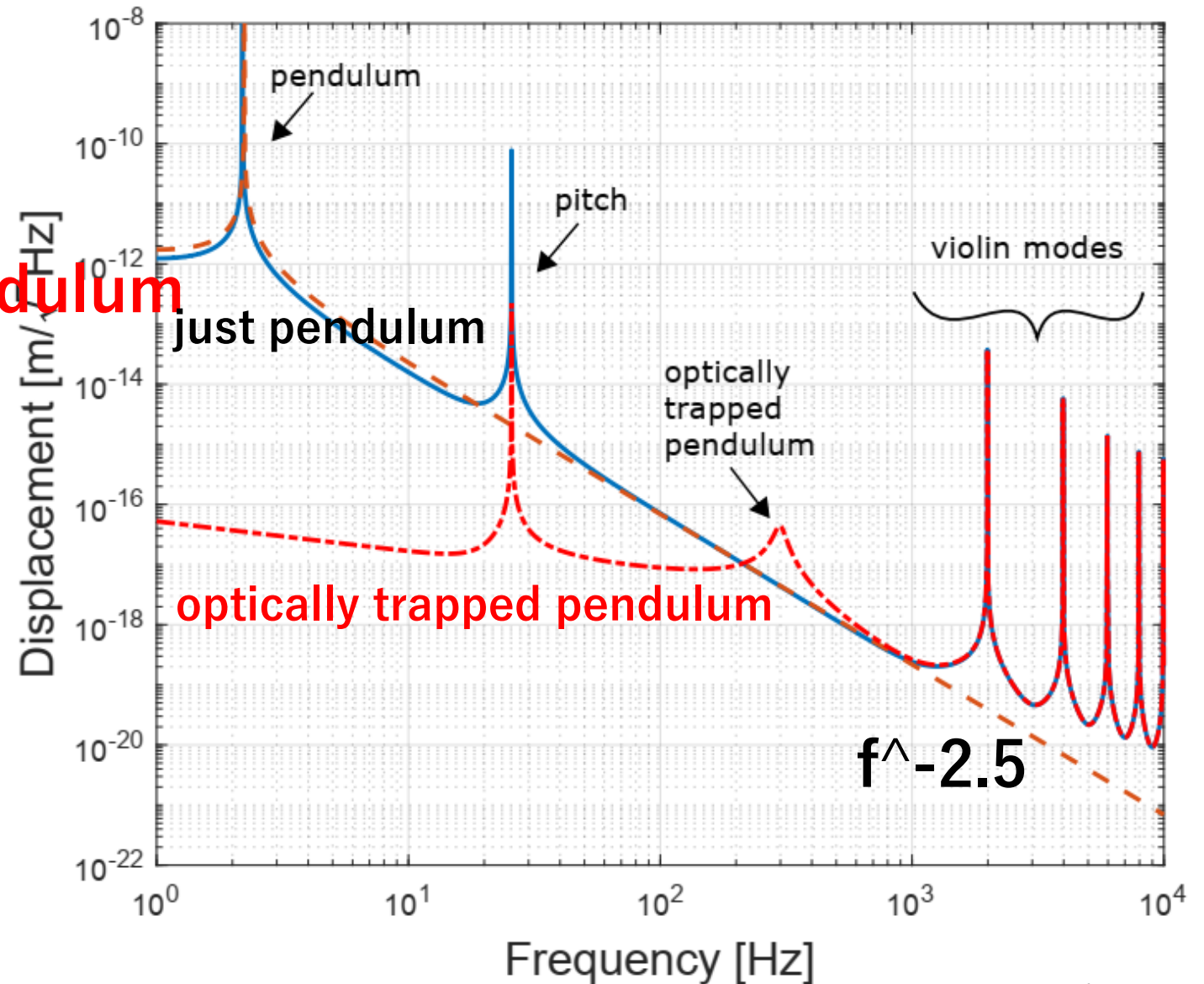
- $n = \frac{1}{\exp\frac{\hbar\omega}{k_B T} - 1} \simeq 0$

→ entropy is about 0

→ 0 K thermal bath



Spectrum of Brownian motion

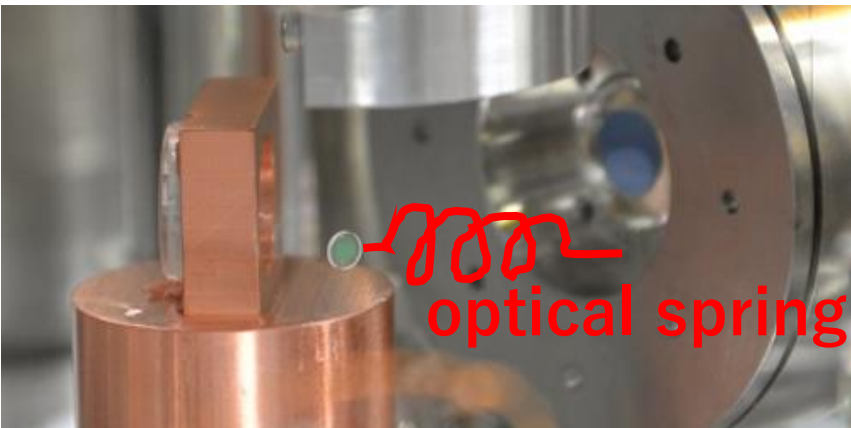
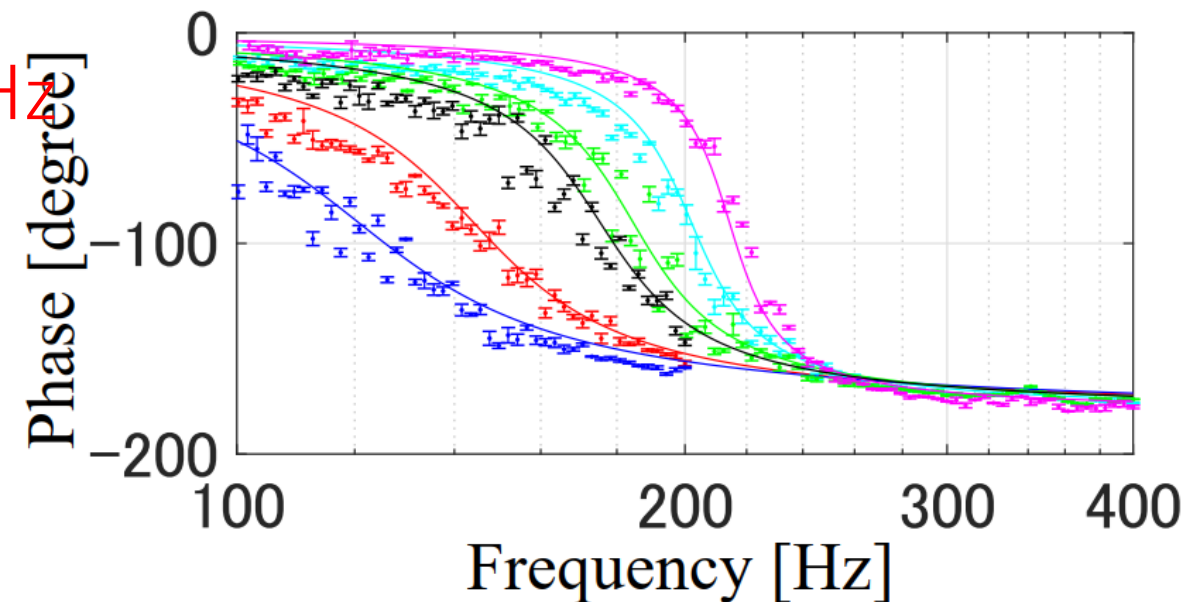
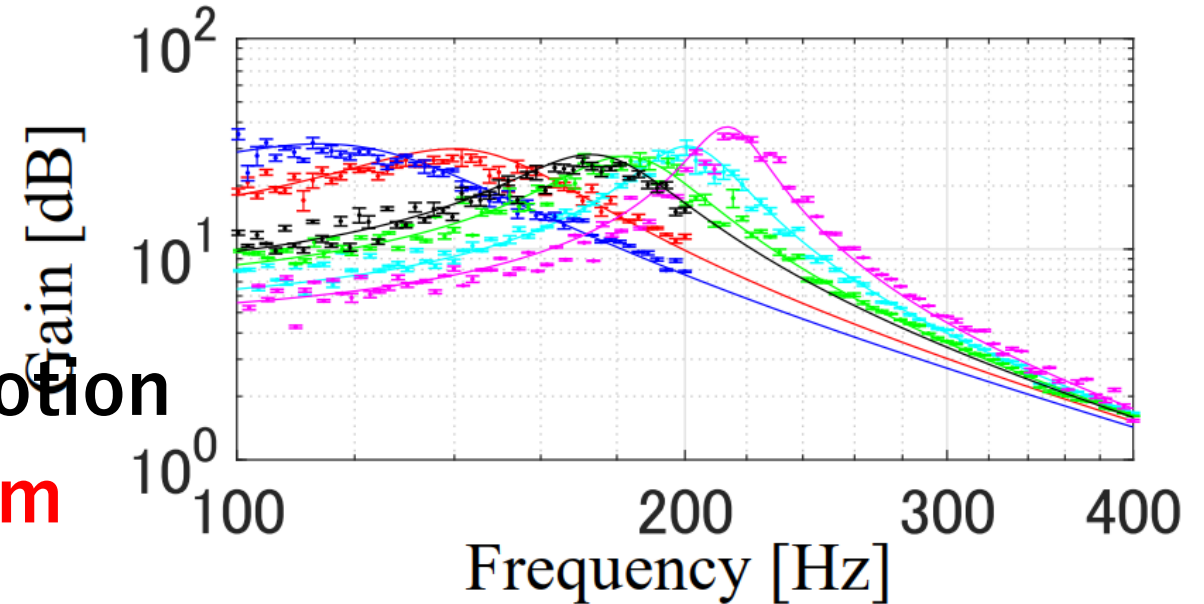


Optical spring

- Cavity with a pendulum
→ measurement of Brown motion
of optically trapped pendulum

Main experiment. 4 Hz → 300 Hz

characterization of optical spring



Setup

total mass ~ 120 kg
main platform ~ 20 kg
damper ~ 10 kg × 2



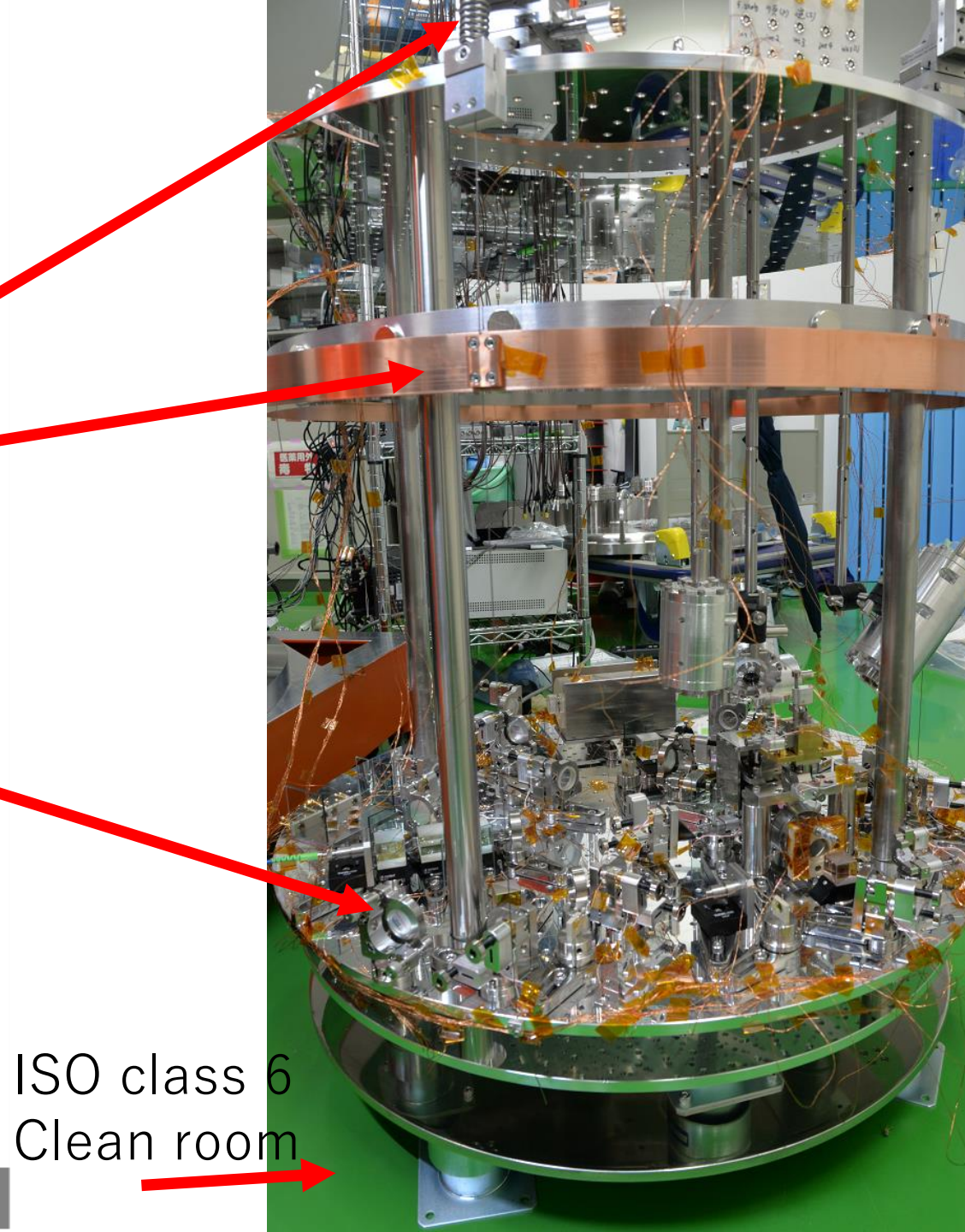
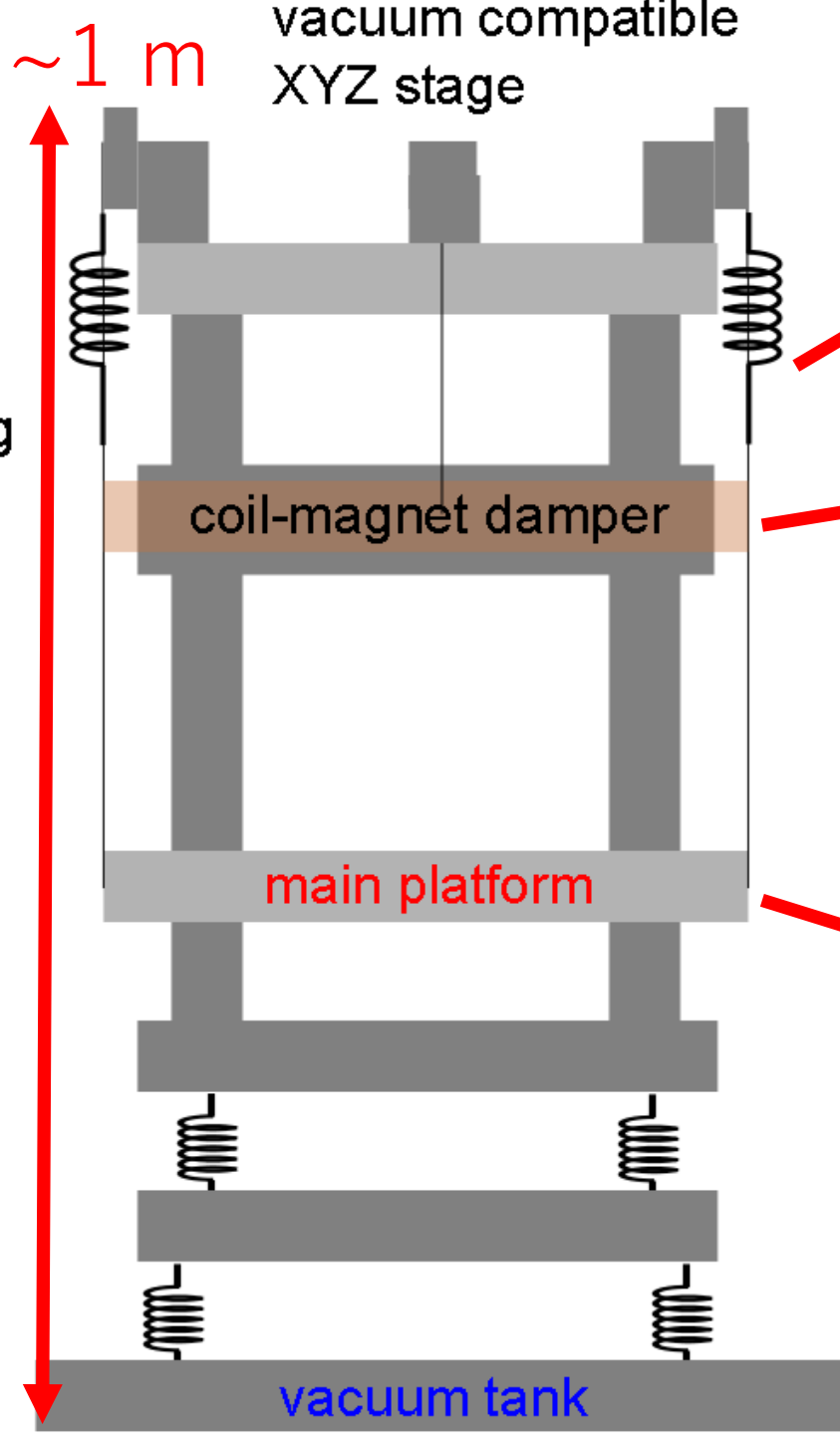
copper



aluminum



SUS



ISO class 6
Clean room

Coil-magnet
actuator

~ 1 m

vacuum compatible
XYZ stage

Int. stab.

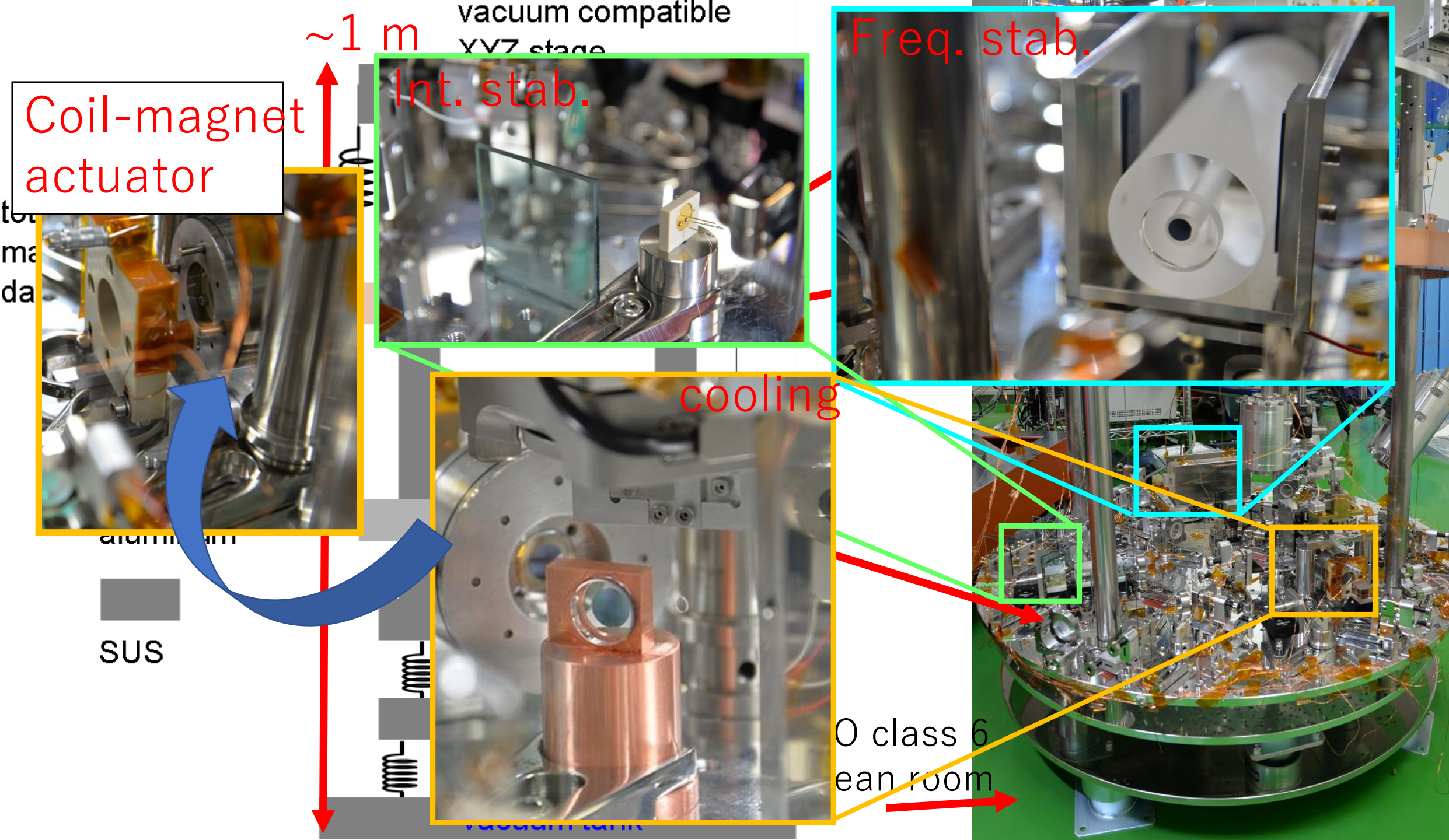
Freq. stab.

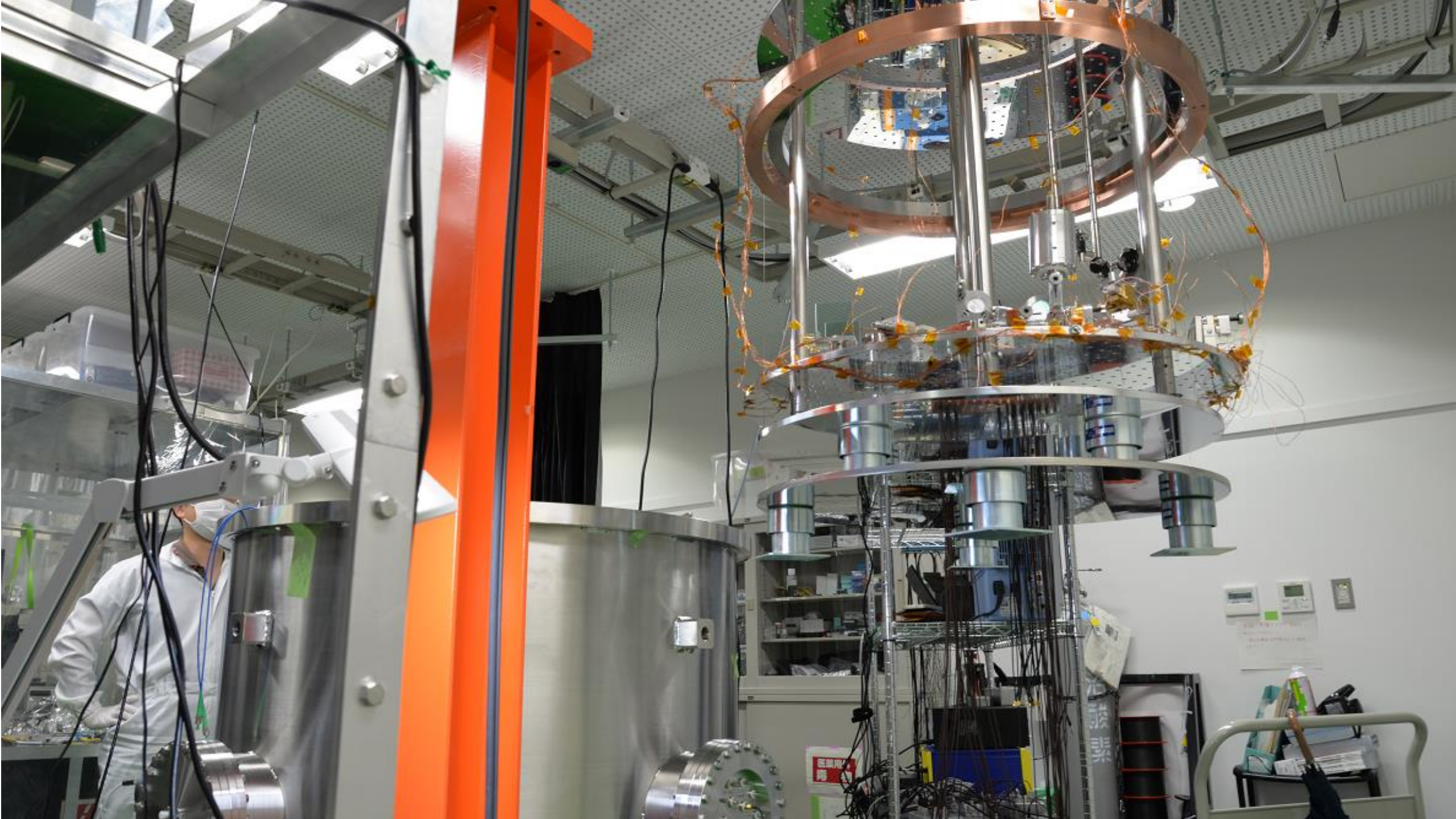
cooling

aluminum

SUS

0 class 6
clean room



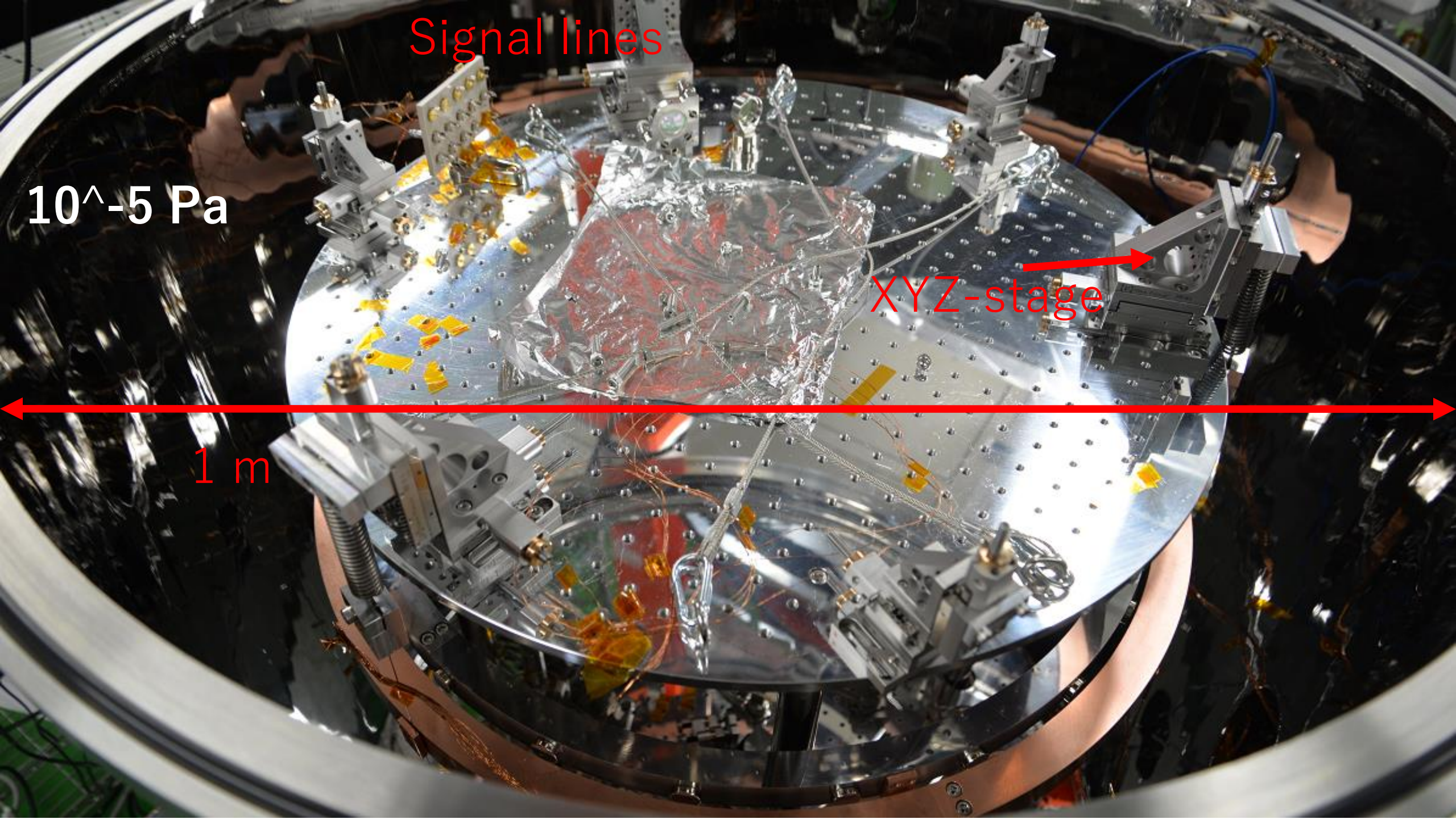


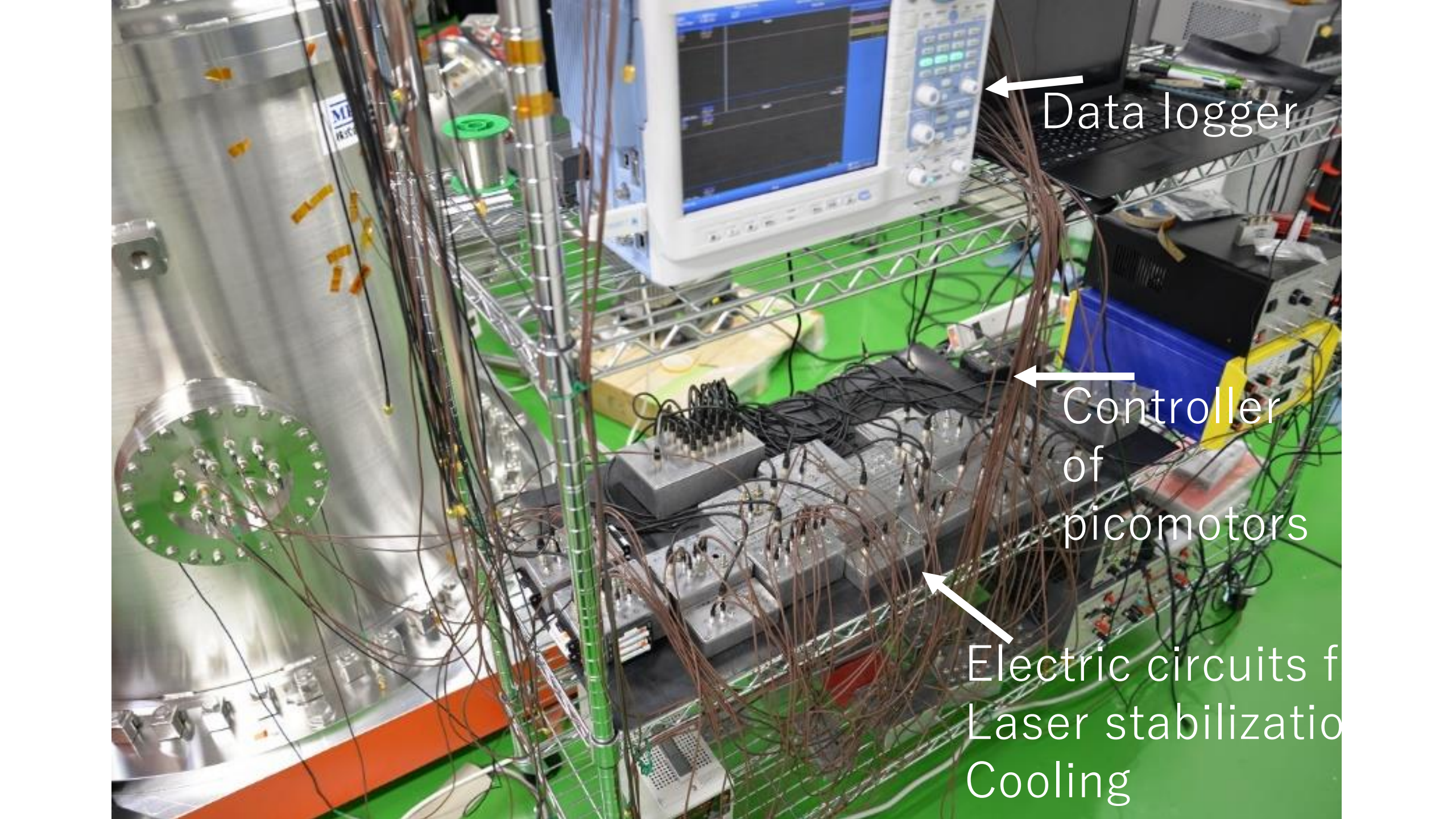
Signal lines

10^{-5} Pa

XYZ-stage

1 m



A photograph of a laboratory setup. On the left is a large, cylindrical metal chamber with a green circular port. To its right, on a metal rack, are several electronic devices. A white oscilloscope with a blue screen is at the top. Below it are several grey circuit boards with many wires connected to them. A blue power supply unit is also visible. The floor is green. Three white arrows point to specific components with text labels.

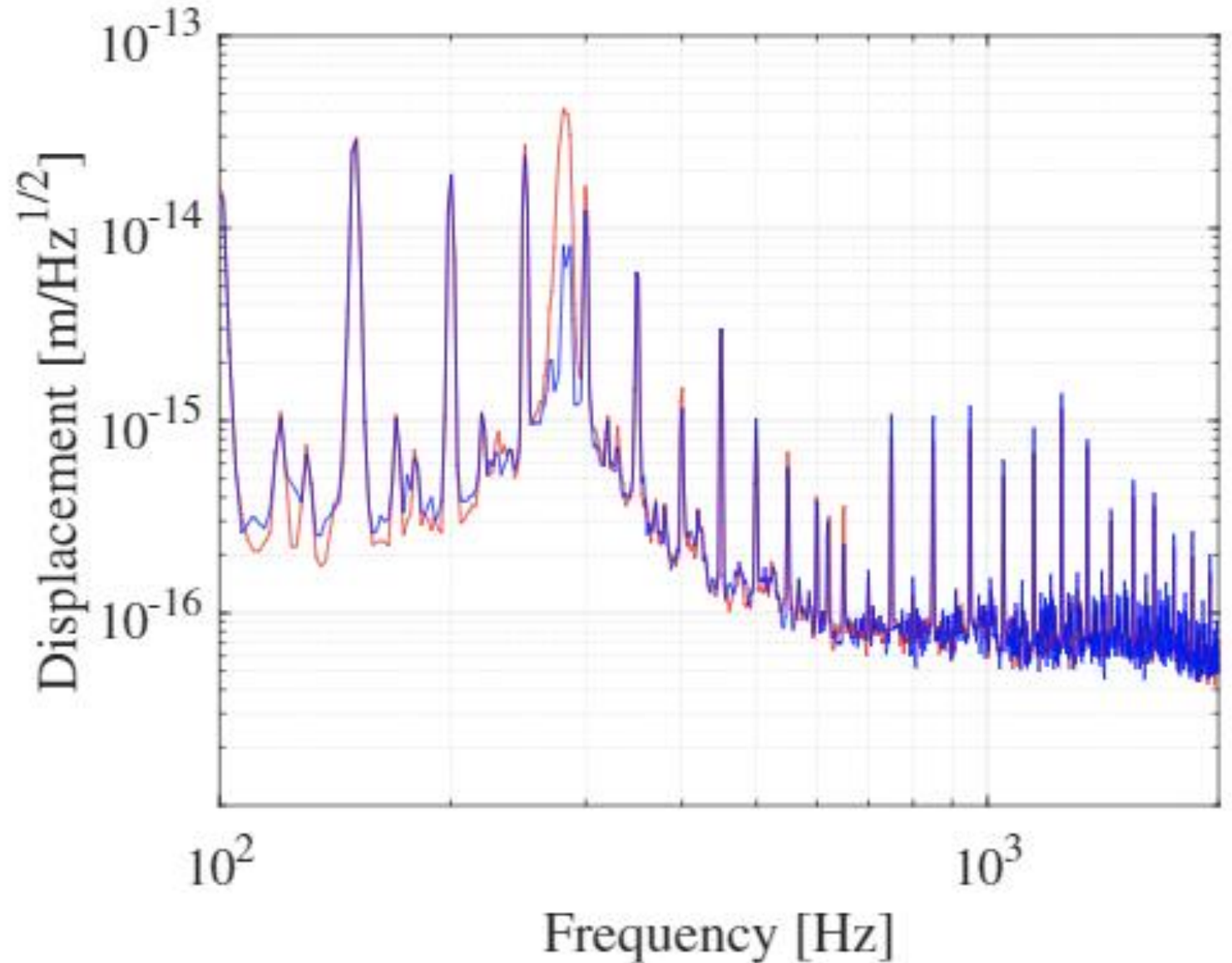
Data logger

Controller
of
picomotors

Electric circuits for
Laser stabilization
Cooling

Diaplacement sensing

Mode temperature
Blue: 420 μK
Red: 11 mK



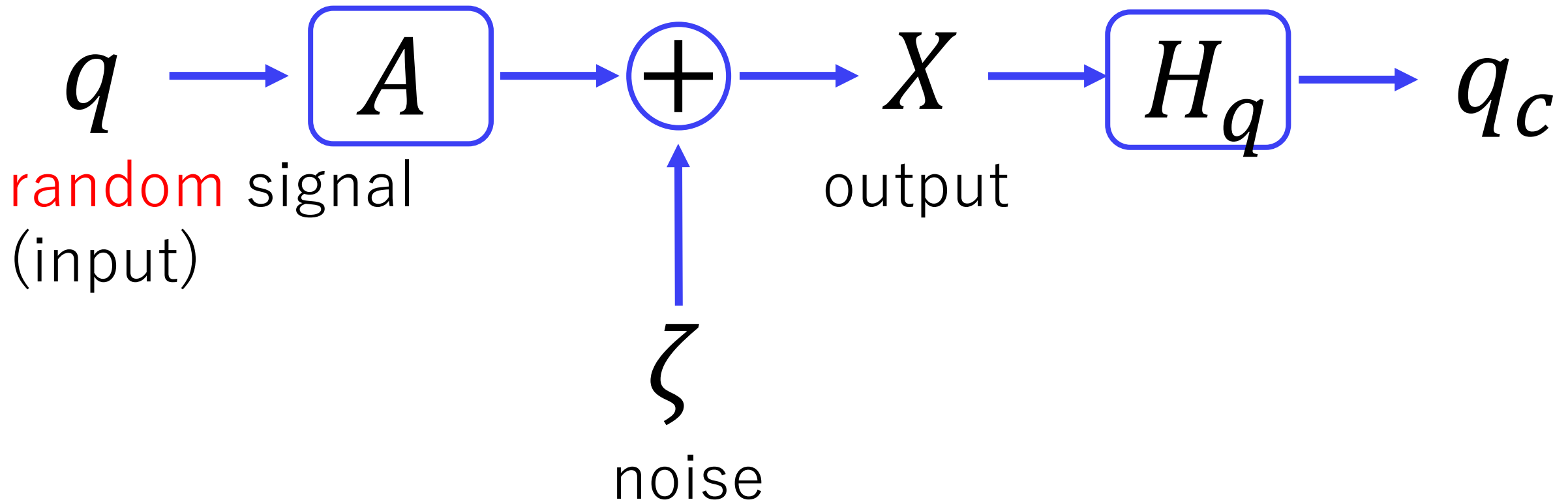
Conditional mechanical squeezing

- $X(t) = Aq(t) + \zeta(t)$
- Conditional position: $q_c = E[q|X]$
- Conditional position variance:
$$V_c(q) = E[(q - q_c)^2] = V(q) - V(q_c) < V(q)$$
- $V_c(q)$ can be maximumly reduced at the expense of increasing $V_c(p)$
→ Uncertainty principle is satisfied

Wiener filter

Stational system
(linear continuous meas.)

$$X(t) = Aq(t) + \zeta(t)$$

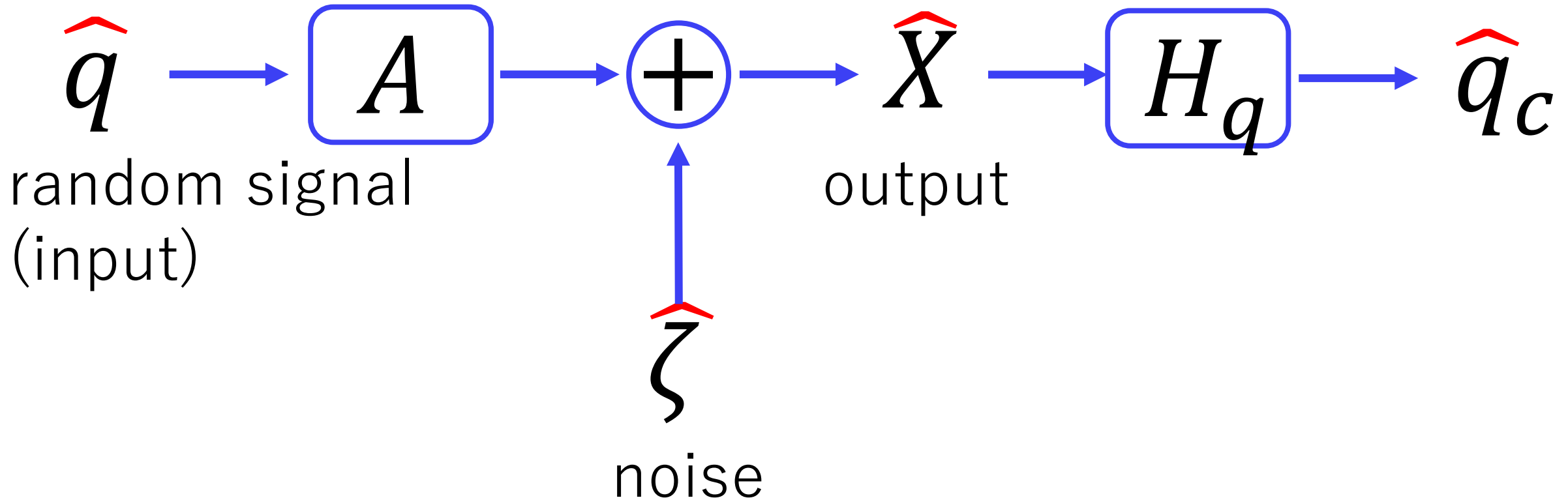


Quantum Wiener filter

$$X(t) = Aq(t) + \zeta(t)$$

Stational system
(linear continuous meas.)

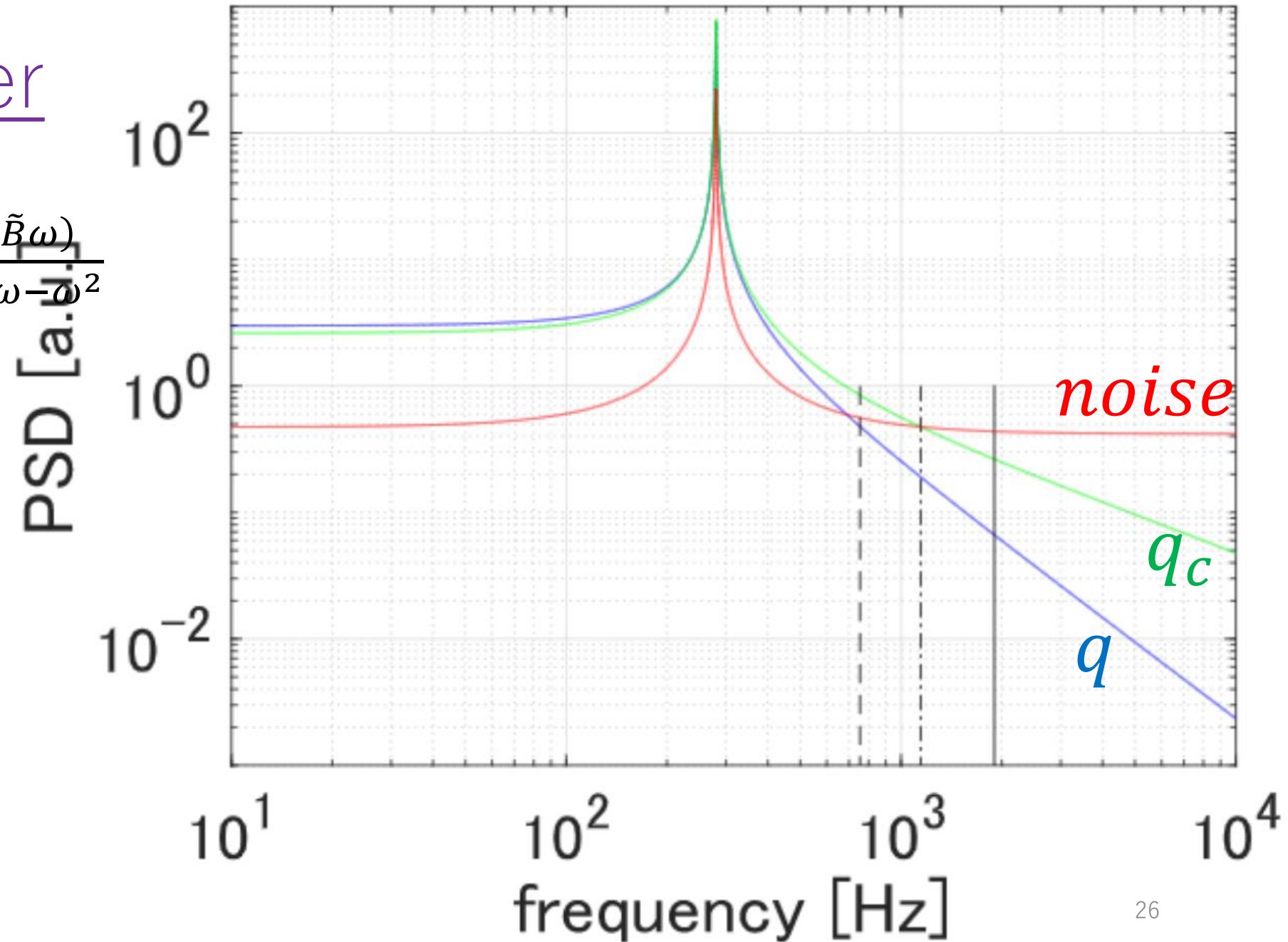
Wiener filter



Quantum noise preserves the heisenberg uncertainty principle 25

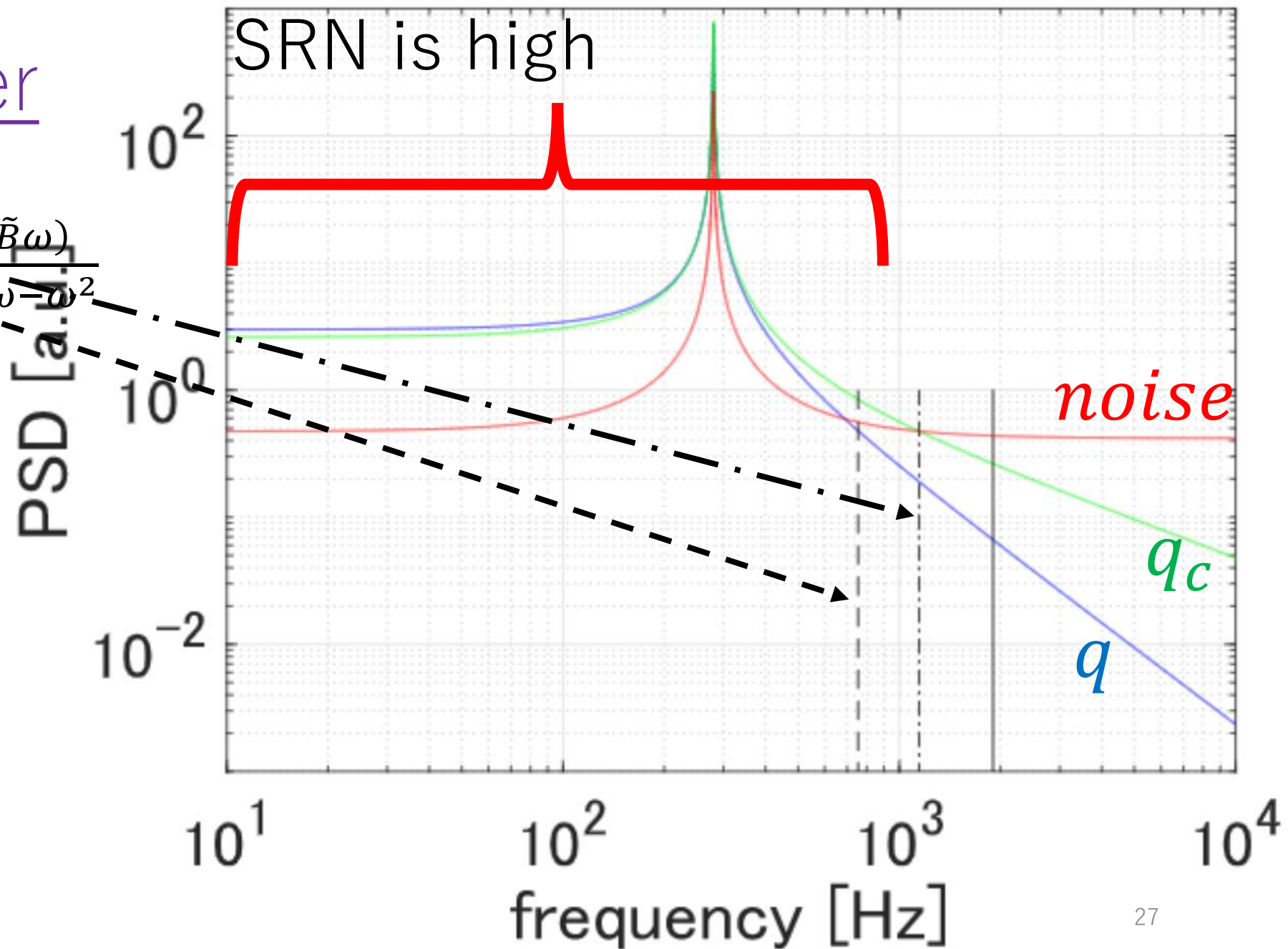
Wiener filter

- $H_q(\omega) = \frac{\tilde{A}(1-i\tilde{B}\omega)}{\Omega' - i\Gamma'\omega - \omega^2}$
- $q_c = H_q X$



Wiener filter

- $H_q(\omega) \approx \frac{\tilde{A}(1-i\tilde{B}\omega)}{\Omega^2 - i\Gamma'\omega - \omega^2}$



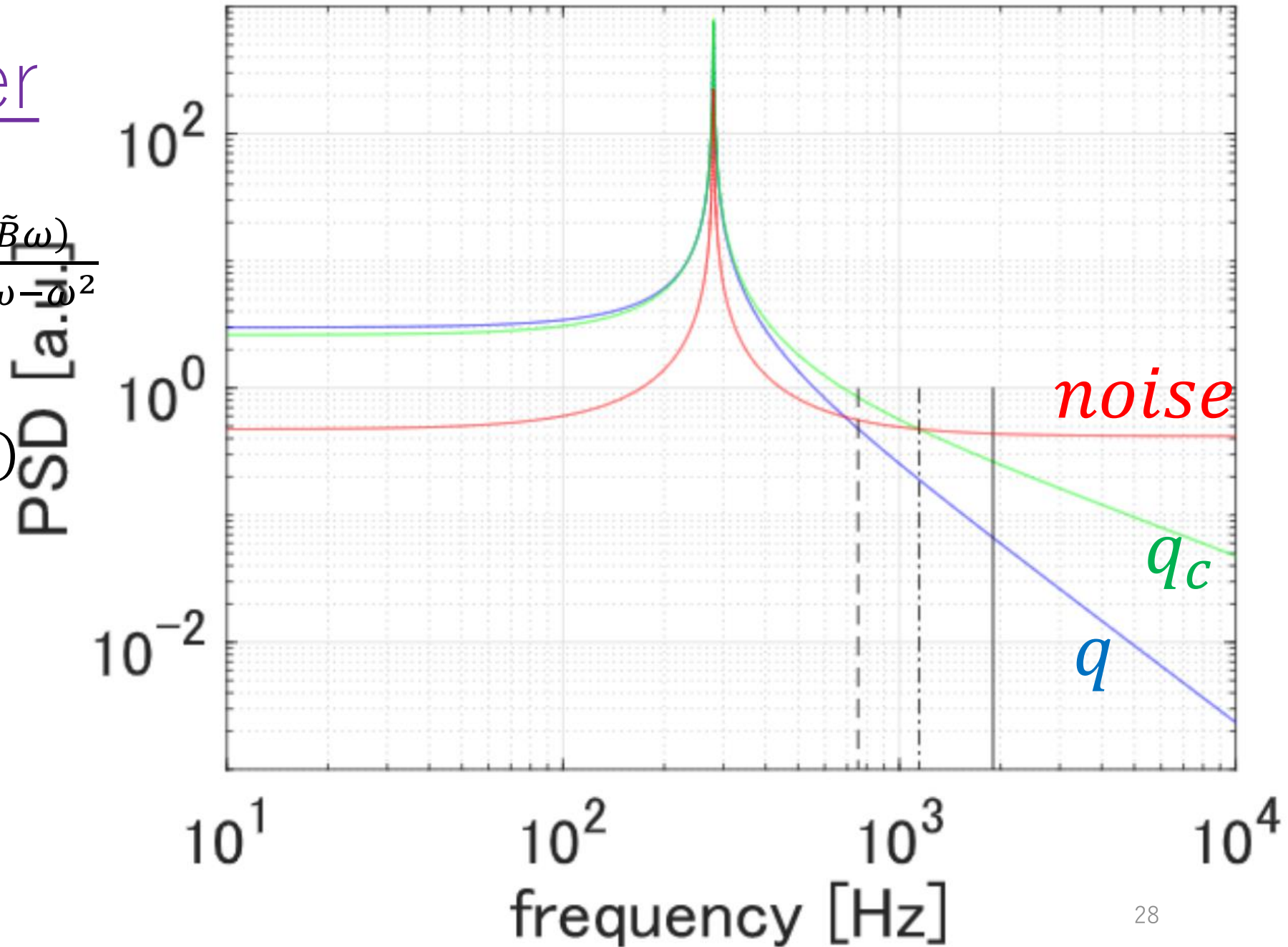
Wiener filter

$$\bullet H_q(\omega) = \frac{\tilde{A}(1-i\tilde{B}\omega)}{\Omega' - i\Gamma'\omega - \omega^2}$$

Conditional variance

$$V_c(q) = E[(q - q_c)^2]$$
$$= V(q) - V(q_c) < V(q)$$

$$V = \int_{-\infty}^{\infty} S df$$



Wiener filter

- $H_q(\omega) = \frac{\tilde{A}(1-i\tilde{B}\omega)}{\Omega' - i\Gamma'\omega - \omega^2}$

Conditional variance

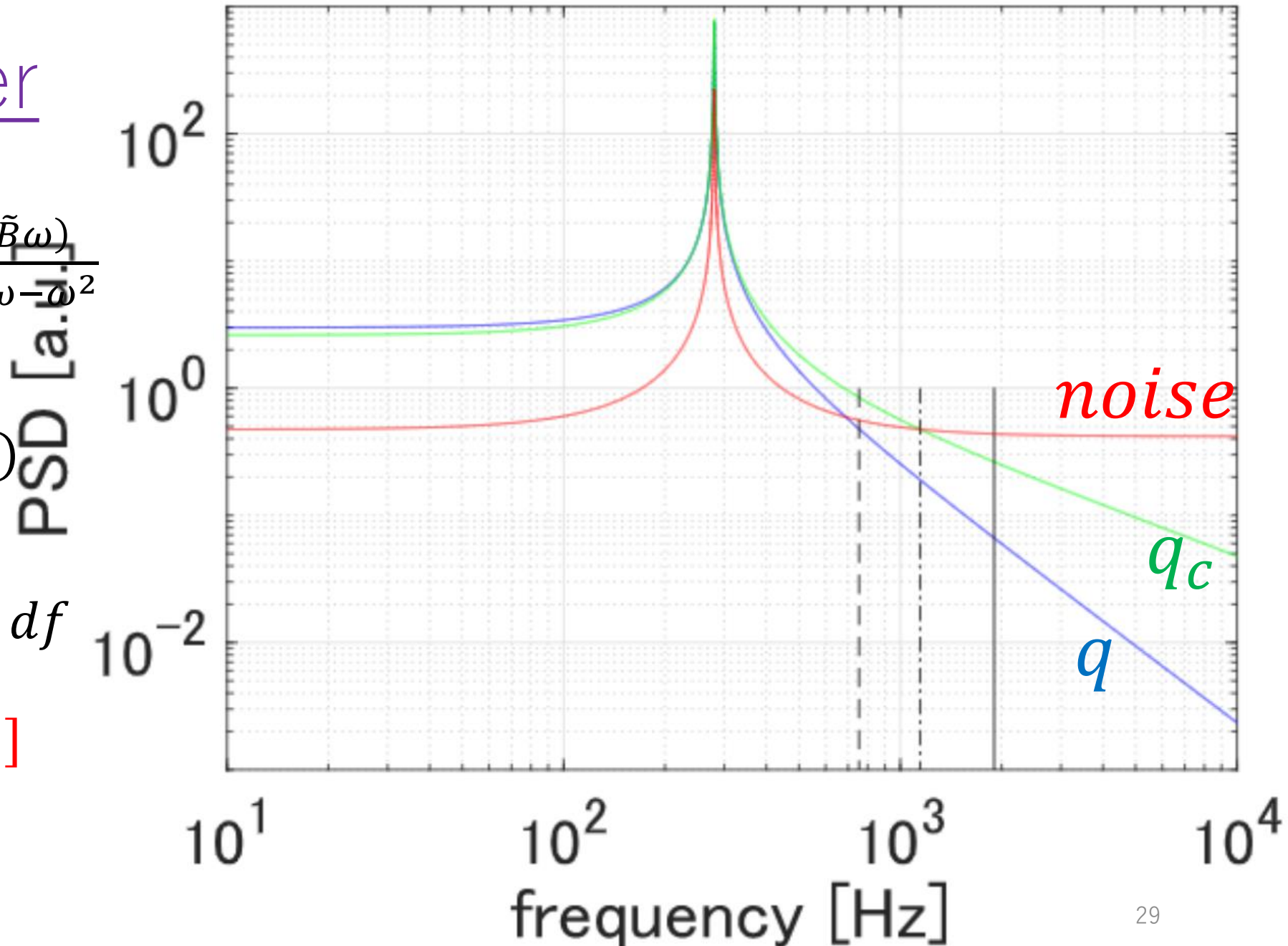
$$V_c(q) = E[(q - q_c)^2]$$

$$= V(q) - V(q_c) < V(q)$$

$$V = \int_{-\infty}^{\infty} S df \simeq \int_{-f_{bw}}^{f_{bw}} S df$$

$$V_c(q) \simeq E[(X/A - q_c)^2]$$

$$X = Aq + \zeta, \quad q \simeq \frac{X}{A}$$



Bandwidth for integration

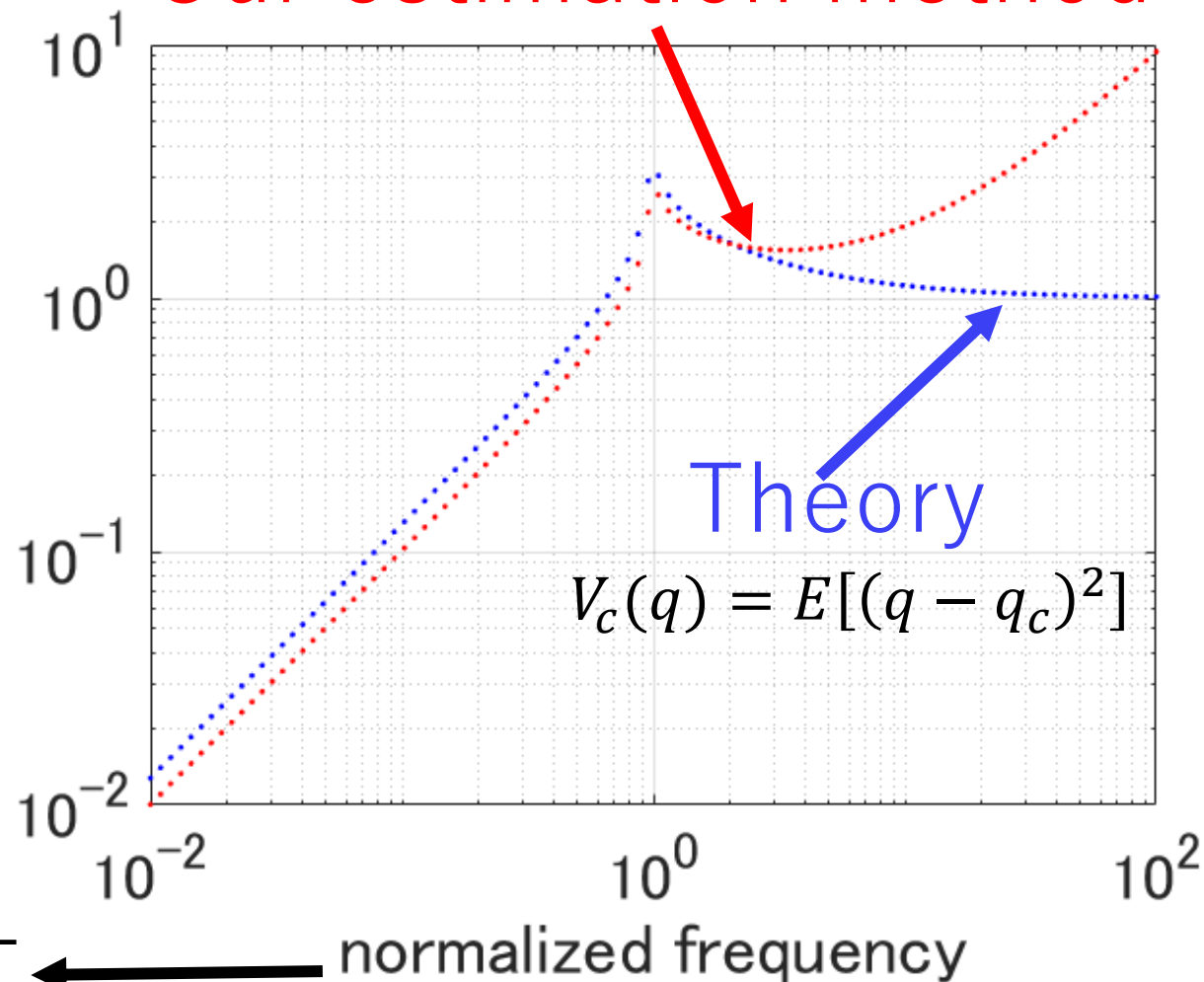
$$V = \int_{-\infty}^{\infty} S df \approx \int_{-f_{bw}}^{f_{bw}} S df$$

$$\frac{\left(\int_{-f_{bw}}^{f_{bw}} S df \right)}{\int_{-\infty}^{\infty} S df}$$

normalized variance

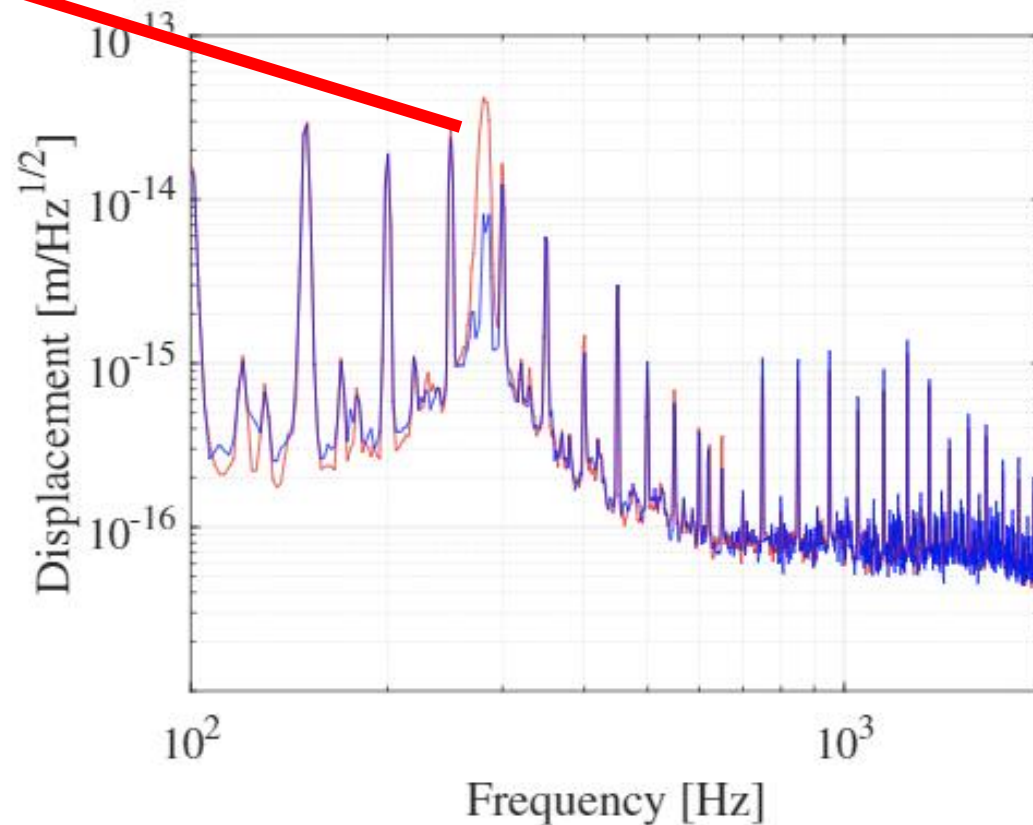
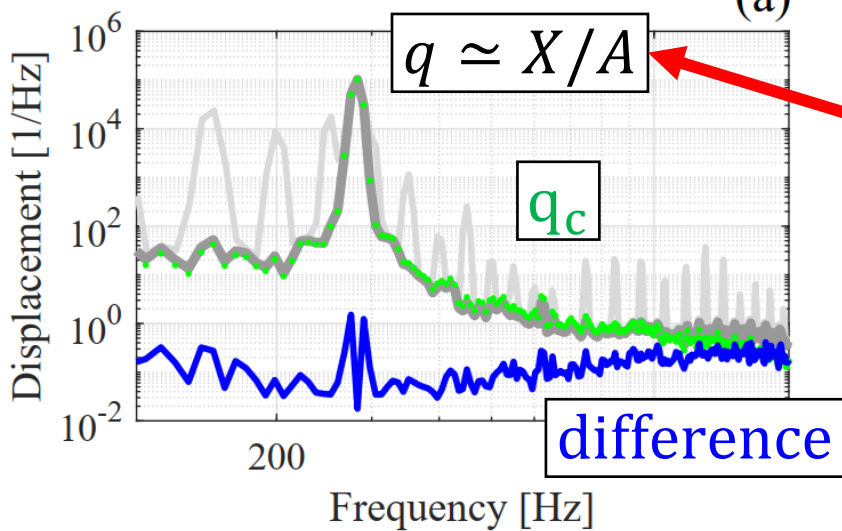
$$\frac{f_{bw}}{\frac{\Omega'}{2\pi} + \frac{\Gamma'}{2\pi}}$$

$V_c(q) \approx E[(X/A - q_c)^2]$
Our estimation method

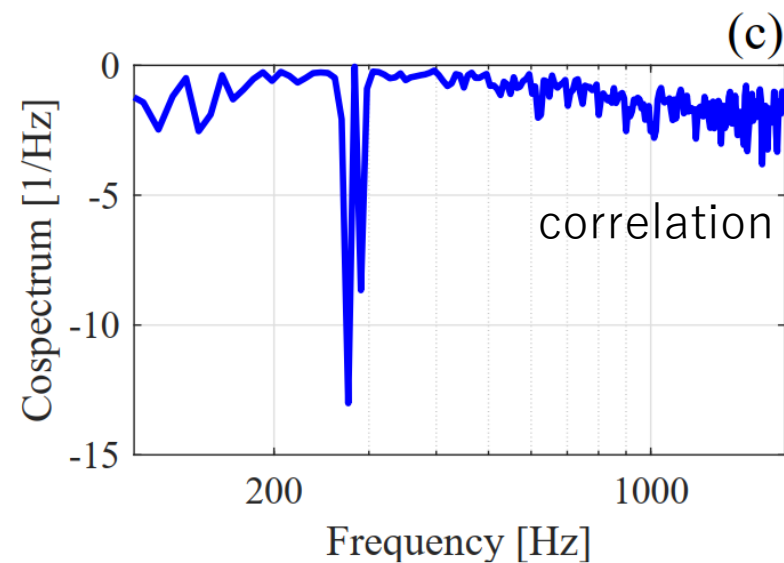
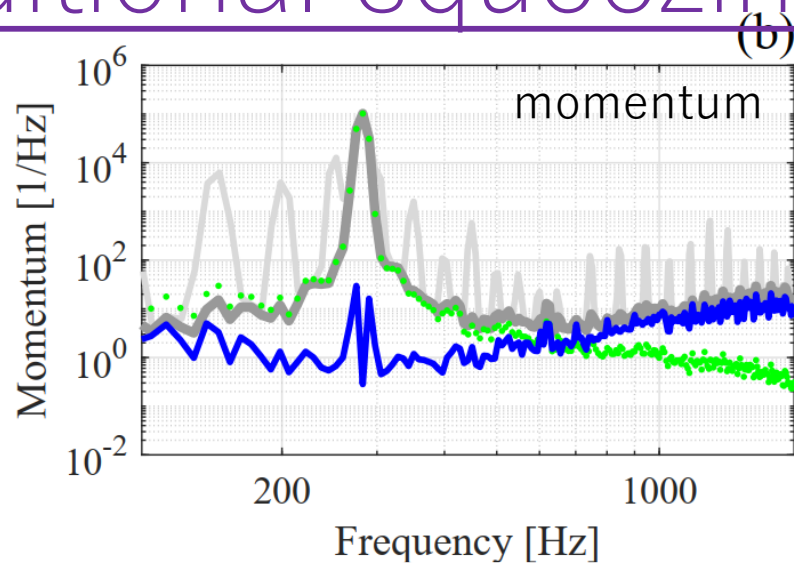
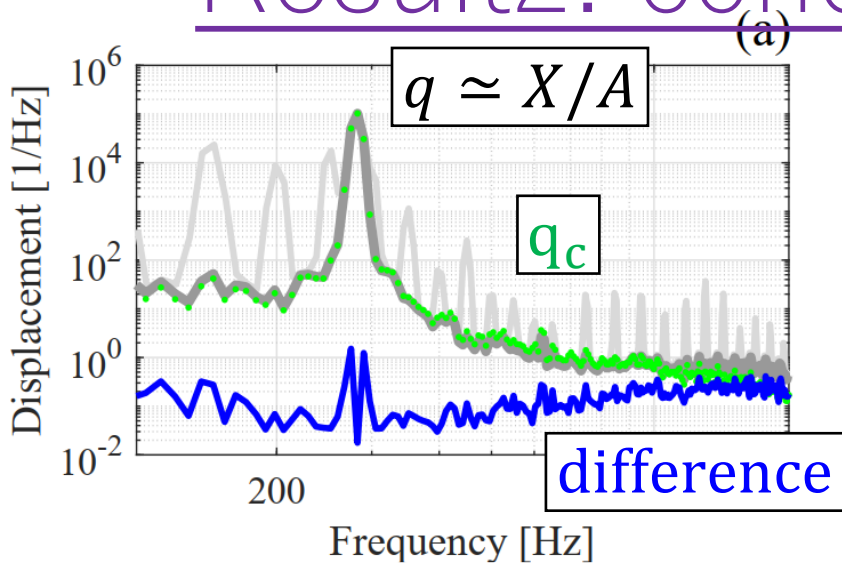


Result2: conditional saueezing

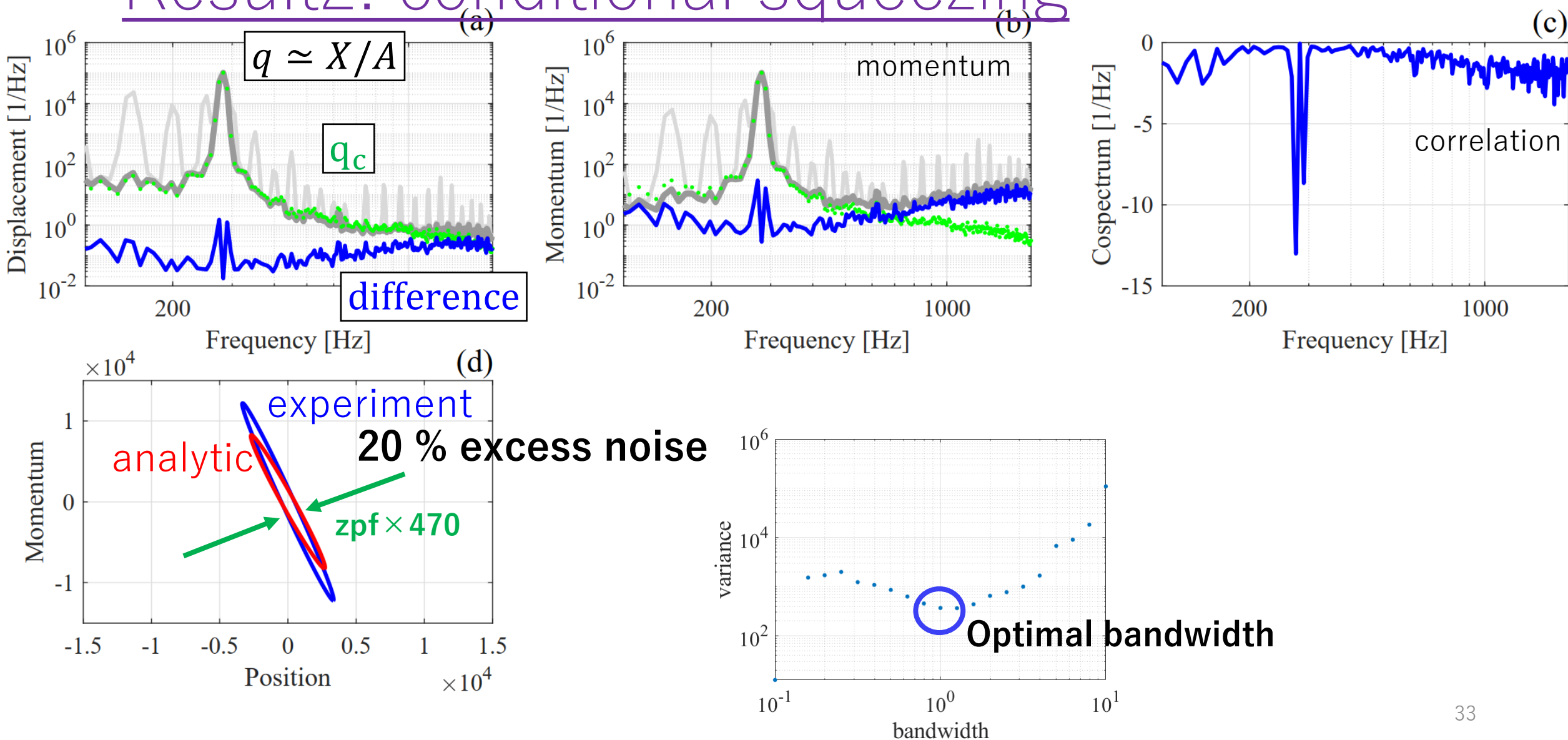
**notch
filtering**



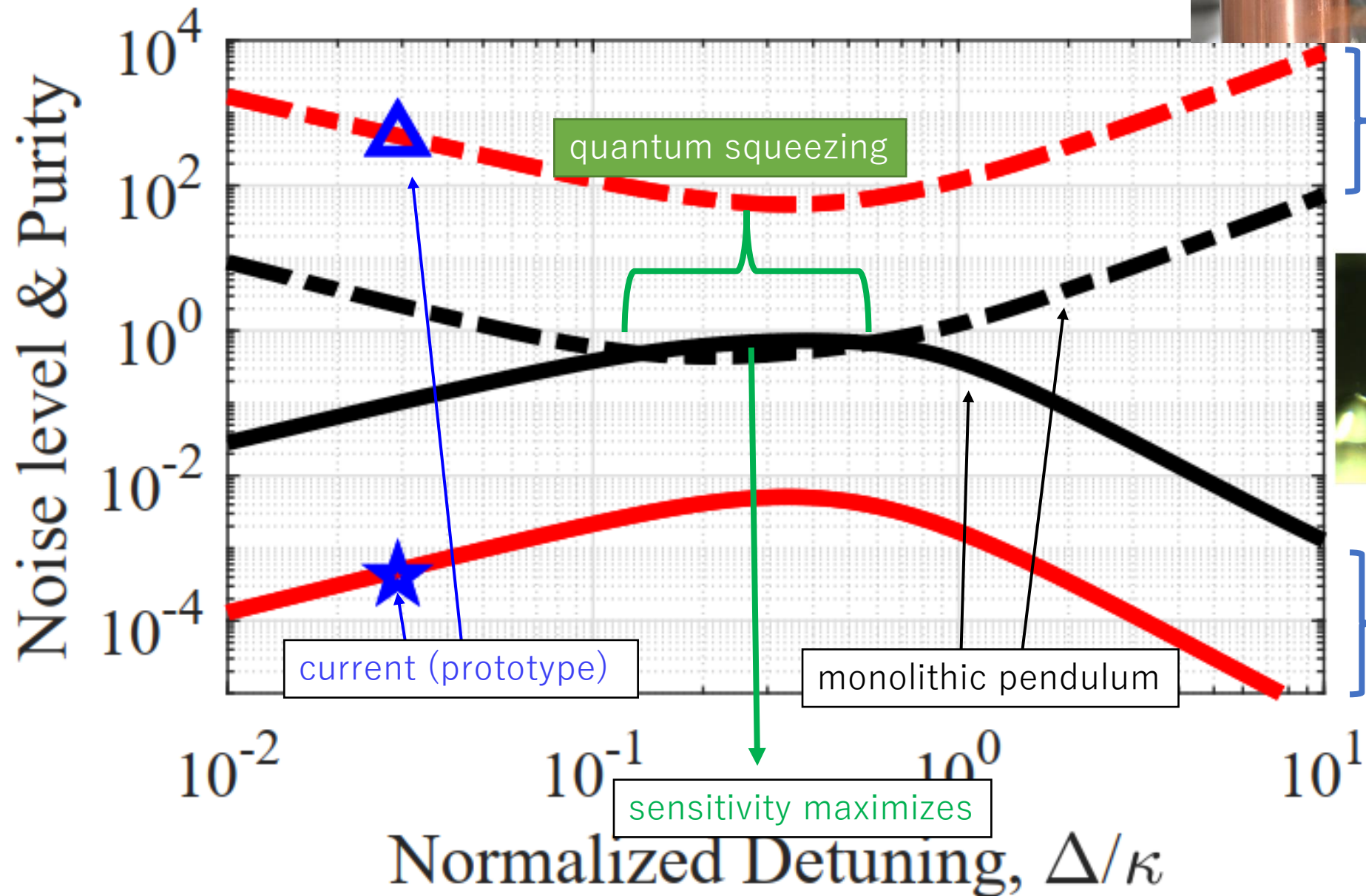
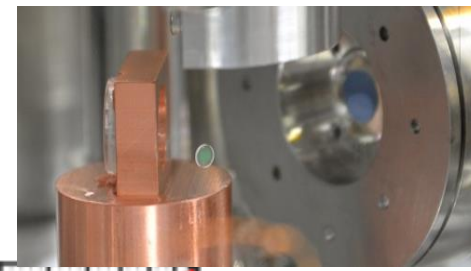
Result2: conditional squeezing



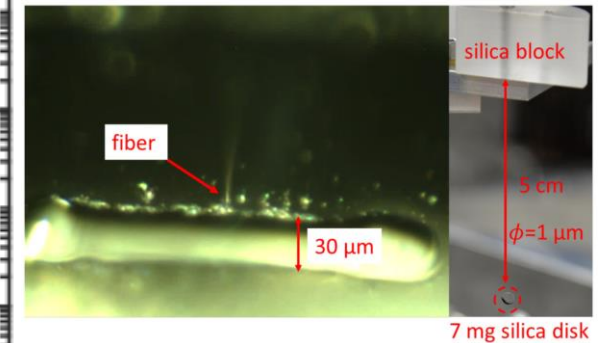
Result2: conditional squeezing



Future: monolithic pendulum



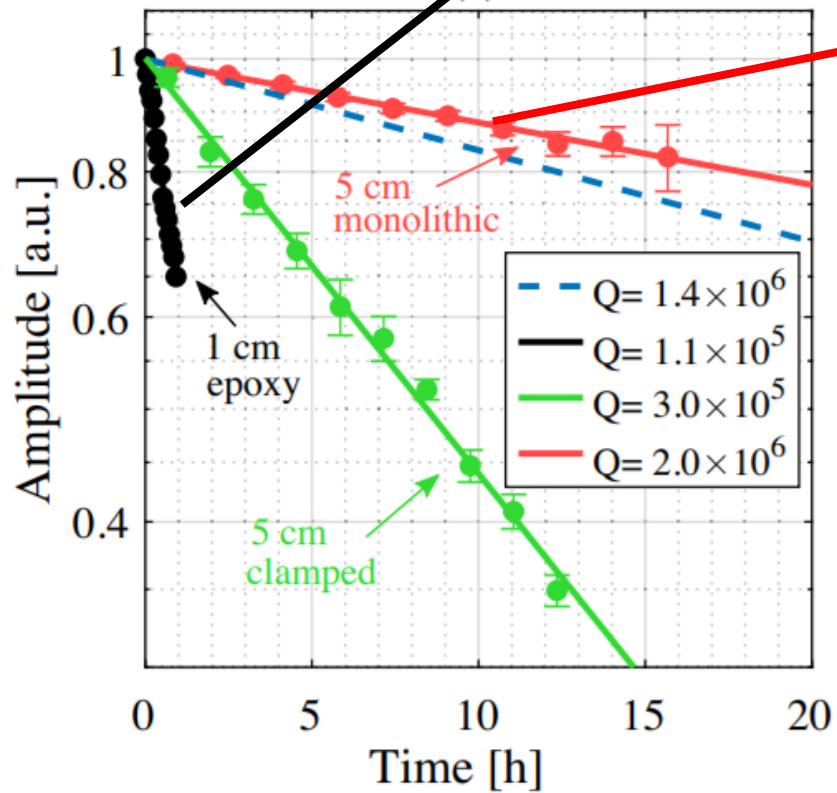
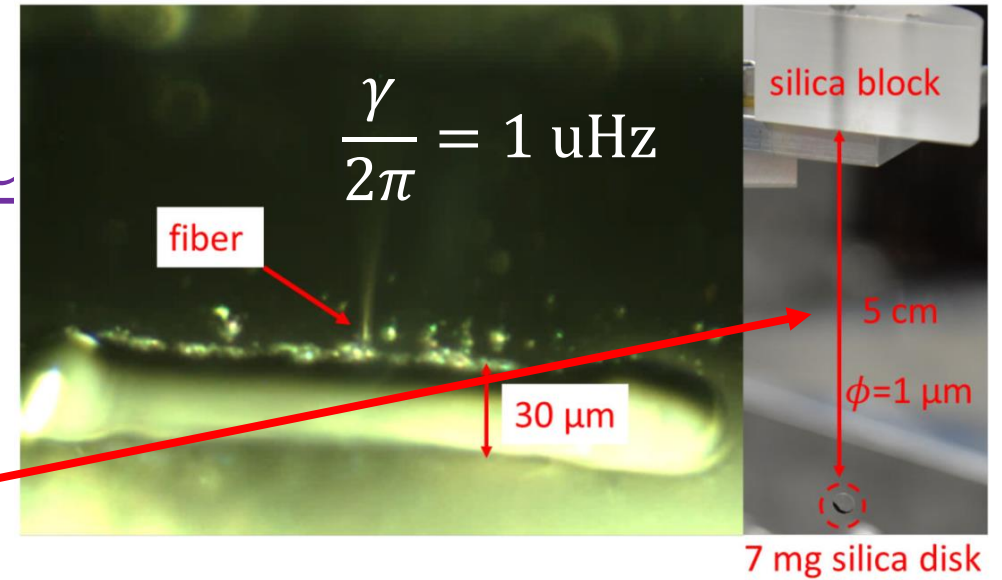
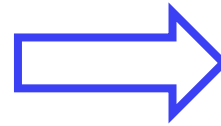
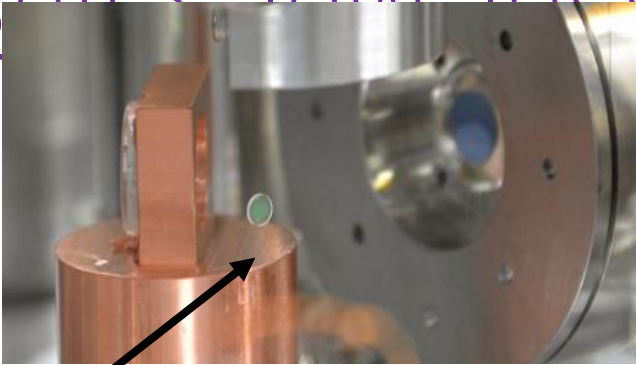
squeezed noise level



Purity: $\frac{1}{\sqrt{\det |V|}}$

Result 3: low loss pendulum

$$\frac{\gamma}{2\pi} = 40 \text{ uHz}$$



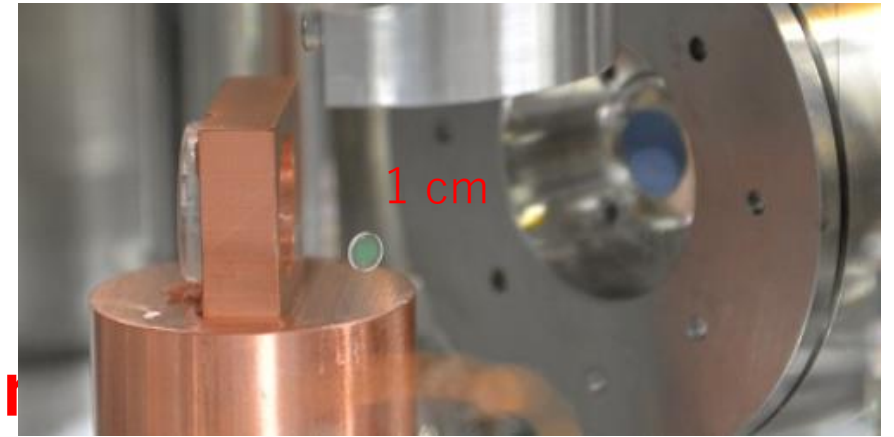
$$\sqrt{S_{qq}^{imp}} \simeq 10^{-17} \frac{\text{m}}{\sqrt{\text{Hz}}}$$

$$n_{imp} = \frac{S_{qq}^{imp}}{2S_{qq}^{zpf}} \simeq 3 \times 10^{-8} \rightarrow 2 \times 10^{-11} (@500 \text{ Hz})$$

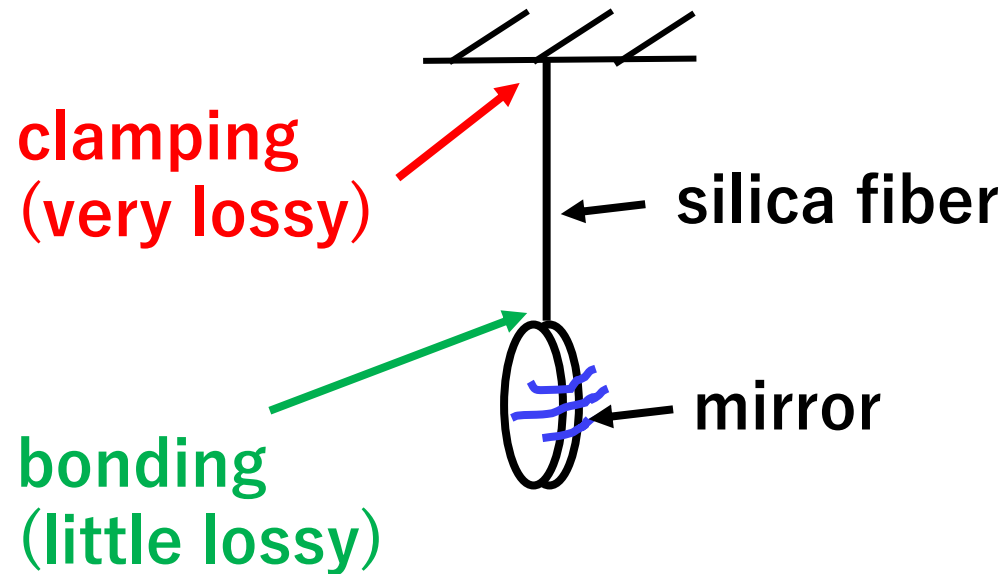
$$\text{Requirement: } n_{imp} \sim 2 \times 10^{-11}$$

Old pendulum

- Simple pendulum **with small dissipation**

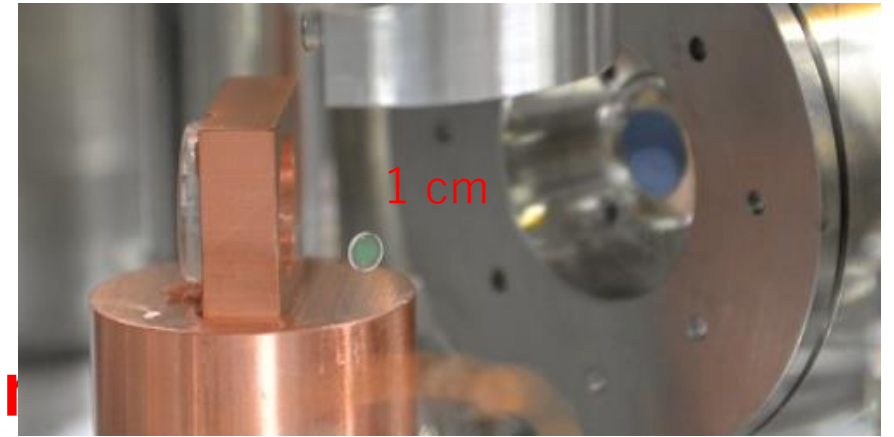


prototype(2015~2019)
resonance ~ 4 Hz
 $Q \sim 1e5$
dissipation $\sim 4e-5$ Hz

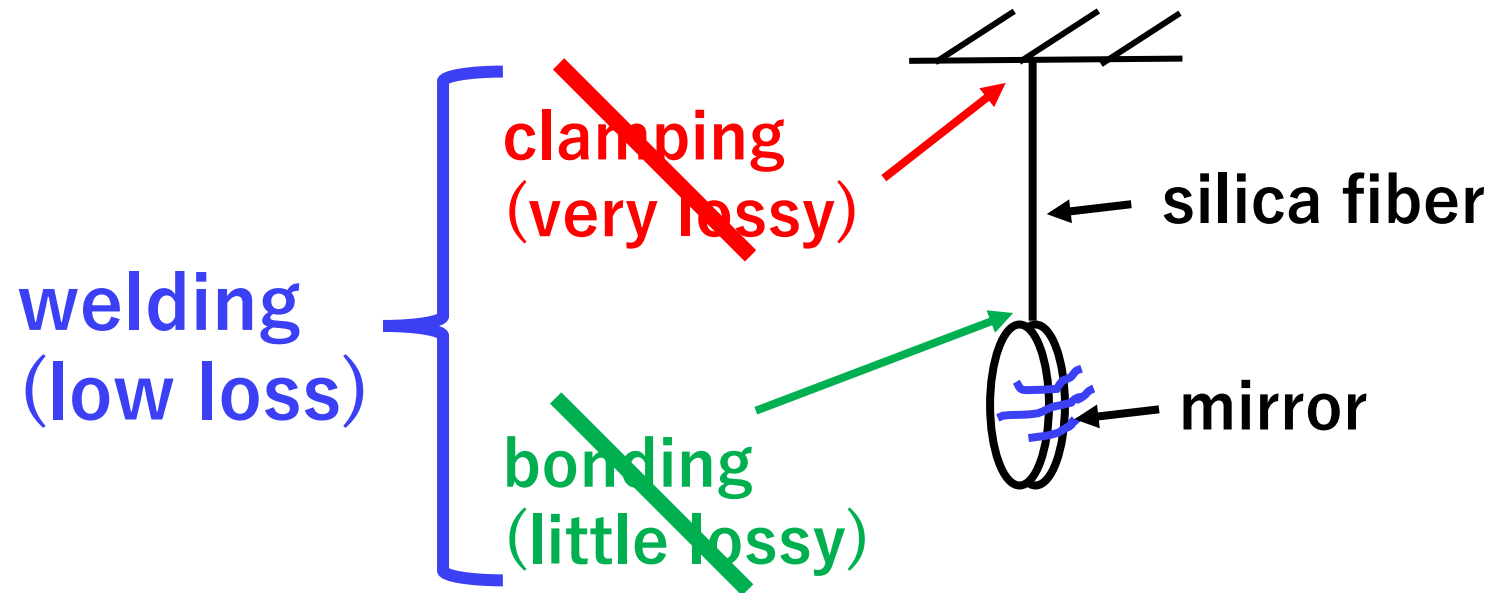


New pendulum: monolithic

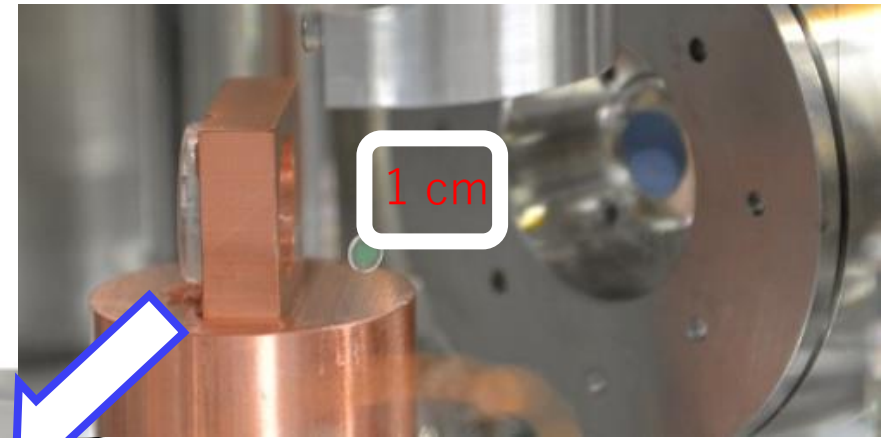
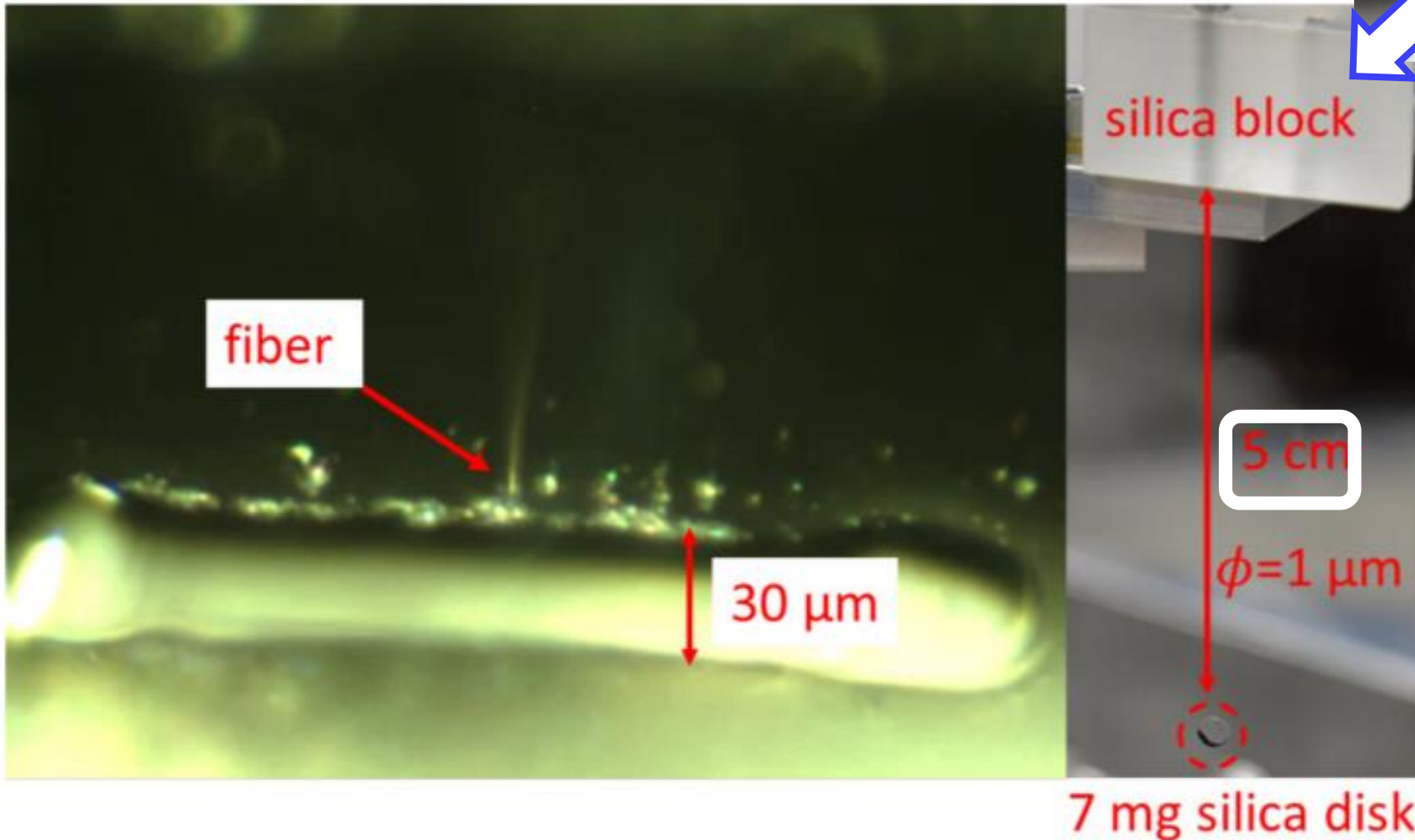
- Simple pendulum **with small dissipation**



prototype(2015~2019)
resonance ~ 4 Hz
 $Q \sim 1e5$
dissipation $\sim 4e-5$ Hz



Monolithic pendulum



prototype(2015~2019)
resonance $\sim 4\ \text{Hz}$
 $Q \sim 1\text{e}5$
dissipation $\sim 4\text{e-}5\ \text{Hz}$

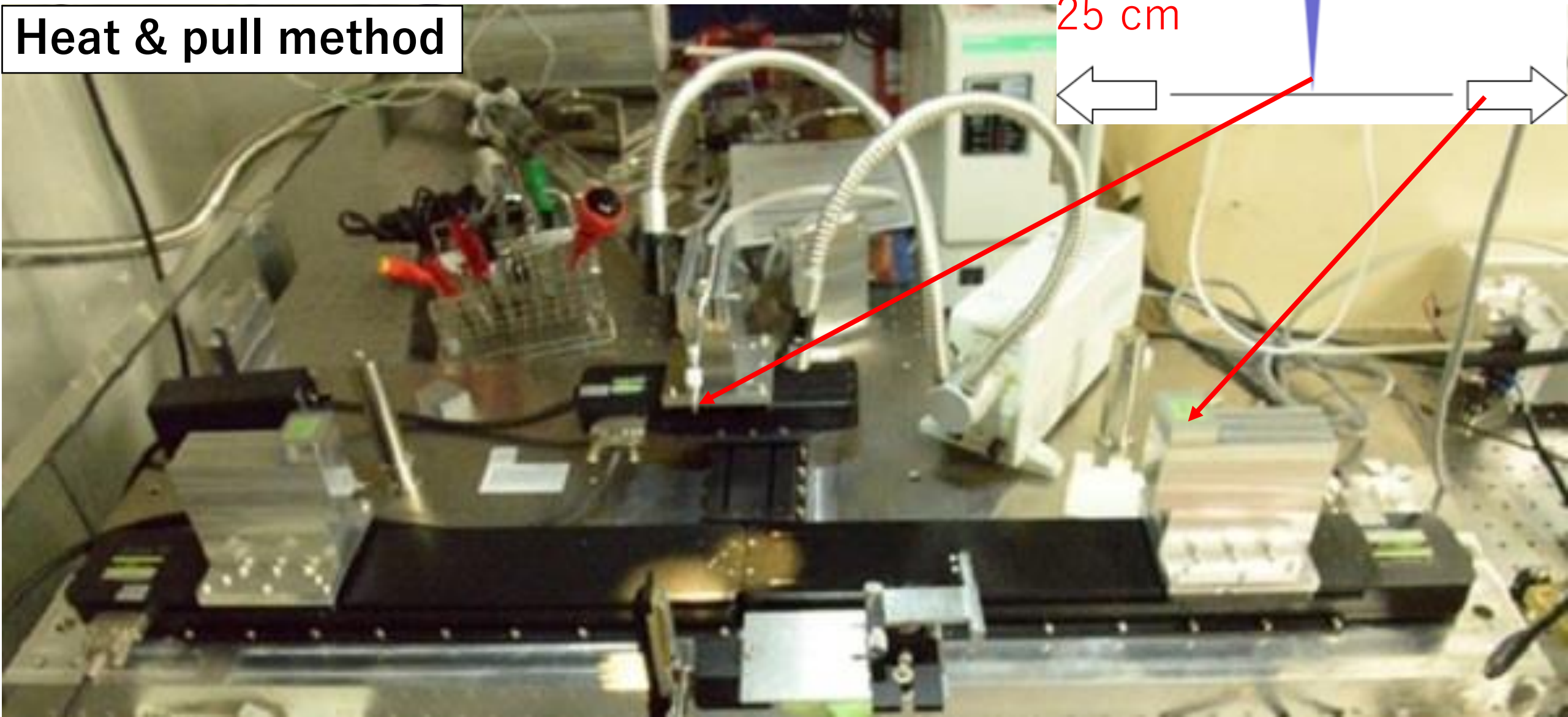
monolithic(2020~)
resonance $\sim 2\ \text{Hz}$
 $Q \sim 2\text{e}6$
dissipation $\sim 1\text{e-}6\ \text{Hz}$

thin (1 μm) & long (5 cm) silica fiber

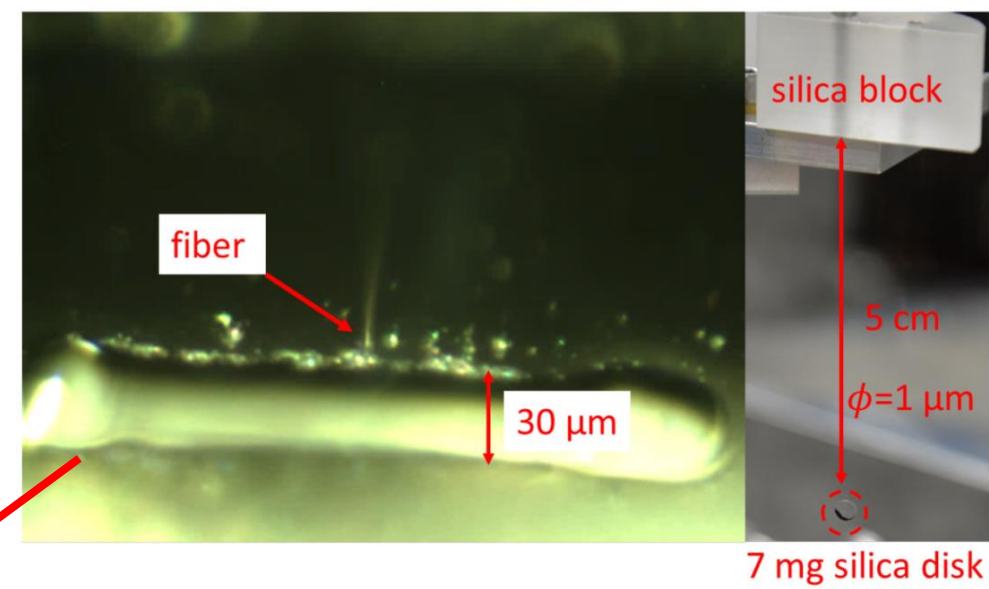
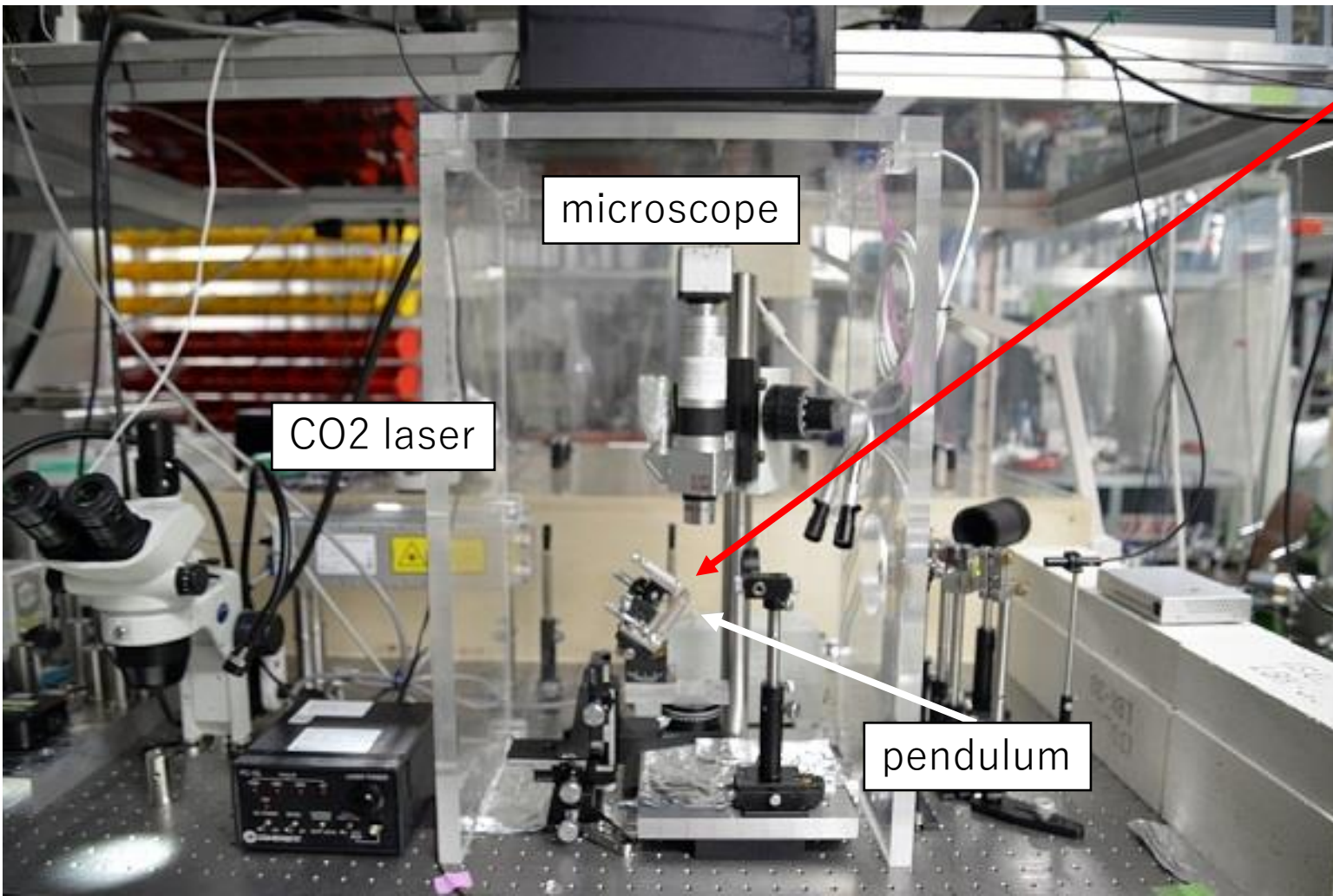
Heat & pull method

initial: 125 μm
final: 1 μm

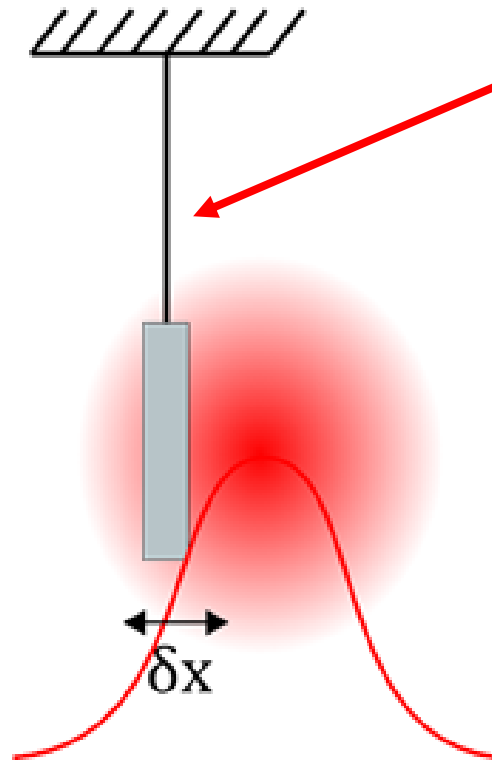
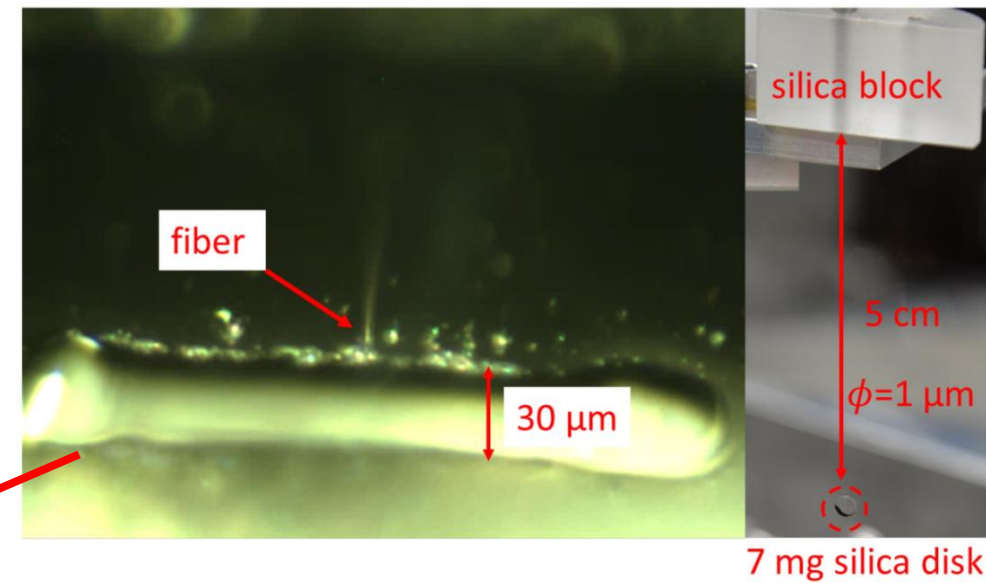
Pull length
25 cm



Monolithic pendulum



Shadow sensing measurement of ringdown (damped oscillation) by shadow sensing



on a vibration isolation
inside a vacuum chamber ($\sim 1\text{e-}5 \text{ Pa}$)

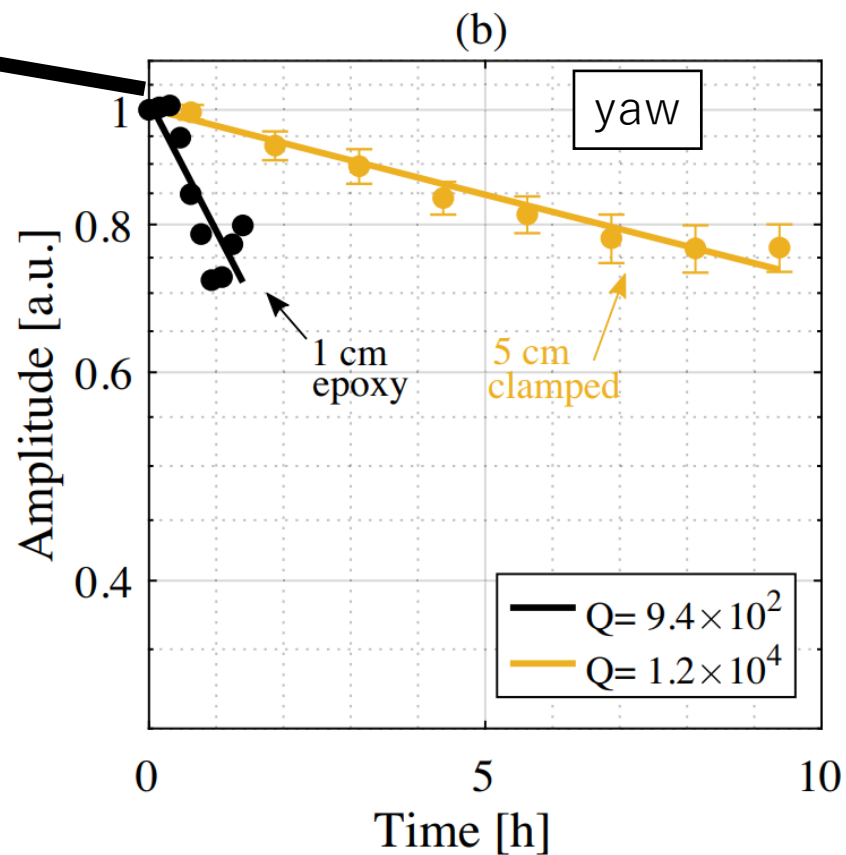
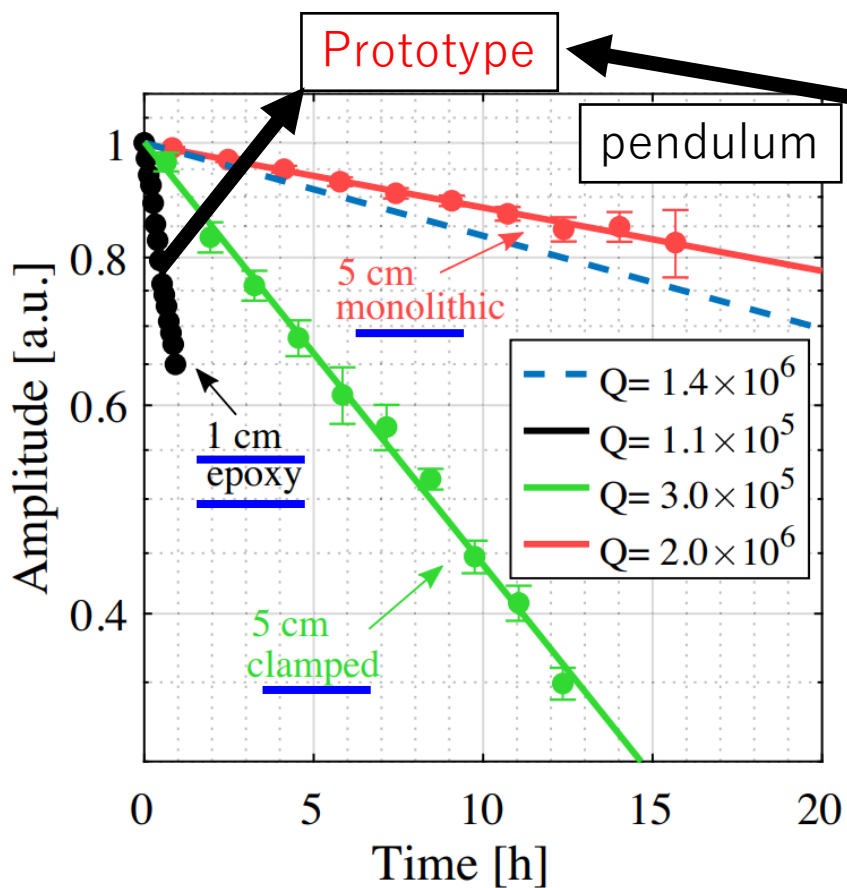
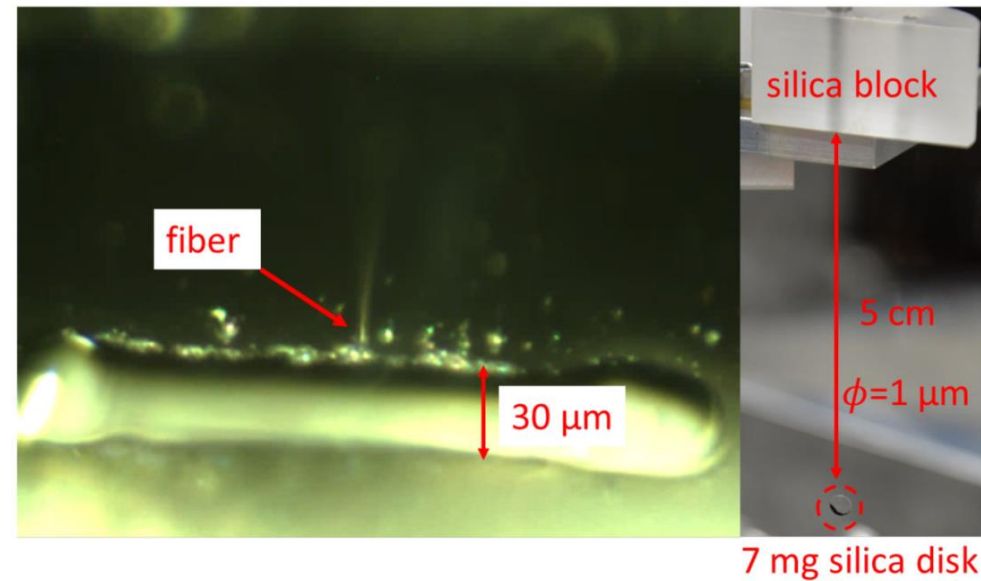
Using the linear range of a
Gaussian beam, we detect
the “shadow” on a PD

result

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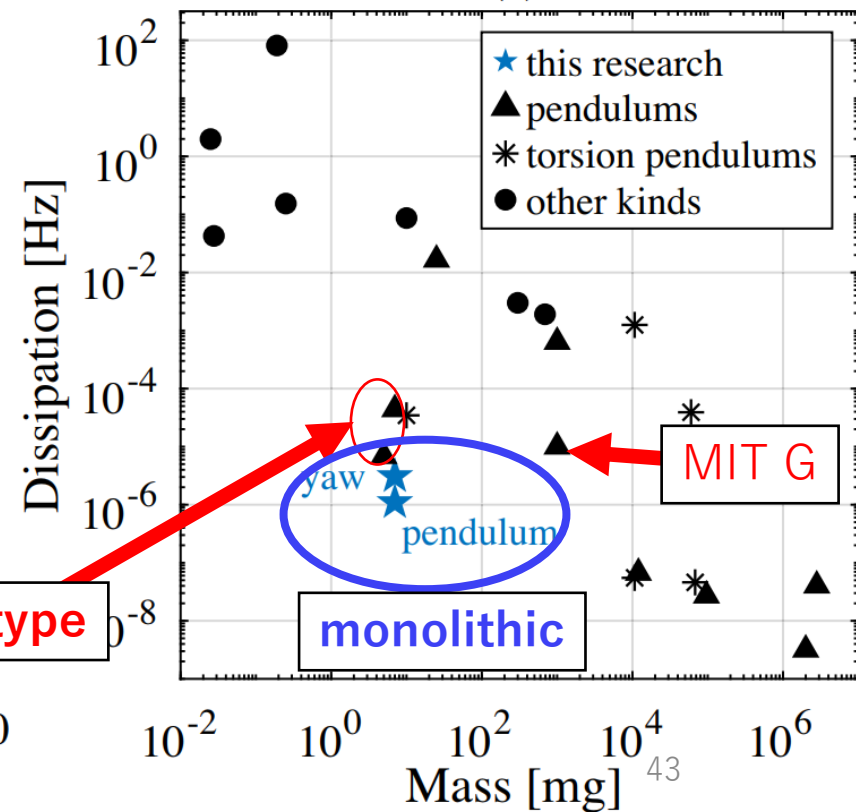
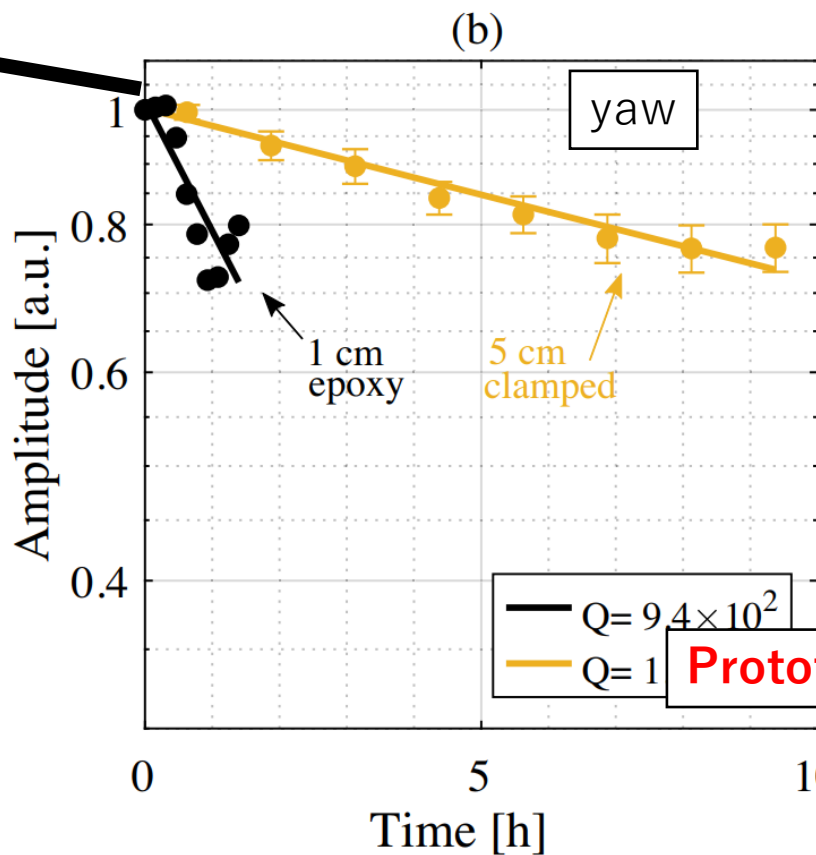
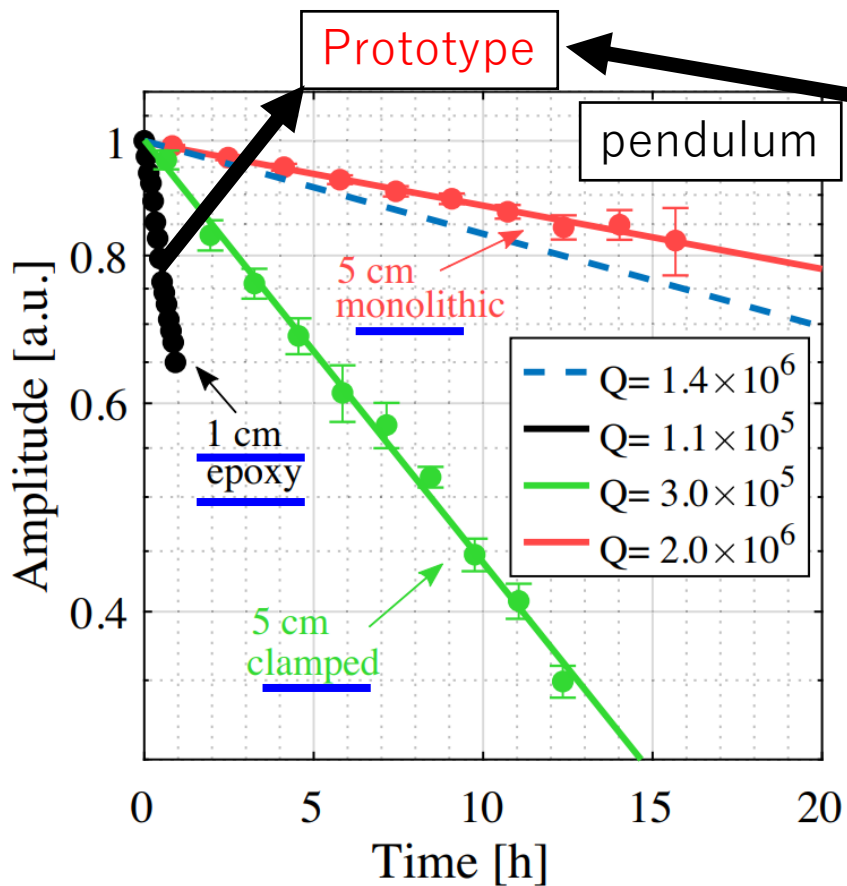
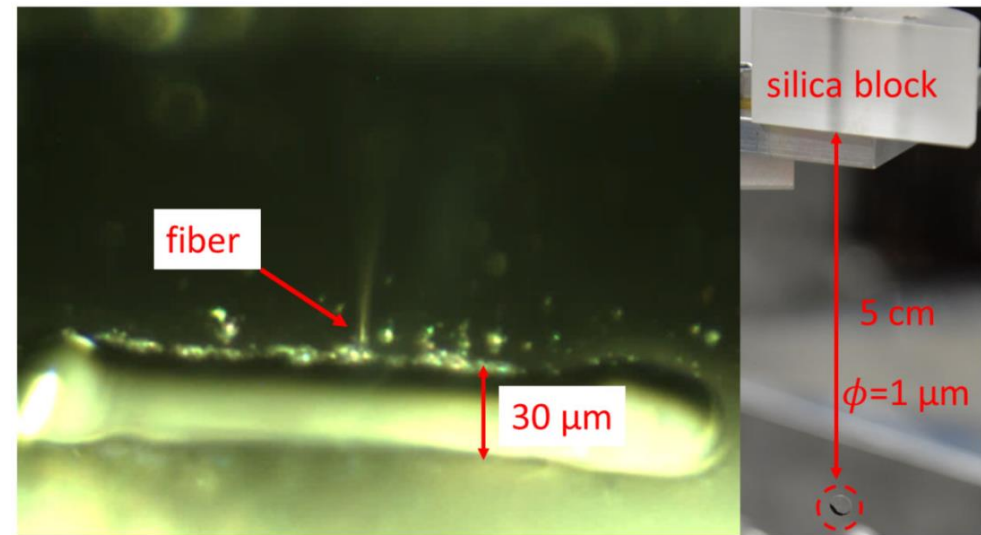


result

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Summary

- Goal: quantum sensing and control of a massive pendulum
- Approach: measurement-based control, development of low-loss pendulum, optical spring
- Current status: conditional squeezing ($\sim 400 \times x_{zpf}$)
- Future: quantum squeezing