KEK素核研物構研QUP連携研究会@KEK, 2022年 2月8日 (火)

Sachdev-Ye-Kitaev模型の ダイナミクスと量子情報

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based on

T.Nosaka (Riken) - TN,JHEP 02 (2020) 081, arXiv 1912.12302

T.Nosaka - TN,JHEP 02 (2021) 150, arXiv 2009.10759

A.Kulkarni (Princeton U)- TN - S.Ryu (Princeton U), arXiv 2021.13489

今日の話

①Introduction

- **connections between Quantum Gravity, cond-mat and QI**
- **Review of the SYK model**

②Model for SYK teleportation

T.Nosaka (Riken) - TN,JHEP 02 (2020) 081, arXiv 1912.12302

T.Nosaka - TN,JHEP 02 (2021) 150, arXiv 2009.10759

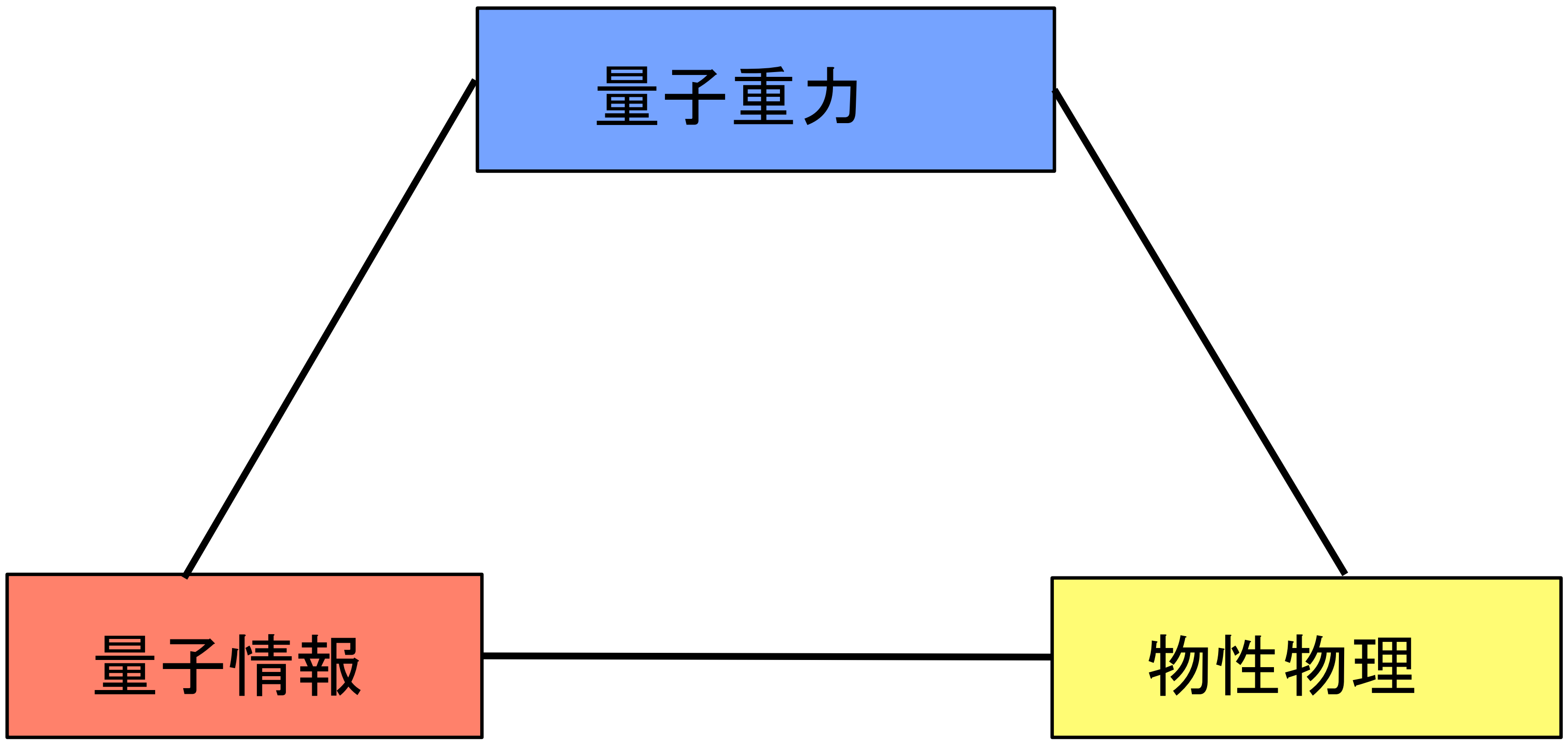
(understand QI and QG using cond-mat)

③SYK Lindbladian

A.Kulkarni (Princeton U)- TN - S.Ryu (Princeton U), arXiv 2021.13489

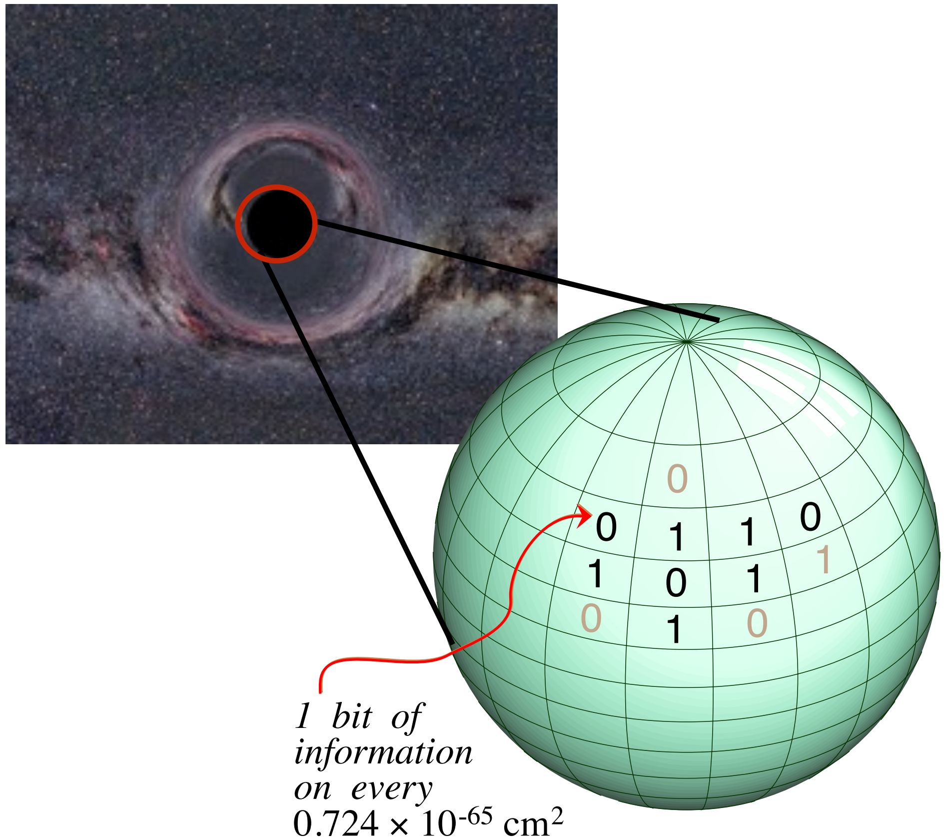
(solvable model of strongly coupled open system motivated from QG)

Connection between QG and QI:



Black Holes (BHs) and information

Black Holes:
Entropy = Area of Horizons



BH entropy formula

ブラックホールのエントロピー公式

[Bekenstein 72] [Hawking 74]

$$S_{BH} = k_B \frac{A}{4\ell_P^2}$$

A : 地平線の面積

$$\ell_P = \sqrt{\frac{\hbar G_N}{c^3}} \approx 1.616 \times 10^{-35}\mathrm{m}$$

→ブラックホールを含めた系で第二法則が成立

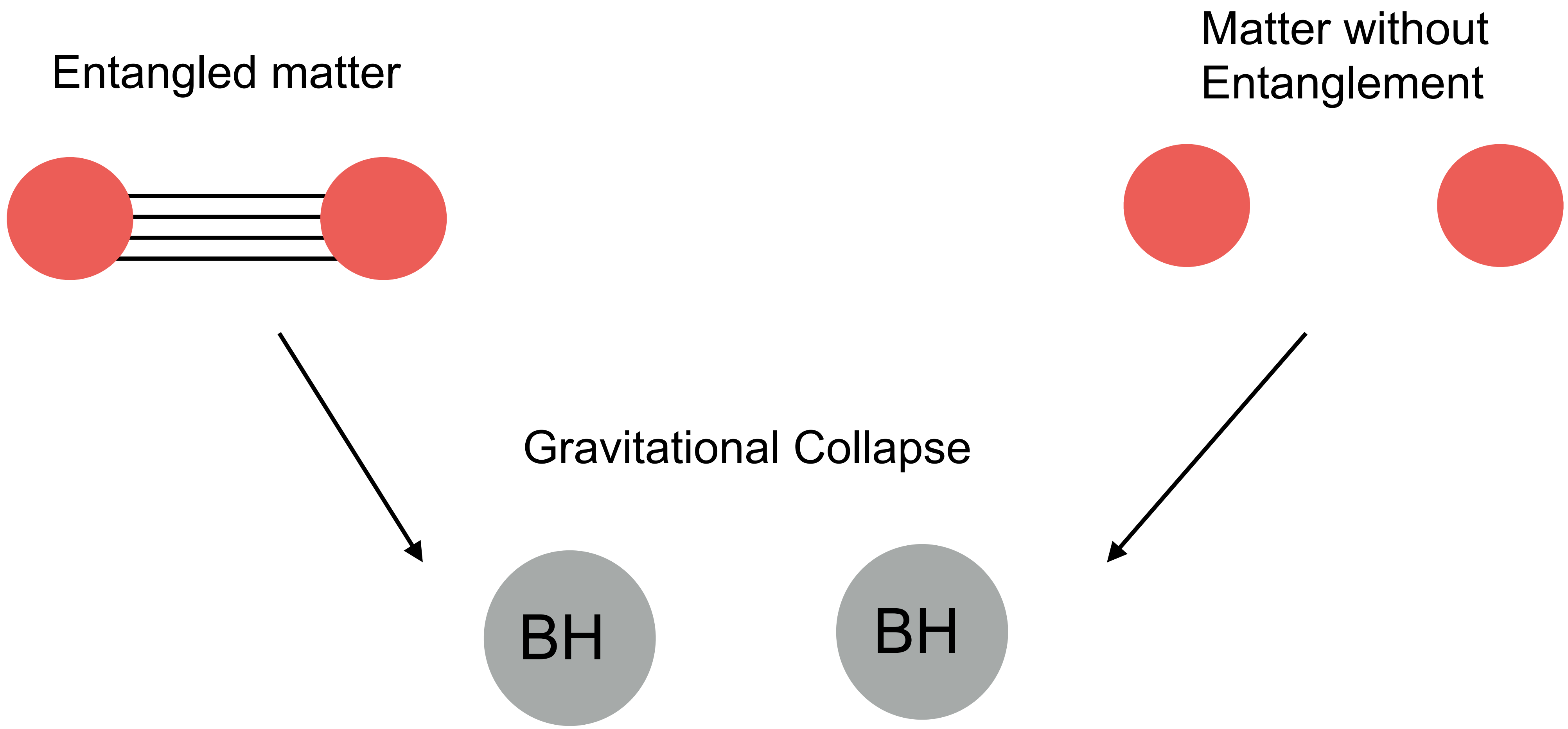
→Holography原理へ [’t Hooft 93] [Susskind 94]

Entanglement and Geometry:

Law of Gravity (Einstein Equation):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} \longrightarrow \phi(r) = G_N \frac{Mm}{r}$$

エネルギーだけが時空の幾何(重力ポテンシャル)を決定



Are they the same two black holes?

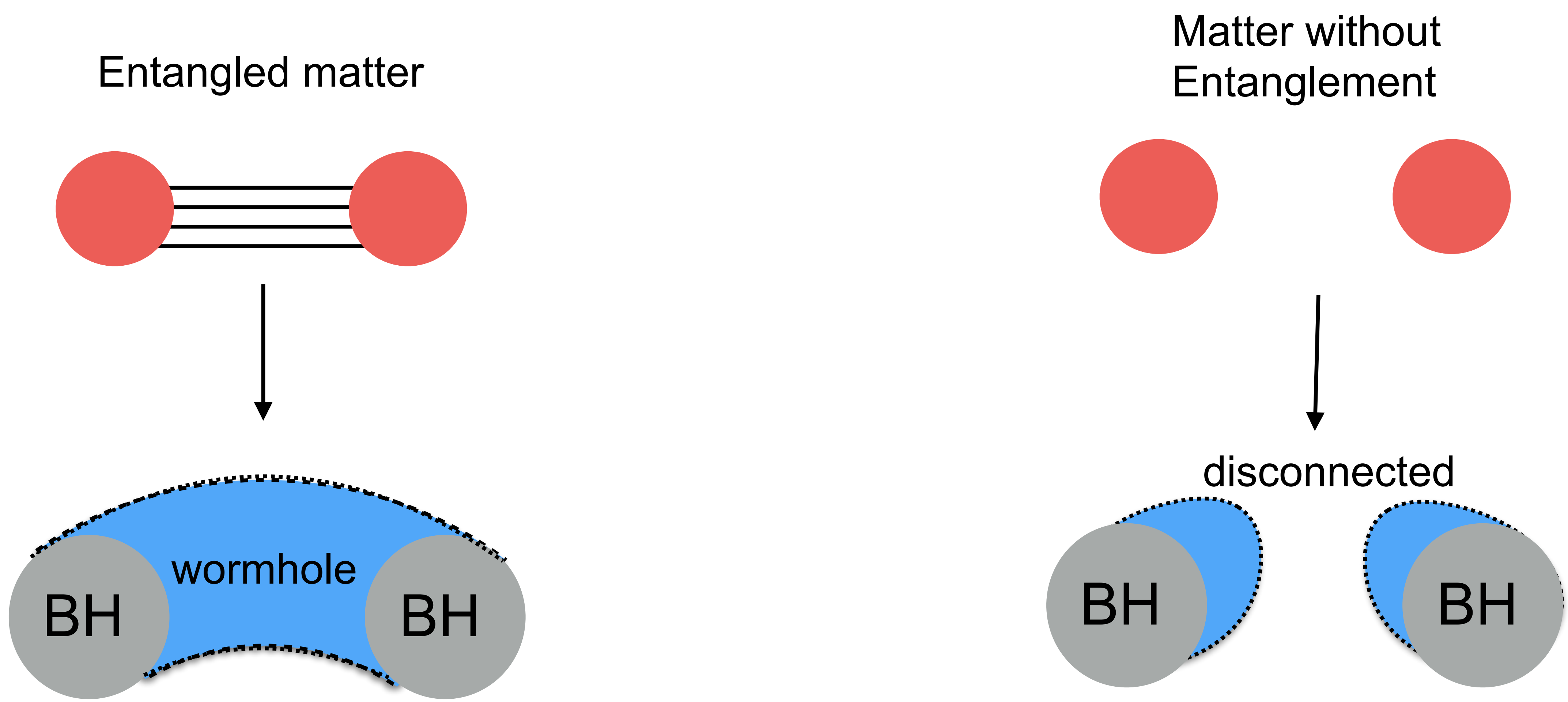
Entanglement and Geometry:

Proposal: ER = EPR

[Raamsdonk, 10]
[Maldacena-Susskind, 13]

エンタングルメントと 時空のつながり(ワームホール)は等価

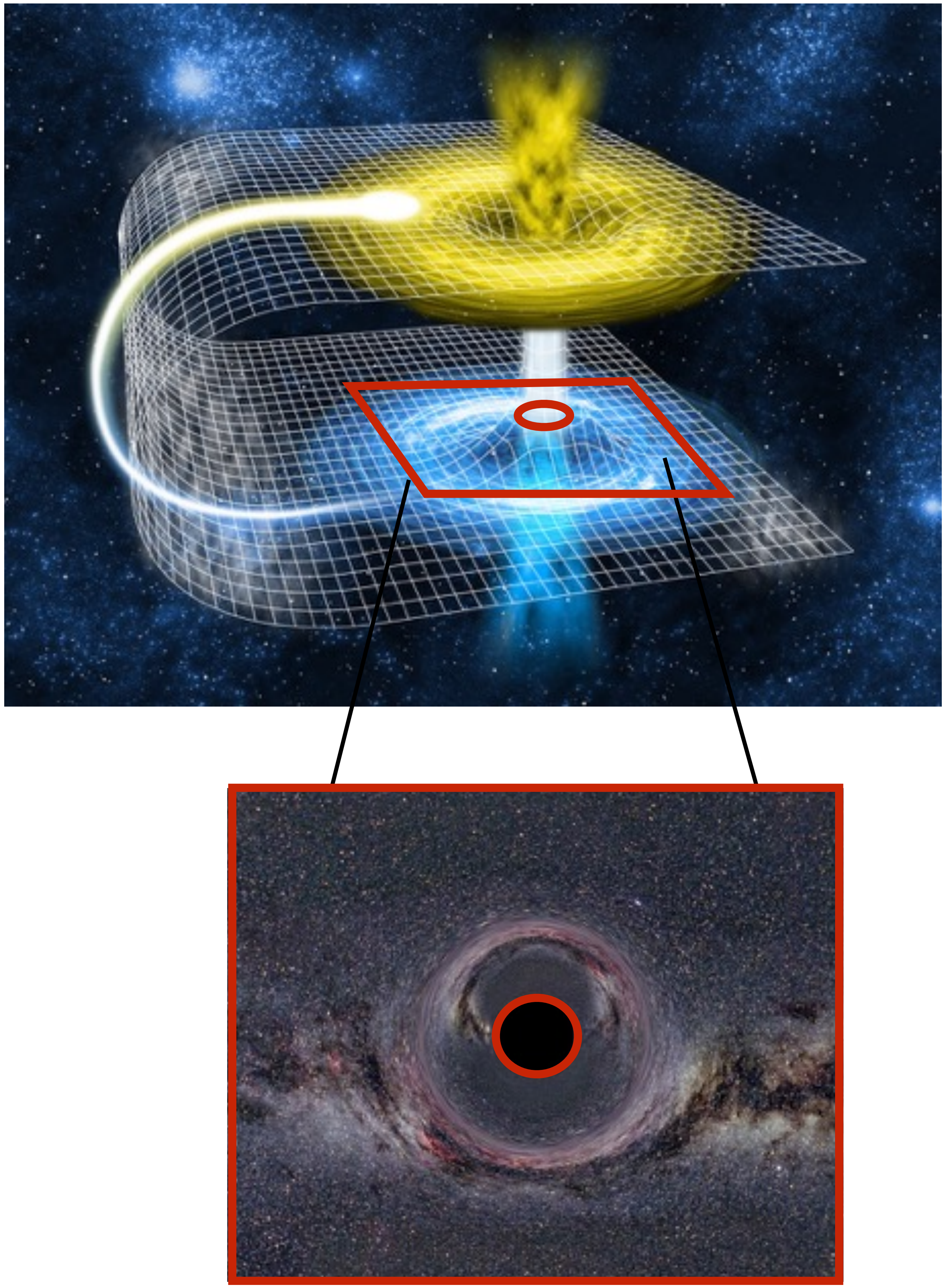
(Entangled Black holes are geometrically connected in the interior)



Area = **Entanglement** Entropy [Ryu-Takayanagi, 06]

Equivalence of wormholes and entangled matter

Wormhole



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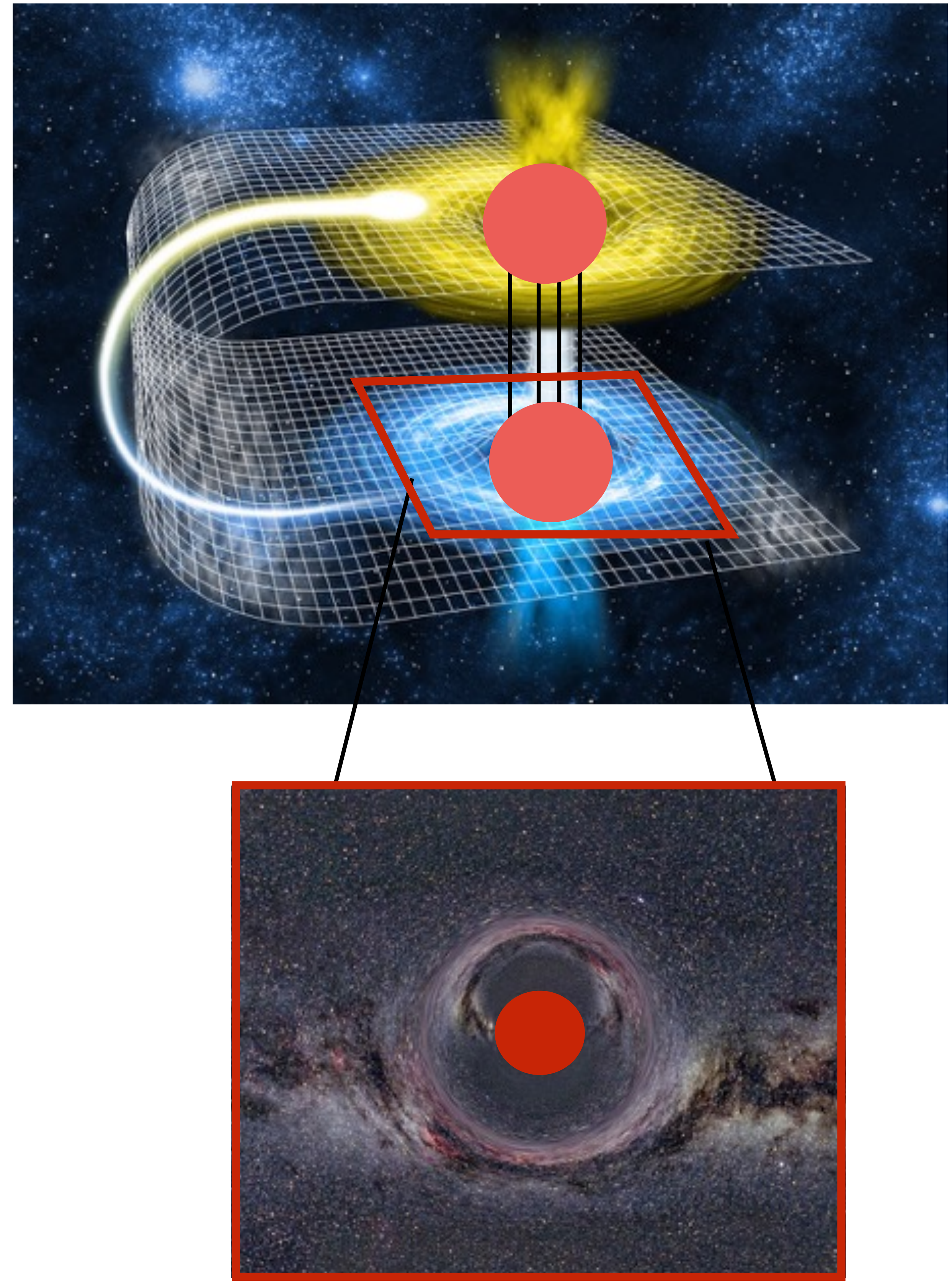
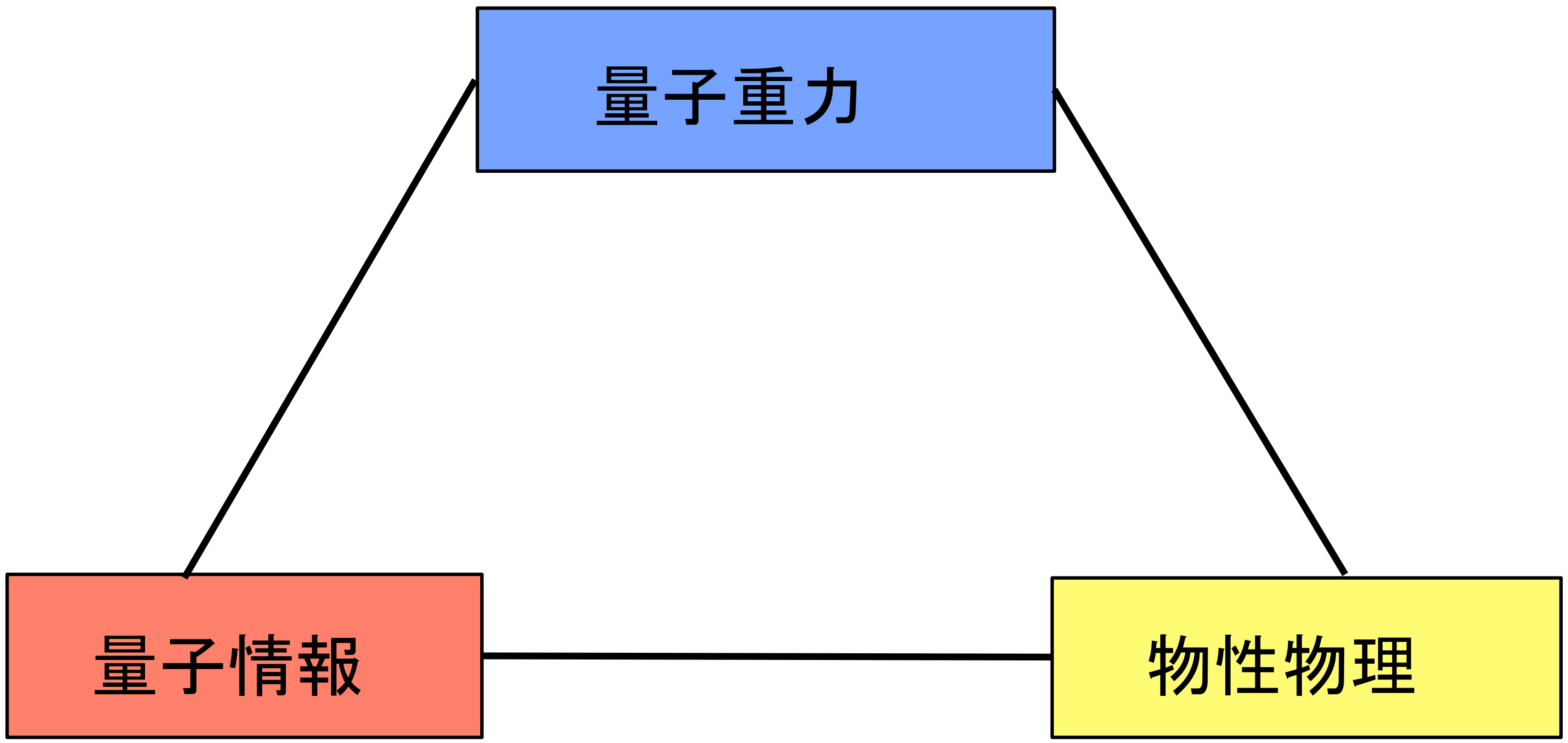
Entangled matter



Connection between QI and cond-mat:



There is much literature…

Entanglement in condensed matter:

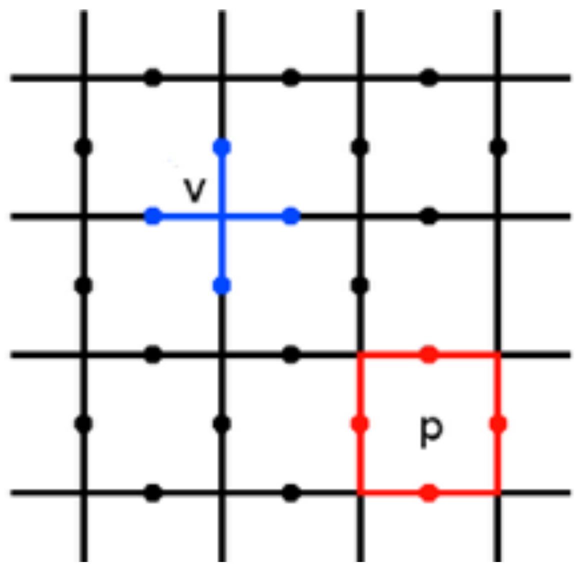
Quantum information plays an important role in condensed matter

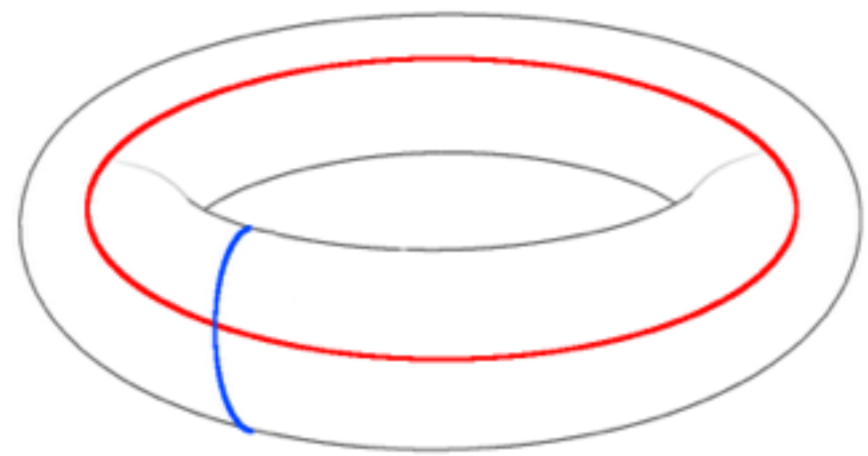
例: Toric code
[Kitaev 97]

$$H_{TC} = -J\sum_v A_v - J\sum_p B_p$$

$$A_v = \prod_{i\in v}\sigma_i^x$$

$$B_p = \prod_{i\in p}\sigma_i^z$$





・EE=トポロジカル相のorder parameter [Levin-Wen 04] [Kitaev-Preskill 05]

・縮退したGround states = logical qubits, encoded in entangled state

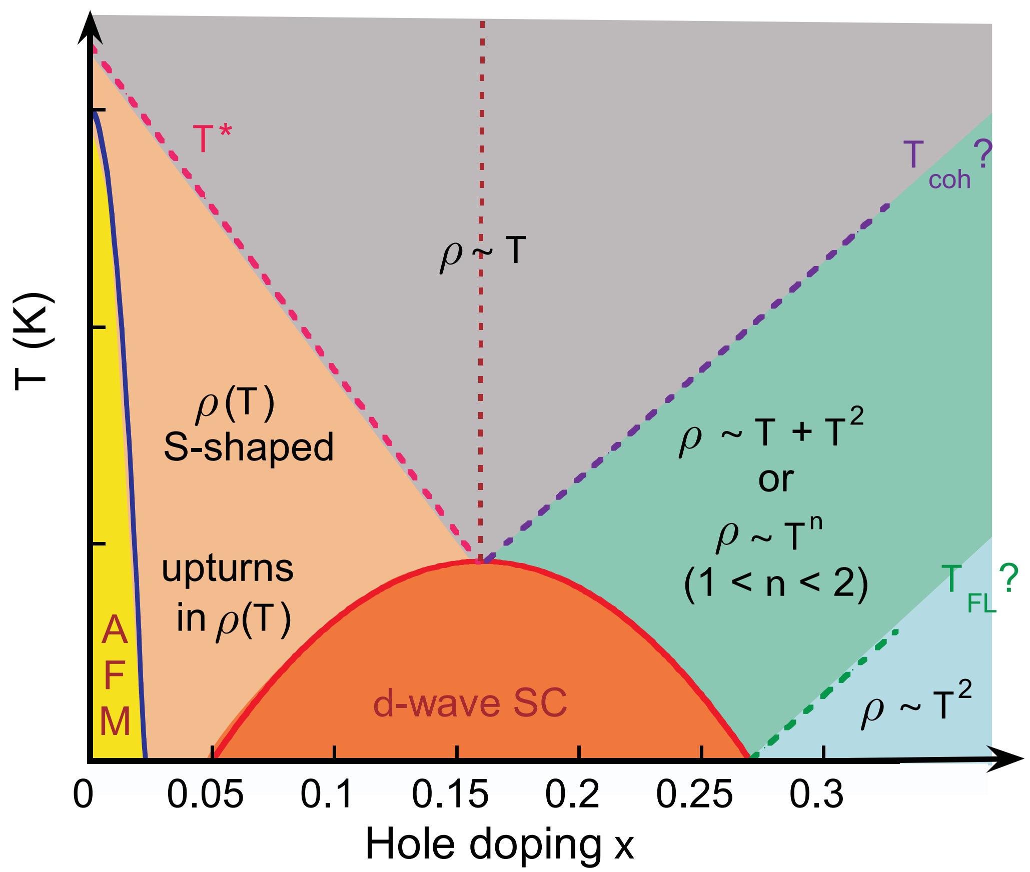
・Wilson loops = logical operators

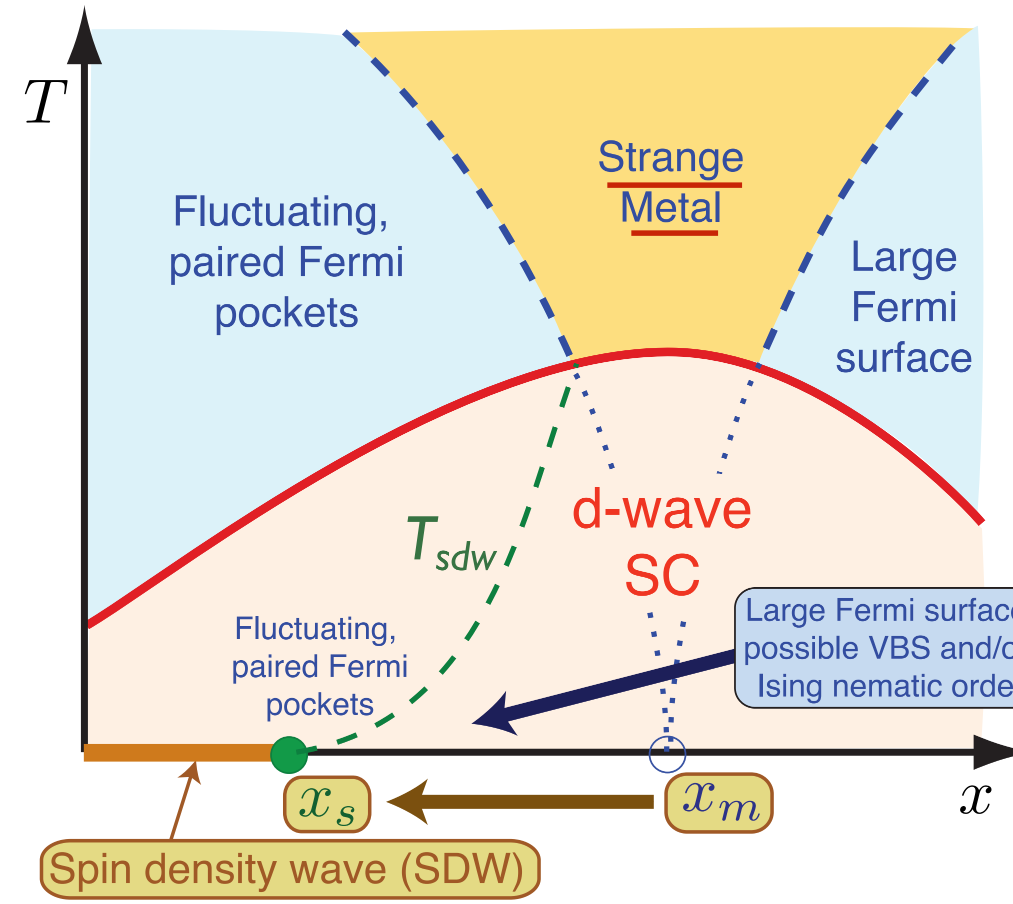
・Anyon 励起 = error

・Gap = robustness against errors

Strong coupled system : Strange Metals

Example of Hole doped CuO2



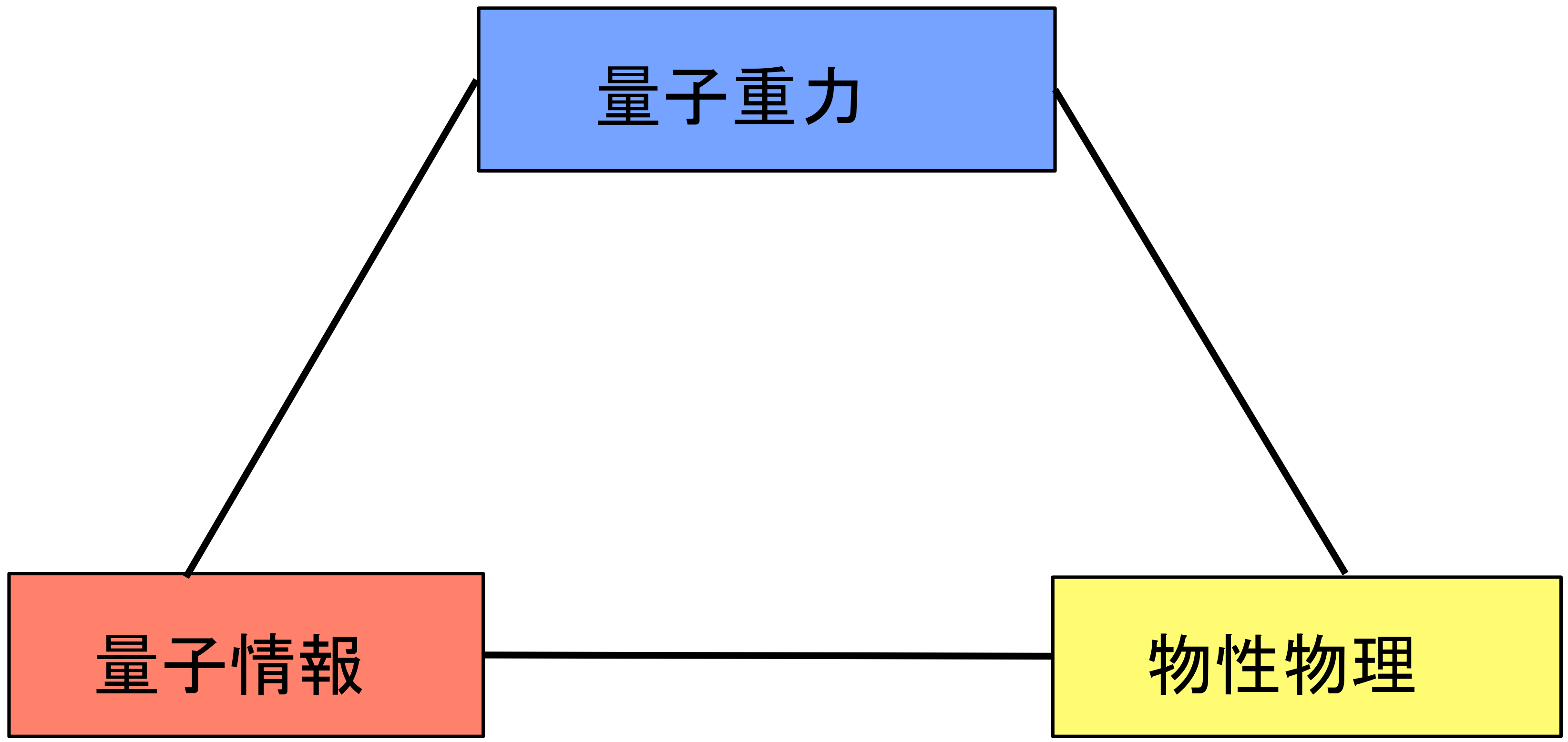


・No quasi particle excitation (フェルミ流体とは違う)

・Highly entangled states による非フェルミ流体の振る舞い

(Pictures taken from Sachdev 0907.0008)

Connection between QG and cond-mat:



There much literature on AdS/CMT…

SYK model:

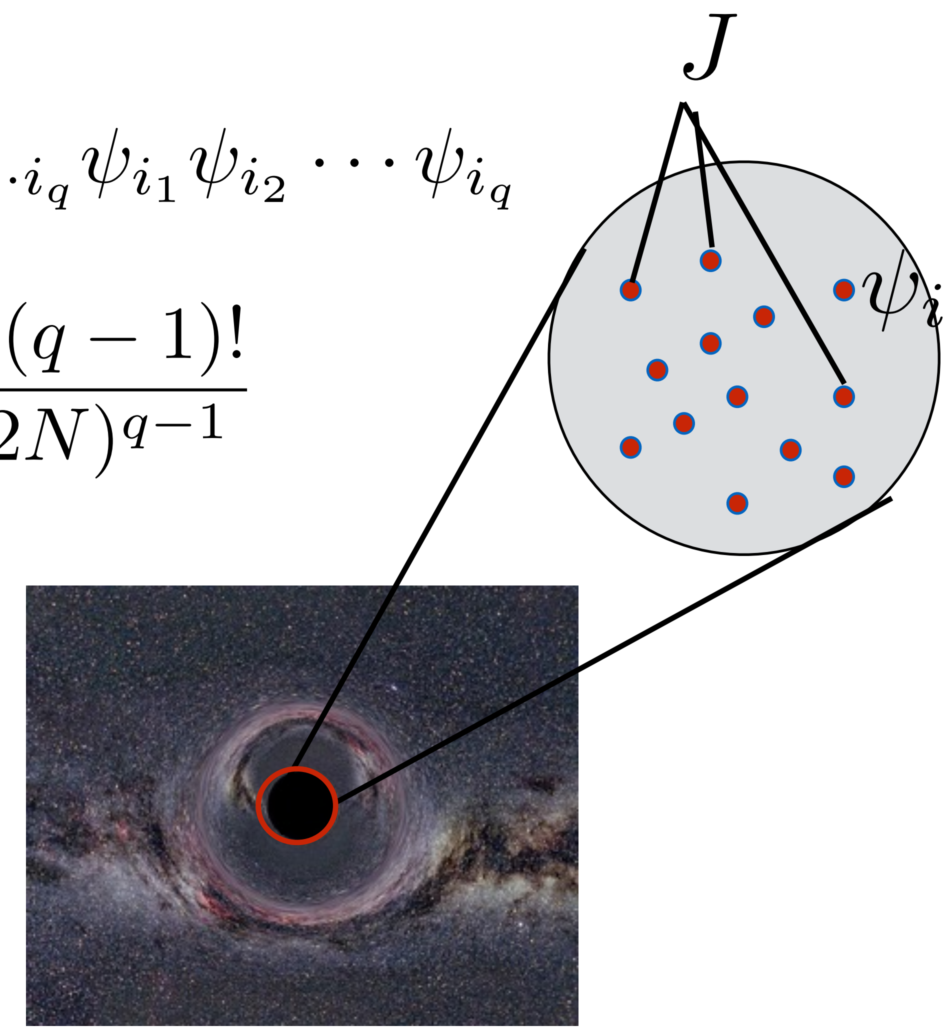
[Sachdev-Ye 93] [Kitaev 14,15]

N Majorana fermions
(= qubits on horizon)

$$\{\psi_i, \psi_j\} = \delta_{ij} \quad (\, \mathrm{dim}\mathcal{H} = 2^{\frac{N}{2}})$$

Hamiltonian: $H_{SYK} = i^{\frac{q}{2}} \sum_{i_1 < i_2 < \cdots < i_q} J_{i_1 i_2 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$

with $\langle J_{i_1 i_2 \cdots i_q} \rangle_J = 0$ and $\langle J^2_{i_1 i_2 \cdots i_q} \rangle_J = \dfrac{\mathcal{J}^2 (q-1)!}{q(2N)^{q-1}}$



- Similar to Holographic Non Fermi Liquid (AdS/CMT), concrete Hamiltonian
- Solvable in the large N limit
- Exact diagonalization at finite N
- ✔ cond-mat ✔ quantum information ?general relativity
- Have the 2d dilaton gravity sector

[Maldacena-Stanford, 16] [Maldacena-Stanford-Yang, 16]

large N limit:

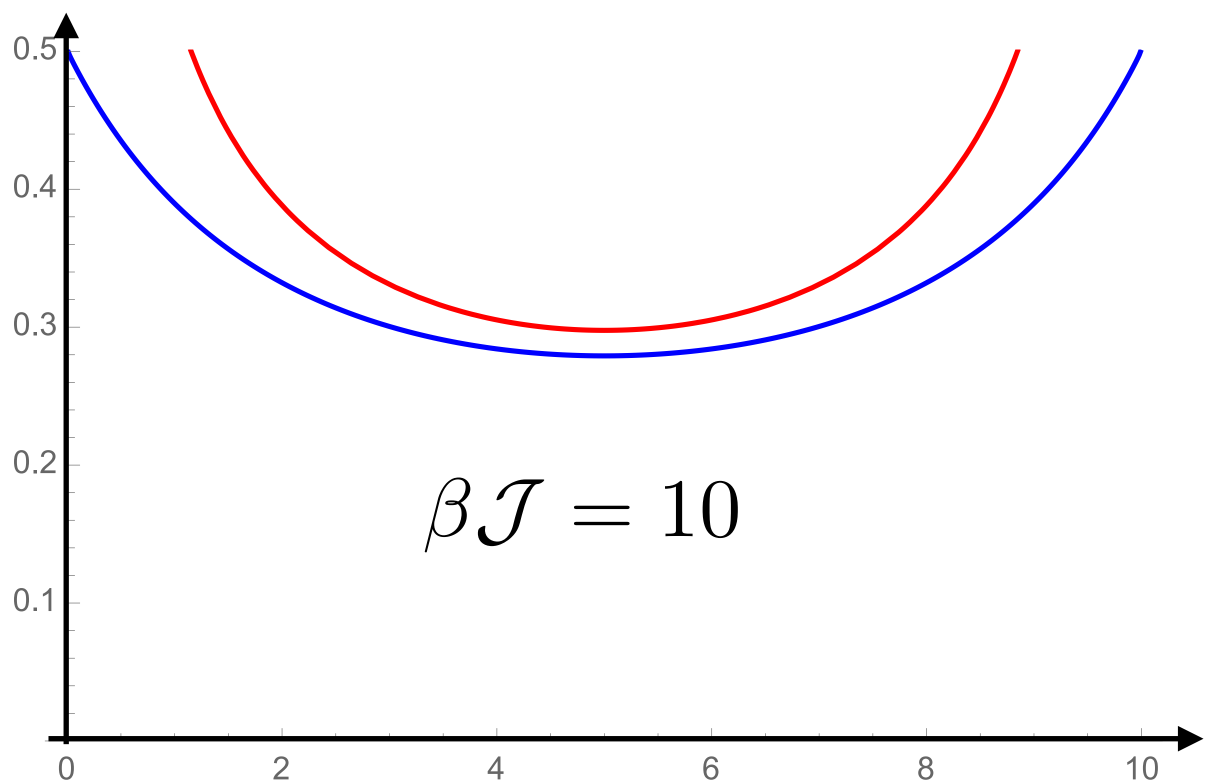
disorder average と Hubbard-Stratonovich変換をすると

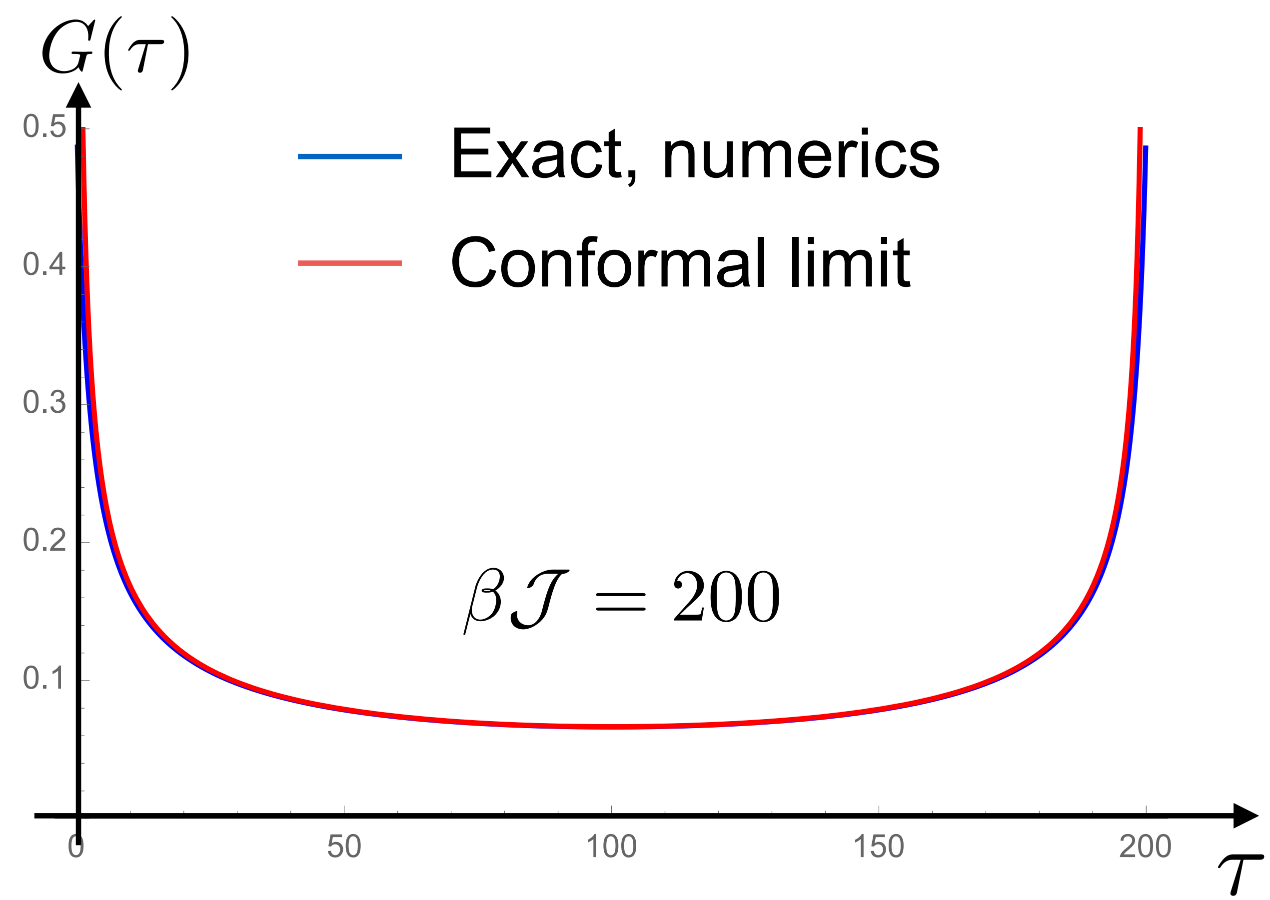
$$S[G,\Sigma] = N\left[\log \mathrm{Pf}(\partial_\tau - \Sigma) - \int d\tau_1 \int d\tau_2 \Sigma(\tau_1,\tau_2) G(\tau_1,\tau_2) - \frac{\mathcal{J}^2}{q} G(\tau_1,\tau_2)^q\right]$$

for $G(\tau_1,\tau_2) = \frac{1}{N}\langle \psi_i(\tau_1)\psi_i(\tau_2)\rangle$ and $\Sigma(\tau_1,\tau_2)$

→ large Nでsaddle pointをとることで解ける

・Numerical solution at q=4





Universality: Conformal symmetry breaking

・SYK model は 破れた reparametrization (conformal) 対称性を持つ:

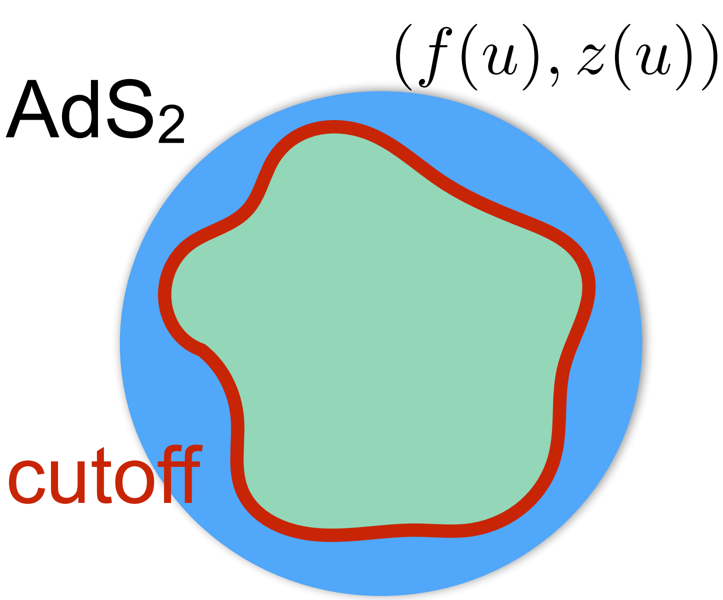
$$G(\tau_1, \tau_2) \;\rightarrow\; [f'(\tau_1) f'(\tau_2)]^{\Delta} G(f(\tau_1), f(\tau_2))$$

(correlation function)

The effective action for Nambu-Goldstone modes $f(\tau)$ is

$$S = -\frac{N\alpha_S}{\mathcal{J}} \int \{f(\tau), \tau\} \qquad \{f(\tau), \tau\} = \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2}\left(\frac{f''(\tau)}{f'(\tau)}\right)^2$$

・同じ対称性の破れは 2d dilaton gravityにも存在



・ある種(near extremal)のBHのダイナミクスは reparametrizationのダイナミクスで記述される

[Maldacena-Stanford-Yang, 16]

Universality: Conformal symmetry breaking

4d near extremal black holes in standard model

Holographic Non-Fermi Liquid (BH in AdS)

SYK model

2d dilation gravity

$$\frac{1}{16\pi G}\int \phi\sqrt{g}(R+2)+\frac{1}{8\pi G}\int_{bdy}\phi_b K$$

low energy

Schwarzian action: $$S=-C\int du\{f(u),u\}$$

$$\{f(u),u\}=\frac{f'''}{f'}-\frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

reparametrizationの"Non Linear Sigma Model" .

Many dynamics are governed by the dynamics of reparametrization

Linear in T resistivity (in high Tc SC) from Schwarzian [Guo-Gu-Sachdev, 20]

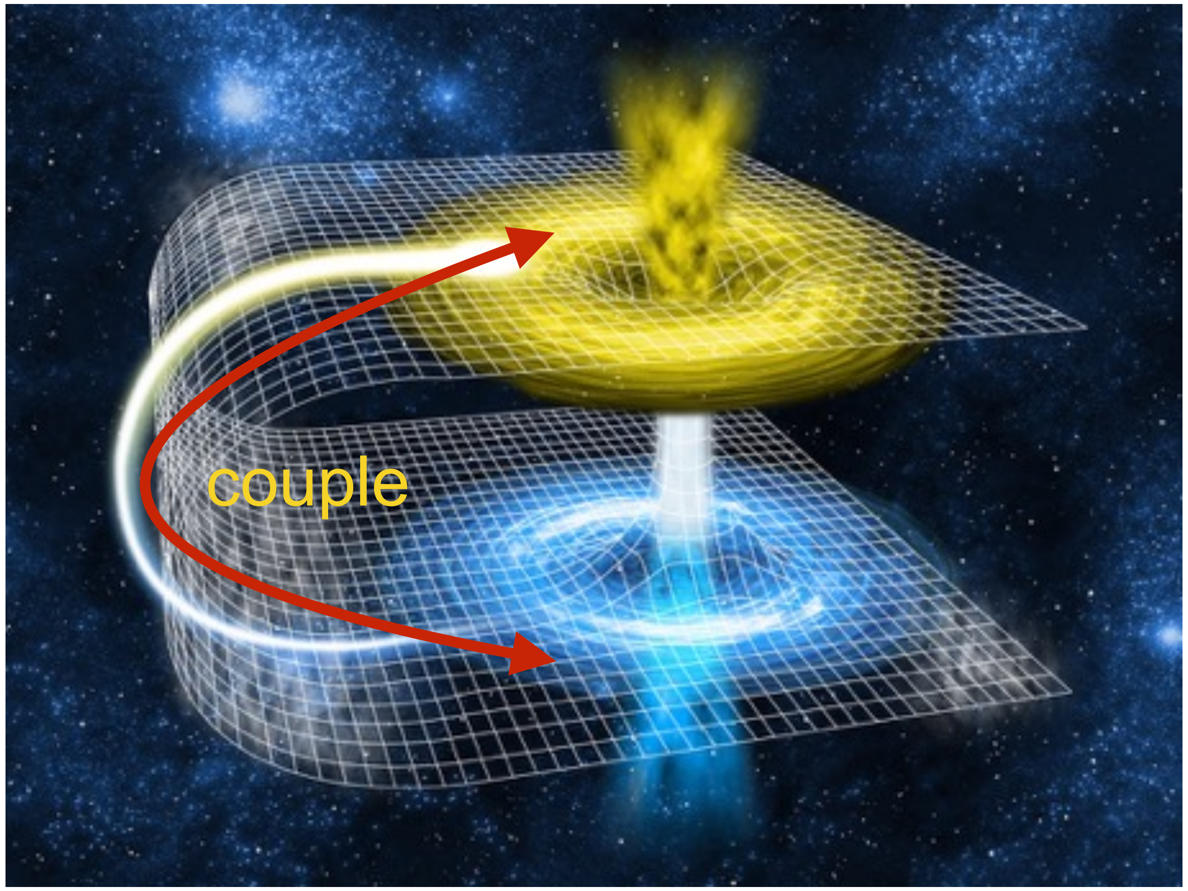
Traversable wormholes

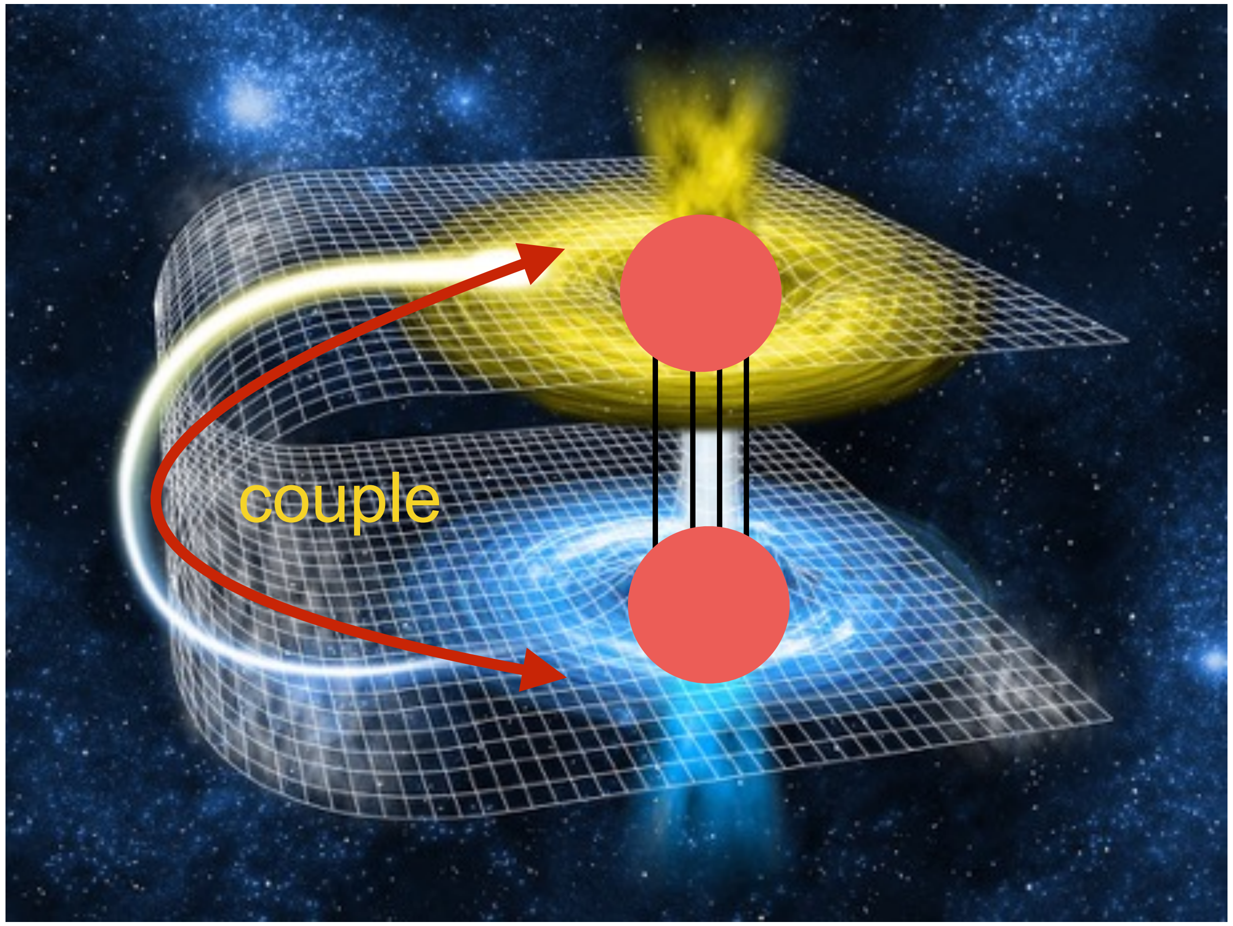
[Gao-Jafferis-Wall, 16]

Entanglement is resource. We can not send messages only with entanglement

Similarly, having wormhole is not enough to send messages

Can we send messages through wormholes? -> traversable wormholes!





A solution can be constructed in the Standard model (smaller than 1/Tev)

in a regime where we can use reparametrization (Schwarzian) modes

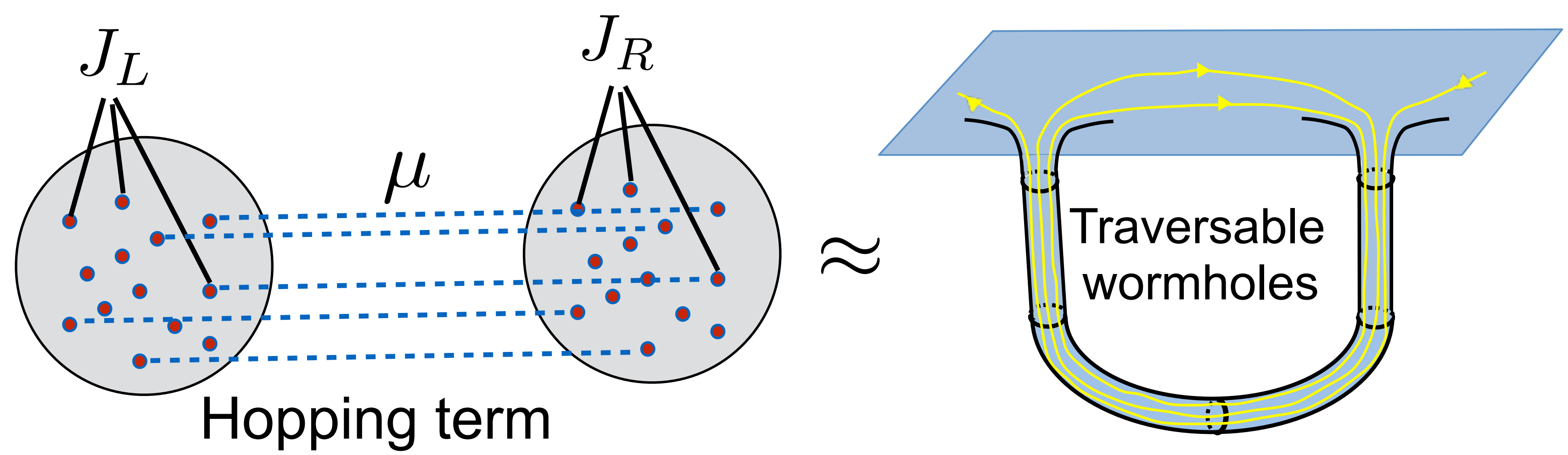
[Maldacena-Milekhin-Popov, 18]

SYK traversable wormholes:

[Maldacena-Qi 18]

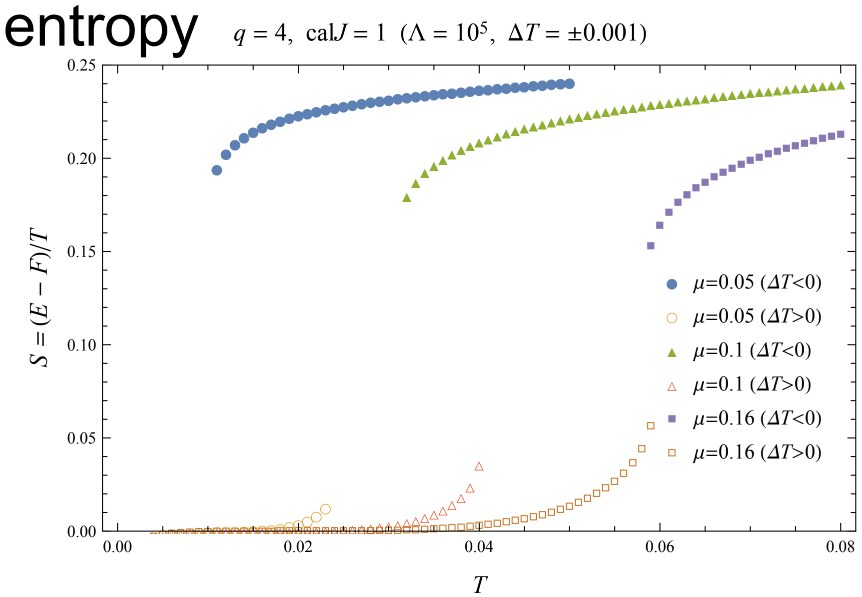
$$H = H_L + H_R + \mu H_{int}$$

$$H_{int} = i \sum_{i=1}^{N} \psi_i^L \psi_i^R$$

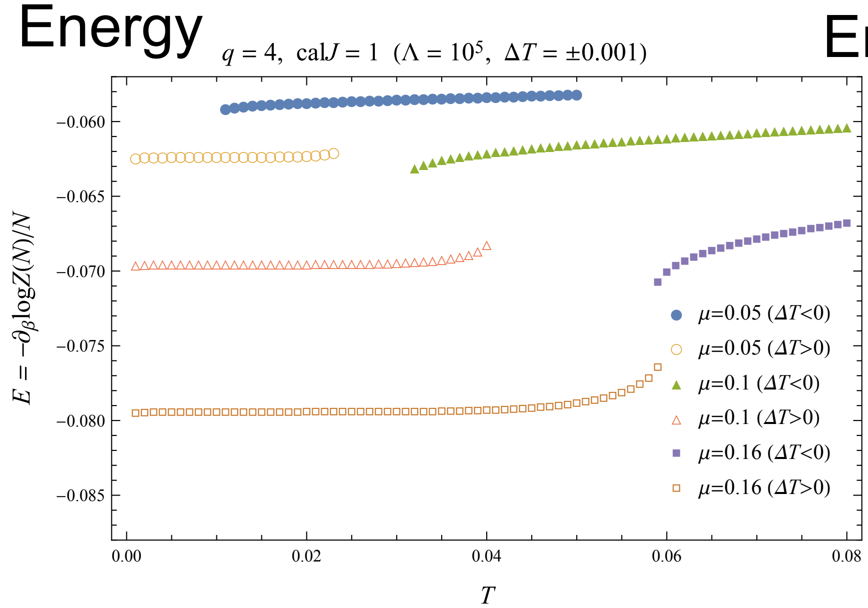


- Gapped phase = Traversable wormholes
- reparametrization (quantum gravity) mode で記述できる
- 高温では, SYK = Non-Fermi Liquid like phase = 2 BH phase

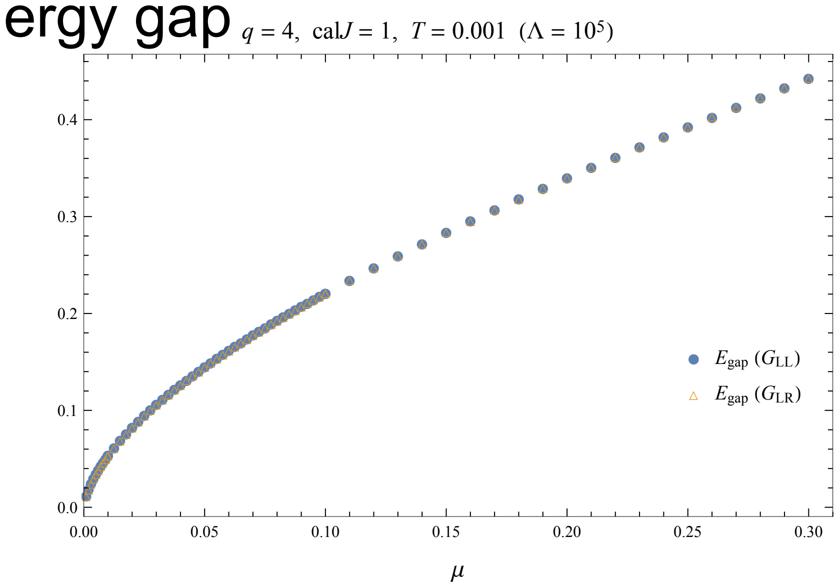
entropy



Energy



Energy gap



①Introduction

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②Model for SYK teleportation

③SYK Lindbladian

direction: QI + cons-mat → QI + QG

based on

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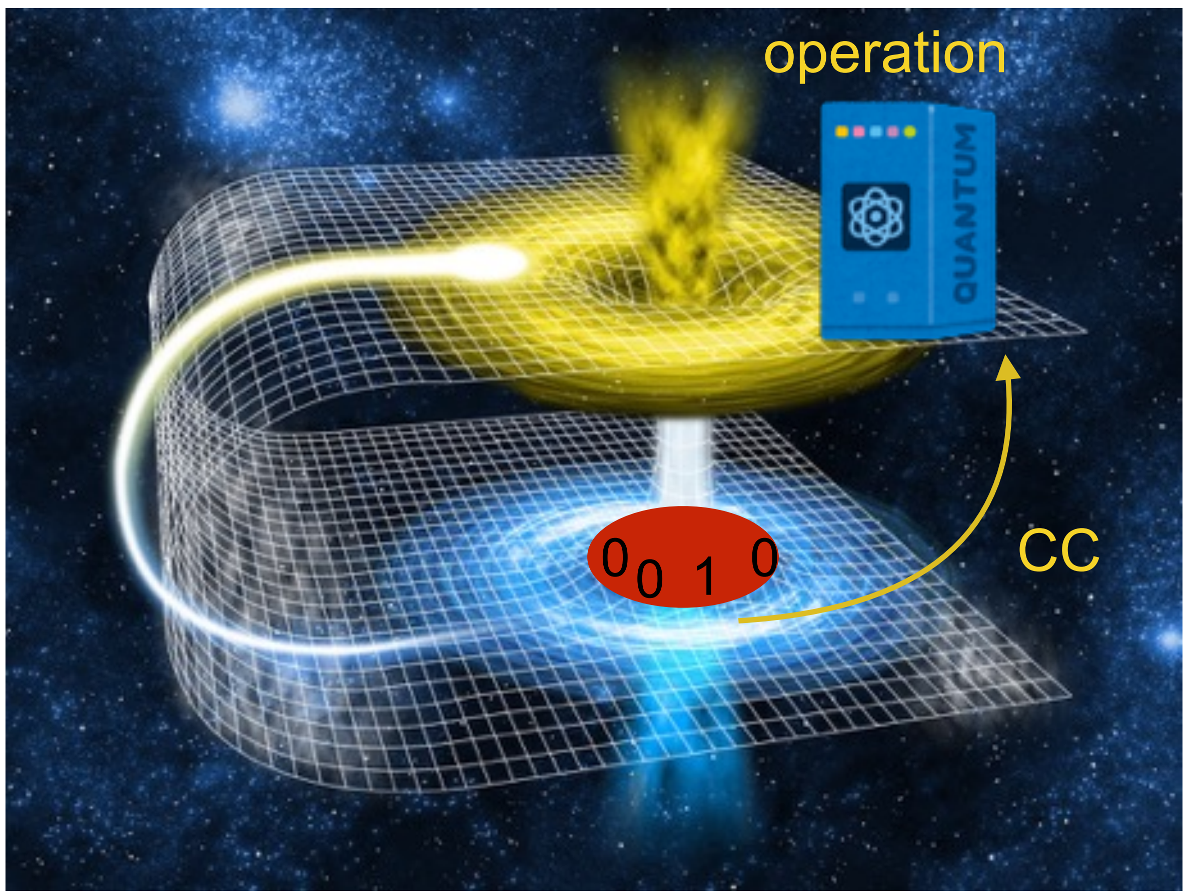
SYK/Holographic teleportation

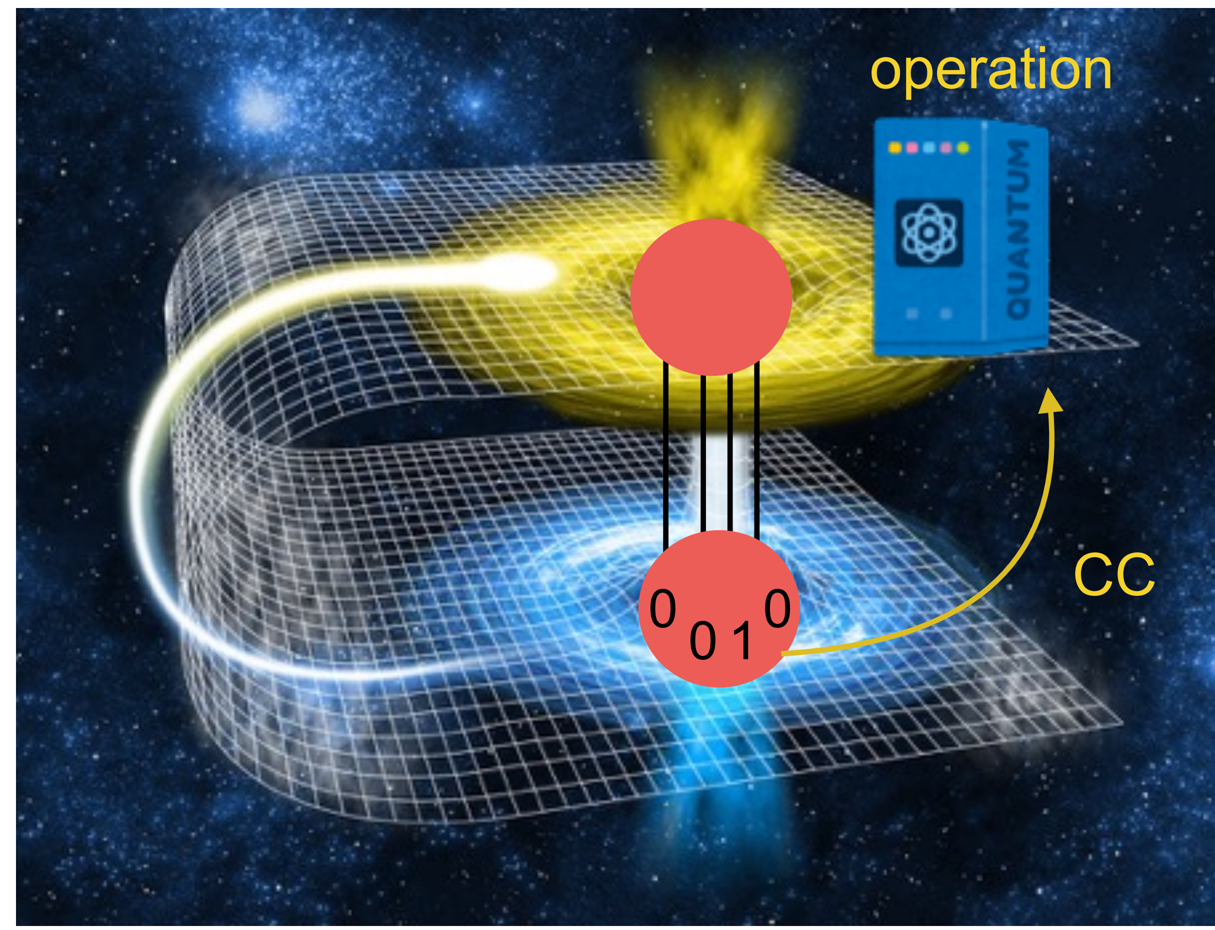
[Shiba-TN-Takayanagi-Watanabe, 16]

[Susskind, 16]

Entanglement is resource.

With measurements and operations, we can teleport quantum information





Split space time

Send results by CC

Do quantum operation

Measure one side on bottom

Send results by CC

Do quantum operation on top

SYK teleportation

[Kourkoulou-Maldacena, 17]
[Gao-Jafferis, 19]

- entangled state:

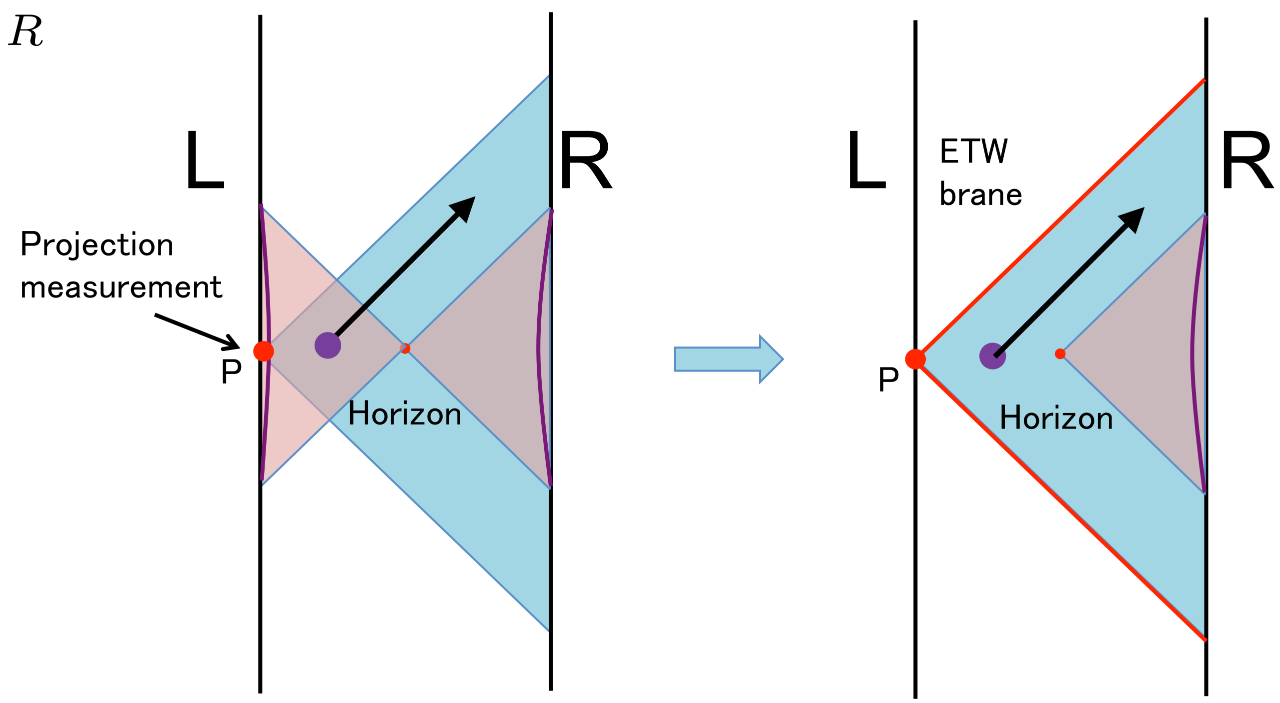
$$|TFD(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle_L \otimes |E_n\rangle_R$$

- measurement basis

$$\psi_{2k-1}|B_{\boldsymbol{s}}\rangle = i s_k \psi_{2k} |B_{\boldsymbol{s}}\rangle \quad \boldsymbol{s} = (s_1, \cdots, s_{\frac{N}{2}})$$

$$|B_{\boldsymbol{s}}(\beta)\rangle = \langle B_{\boldsymbol{s}}|TFD(\beta)\rangle$$

result in pure states in R system

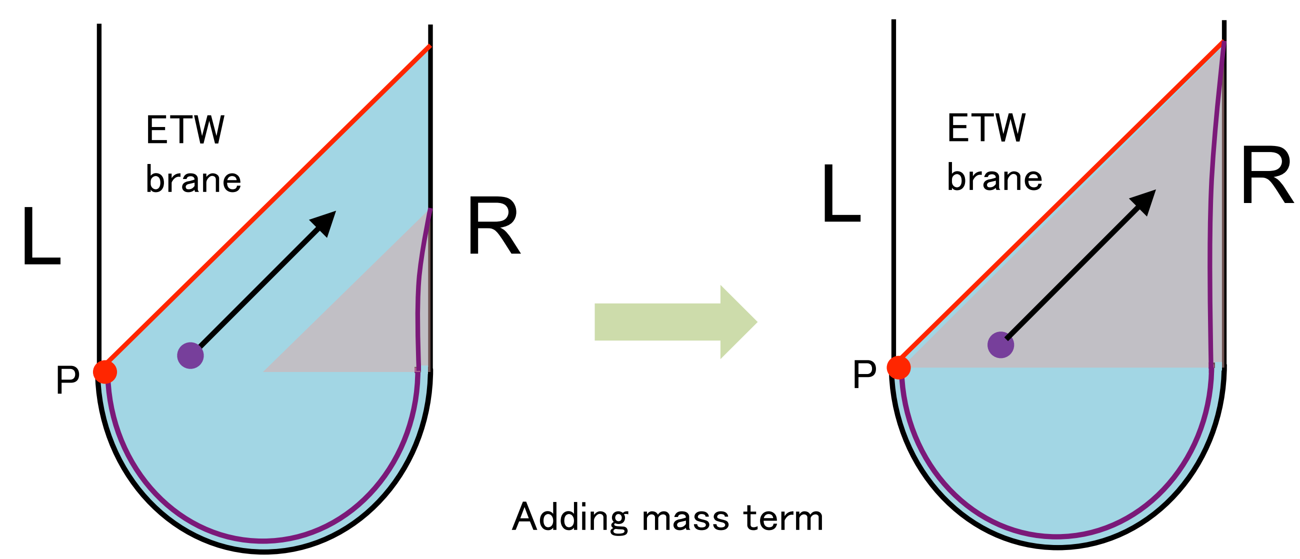


- state dependent deformation

$$H_{SYK} \to H_{\mathrm{def}}$$

$$H_{\mathrm{def}} = H_{SYK} + \mu H_M(\boldsymbol{s})$$

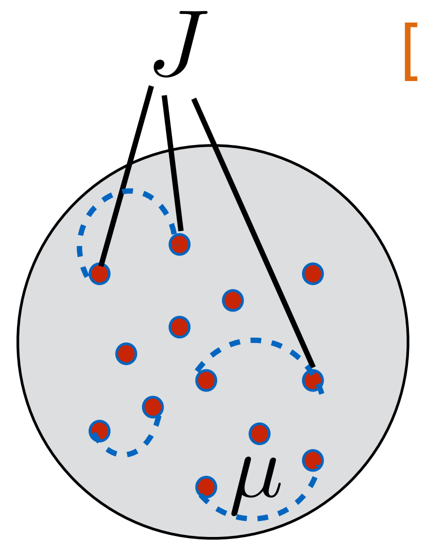
can teleport the QI of L system



[cf: Almheiri-Mousatov-Shanyi, 18] [cf: QET in BH formations, Hotta10]

mass deformed SYK model :

[Kourkoulou-Maldacena, 17]



$$H_{\text{def}} = H_{SYK} + \mu H_M(\boldsymbol{s})$$

$$H_{SYK} = i^{\frac{q}{2}} \sum_{i_1 < \cdots < i_q} J_{i_1 \cdots i_q} \psi_{i_1} \cdots \psi_{i_q}$$

$$H_M(\boldsymbol{s}) = i \sum_{k=1}^{N/2} \underline{s_k} \psi_{2k-1} \psi_{2k}$$

($H_M \to i \sum_{i=1}^{\frac{N}{2}} (\hat{\sigma}_k \psi_{2k-1} \psi_{2k} + B_k \hat{\sigma}_k)$: SYK with controller)

measurement basis

$$\psi_{2k-1} |B_{\boldsymbol{s}}\rangle = i\underline{s_k} \psi_{2k} |B_{\boldsymbol{s}}\rangle \qquad \underline{\boldsymbol{s} = (s_1, \cdots, s_{\frac{N}{2}})} \qquad k = 1 \sim N/2$$

$|B_{\boldsymbol{s}}\rangle$: simultaneous eigenstates of spin operators $S_k = -2i\psi_{2k-1}\psi_{2k}$

ground state of the mass term

(cf: stabilizer formalism of quantum error correction)

projected state: $|B_{\boldsymbol{s}}(\beta)\rangle = \langle B_{\boldsymbol{s}} | TFD(\beta)\rangle = e^{-\frac{\beta}{2} H_{SYK}} |B_{\boldsymbol{s}}\rangle$

what we did in SYK teleportation

Try to put $|B_{\boldsymbol{s}}(\beta)\rangle$ as the ground state of deformed Hamiltonian

Variational approximation by projected state

[Nosaka-TN, 19]

- We want to understand how $|G_{\boldsymbol{s}}(\mu)\rangle$ and $|B_{\boldsymbol{s}}(\beta)\rangle$ are close.

$\rightarrow$ We use the projected states $|B_{\boldsymbol{s}}(\beta)\rangle$

as variational ansatz for the GS of the mass deformed SYK

[cf: 1+1d CFT case, Cardy 17]

- $|B_{\boldsymbol{s}}(\beta = 0)\rangle$ is the ground state of the mass term H_M ($\mu \rightarrow \infty$)
- $|B_{\boldsymbol{s}}(\beta = \infty)\rangle$ is the ground state of the SYK H_{SYK} ($\mu \rightarrow 0$)

- direct computation of the overlap

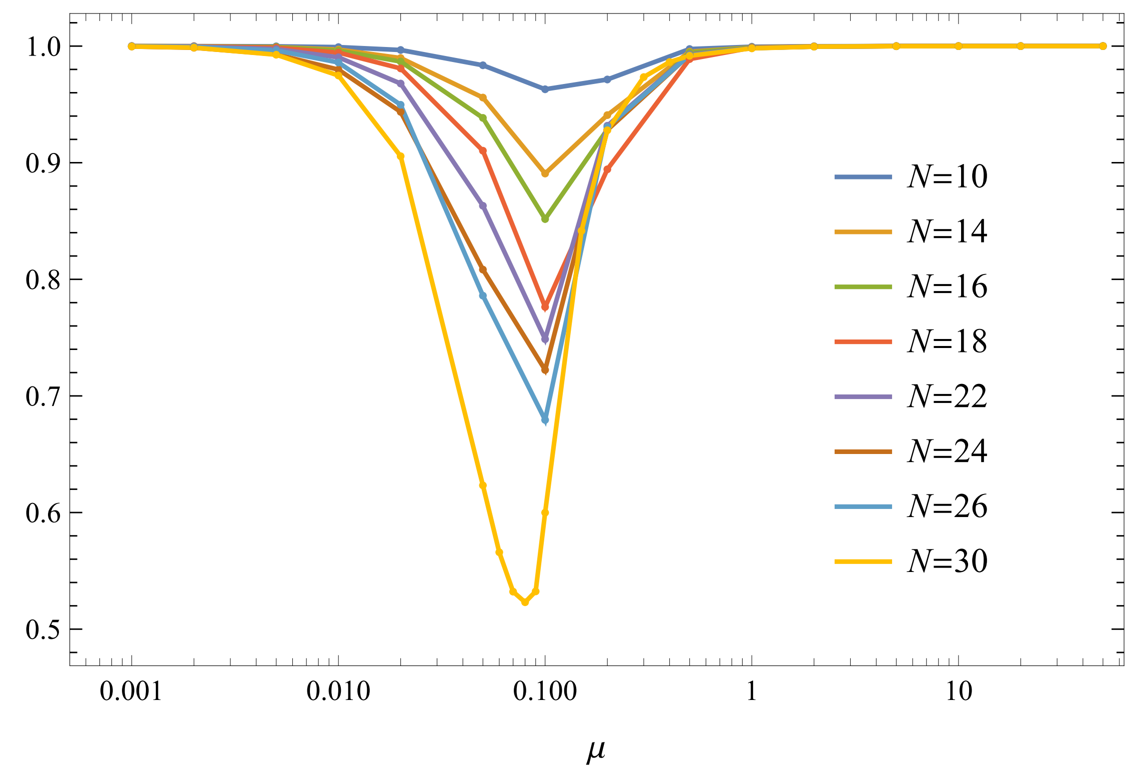
$$|\langle G_{\boldsymbol{s}}(\mu)|B_{\boldsymbol{s}}(\beta(\mu))\rangle|$$

$\beta(\mu)$: maximize overlap

- at large q limit, the overlap is

$$|\langle G_{\boldsymbol{s}}(\mu)|B_{\boldsymbol{s}}(\beta(\mu))\rangle| = e^{-\frac{N}{q^2}\cdot 0 - \#\frac{N}{q^3}}$$

We can compute using Liouville action



Variational approximation, Schwarzian

- Energy in the trial wave function is

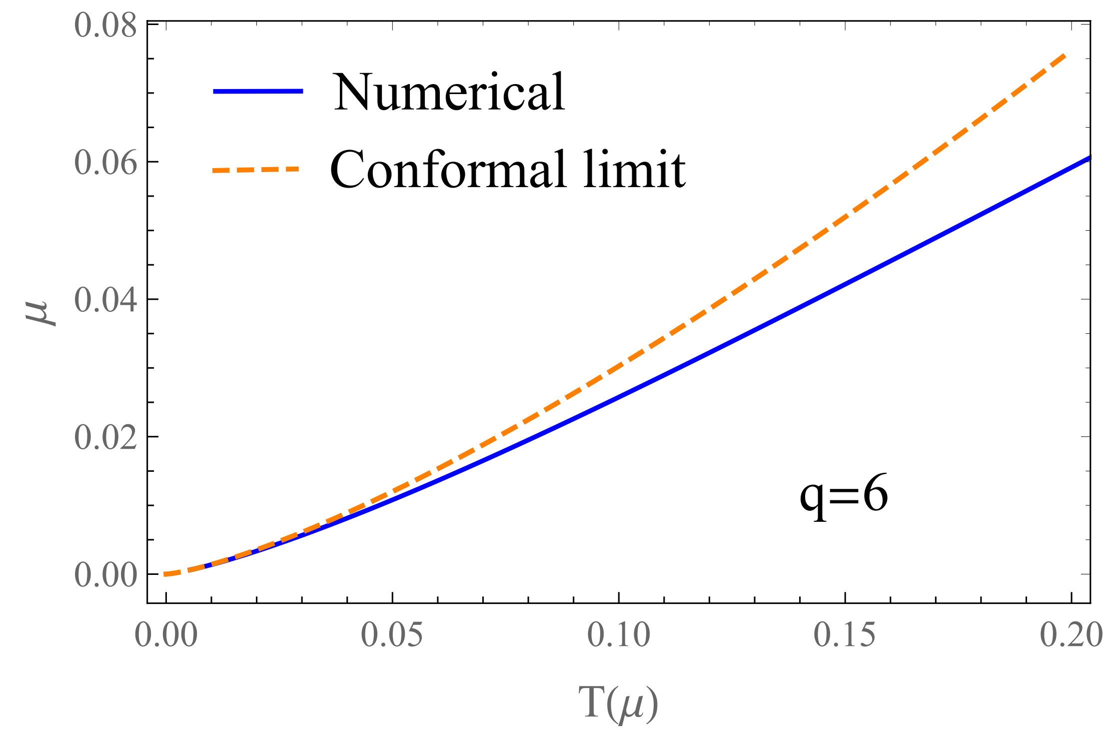
$$\frac{\langle B_{\boldsymbol{s}}(\beta)|H_{\mathrm{def}}|B_{\boldsymbol{s}}(\beta)\rangle}{\langle B_{\boldsymbol{s}}(\beta)|B_{\boldsymbol{s}}(\beta)\rangle} = \underbrace{E_{SYK}}_{\text{SYK thermal energy}} - \mu \underbrace{G_\beta(\beta/2)}_{\text{SYK thermal correlator at } \tau = \frac{\beta}{2}}{}^2$$

$\rightarrow$ vary β , minimize the trial energy $\rightarrow$determine $\beta = \beta(\mu)$

- For large β (small μ), we can use the Schwarzian (quantum gravity)

$$\begin{cases} E_{SYK} = E_0 + \dfrac{2\pi^2 \alpha_S}{\beta^2 \mathcal{J}} \\ G_\beta(\beta/2) = c_\Delta \Big(\dfrac{\pi}{\beta \mathcal{J}}\Big)^{2\Delta} \end{cases}$$

$$\rightarrow \quad \frac{\pi}{\beta(\mu)\mathcal{J}} = \left(\frac{\Delta (c_\Delta)^2 \mu}{\mathcal{J}\alpha_S}\right)^{\frac{1}{2(1-2\Delta)}}$$



Compare observables

(1)Ground state energy $E_0(\mu)$

and $\langle B_{\boldsymbol{s}}(\beta)|H_{\mathrm{def}}|B_{\boldsymbol{s}}(\beta)\rangle$

(2)Spin operator expectation value

$$\langle G_{\boldsymbol{s}}(\mu)|S_k|G_{\boldsymbol{s}}(\mu)\rangle = -2is_k G_{\mathrm{off}}(0)$$

and $\langle B_{\boldsymbol{s}}(\beta)|S_k|B_{\boldsymbol{s}}(\beta)\rangle = 4G_\beta(\beta/2)^2$

(3)Energy in the SYK Hamiltonian

$$\langle G_{\boldsymbol{s}}(\mu)|H_{SYK}|G_{\boldsymbol{s}}(\mu)\rangle$$

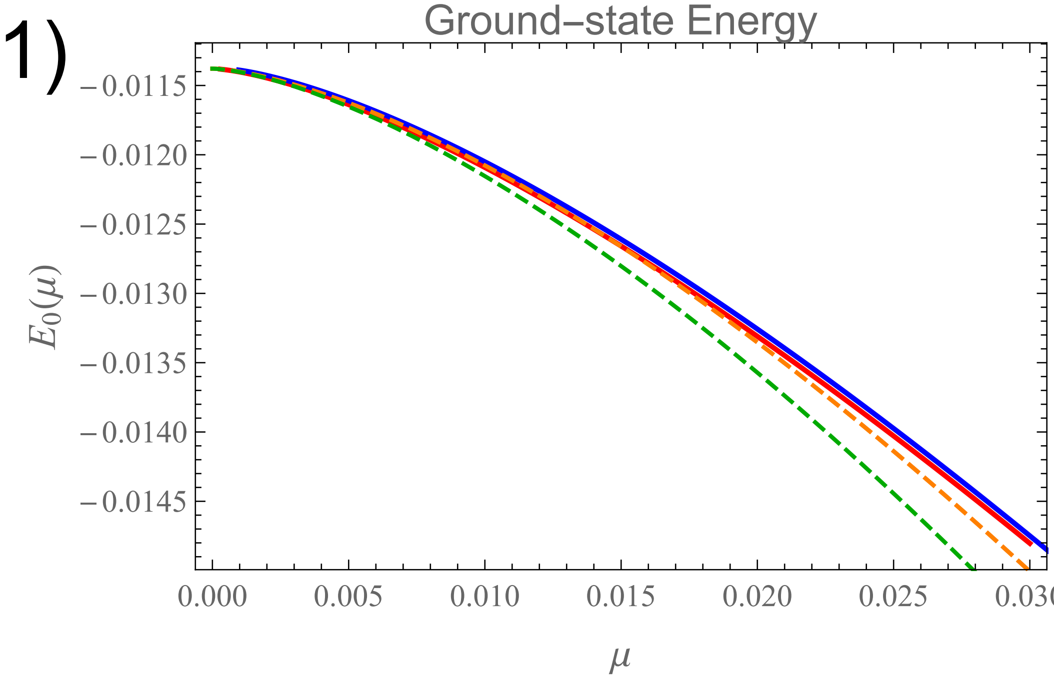
and $\langle B_{\boldsymbol{s}}(\beta)|H_{SYK}|B_{\boldsymbol{s}}(\beta)\rangle$

・same scaling but different coeff. in conformal limit

・at large q, they perfectly agree

・$|G_{\boldsymbol{s}}(\mu)\rangle$ is excited state in SYK

(1)



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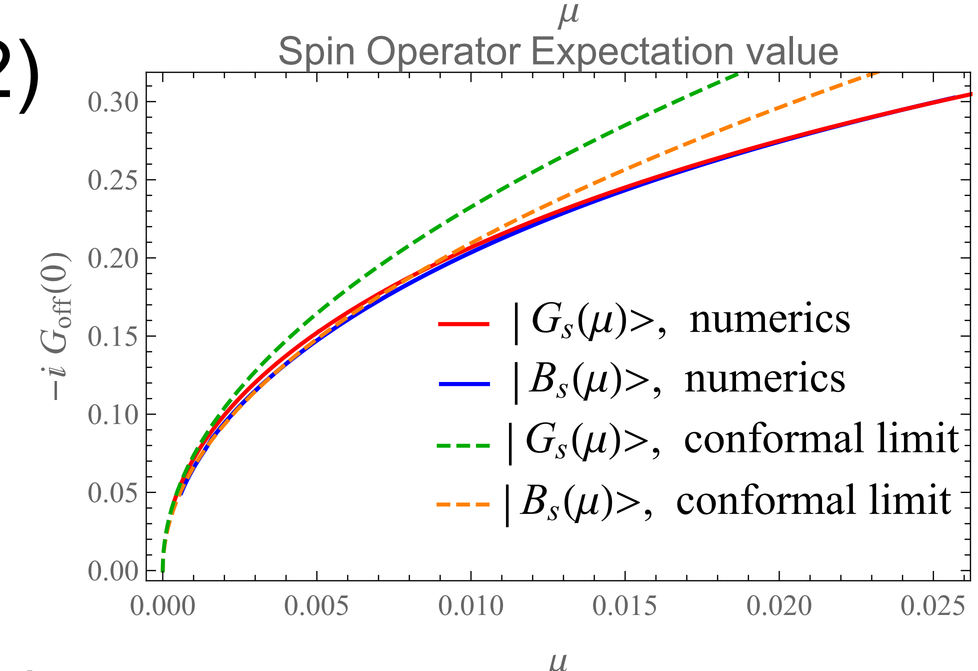

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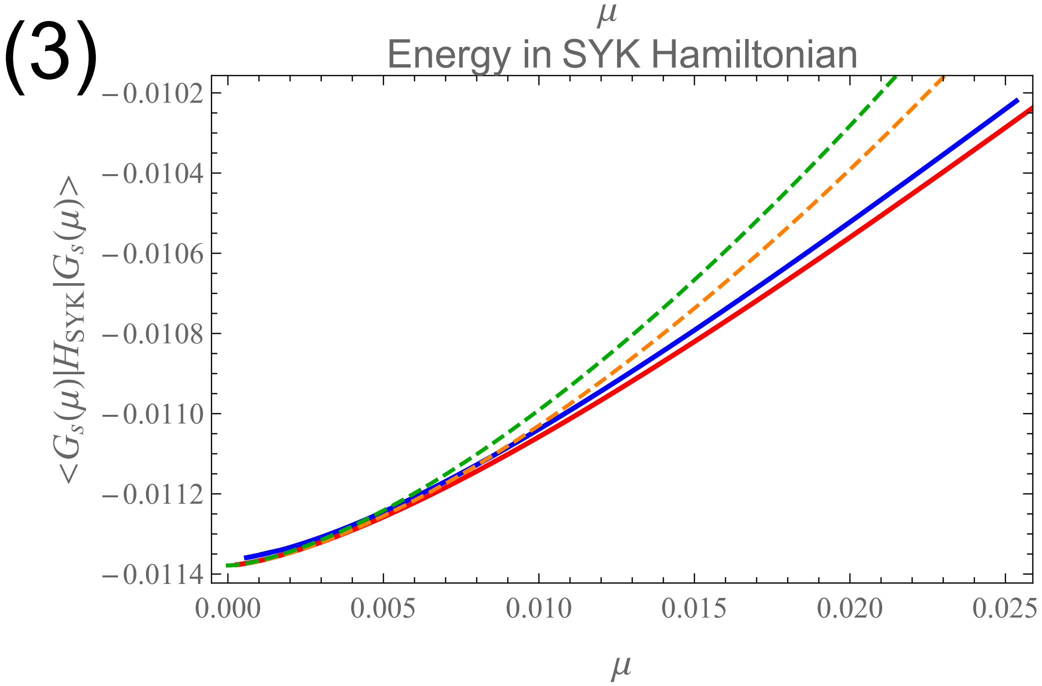
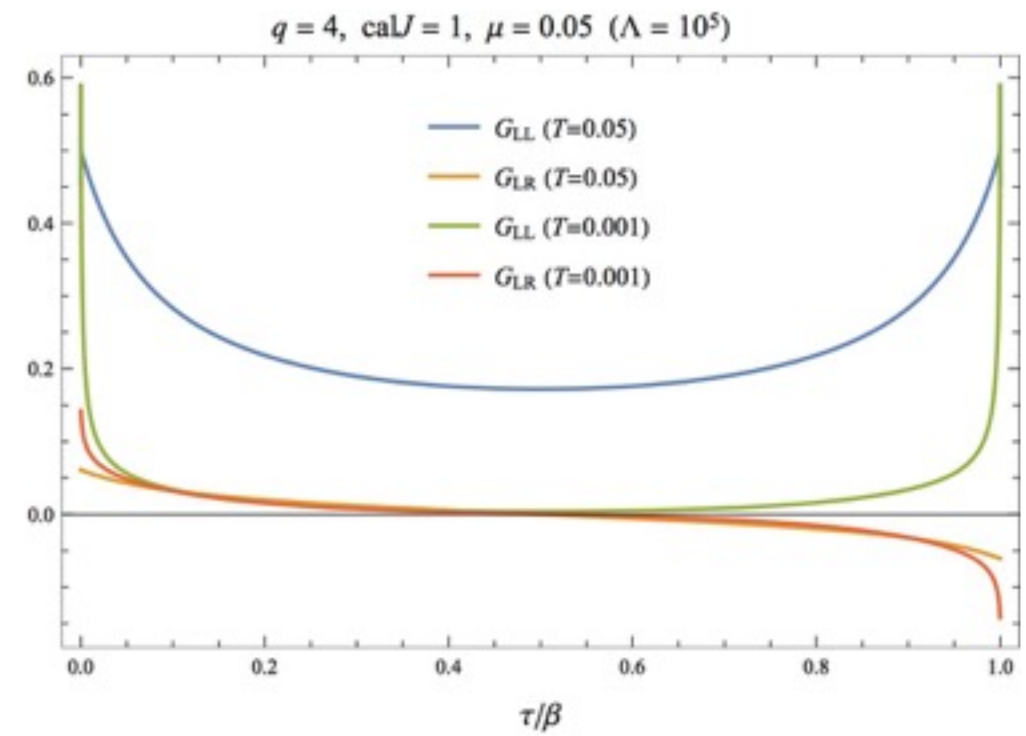


Correlation function: numerical solution

$$G(\tau_1, \tau_2) = \langle \psi_i(\tau_1)\psi_i(\tau_2)\rangle$$
$$G_{\text{off}}(\tau_1, \tau_2) = \langle \psi_{2k-1}(\tau_1)\psi_{2k}(\tau_2)\rangle$$

Imaginary Time

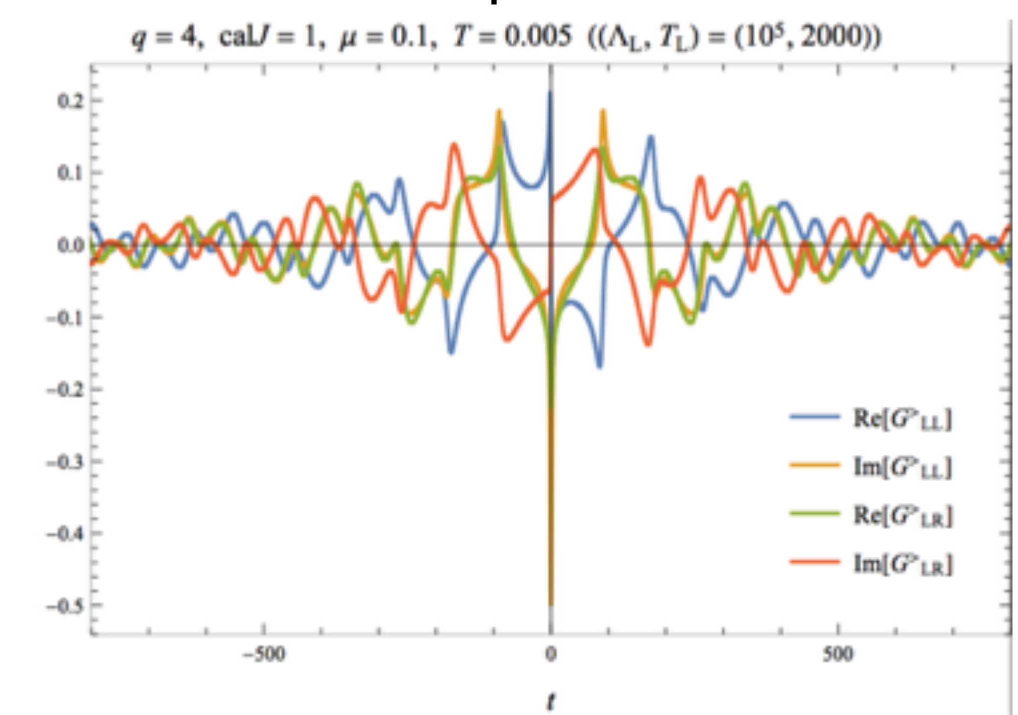


exponential decay (gap) at low temperature

Similar to SYK at hight temperature

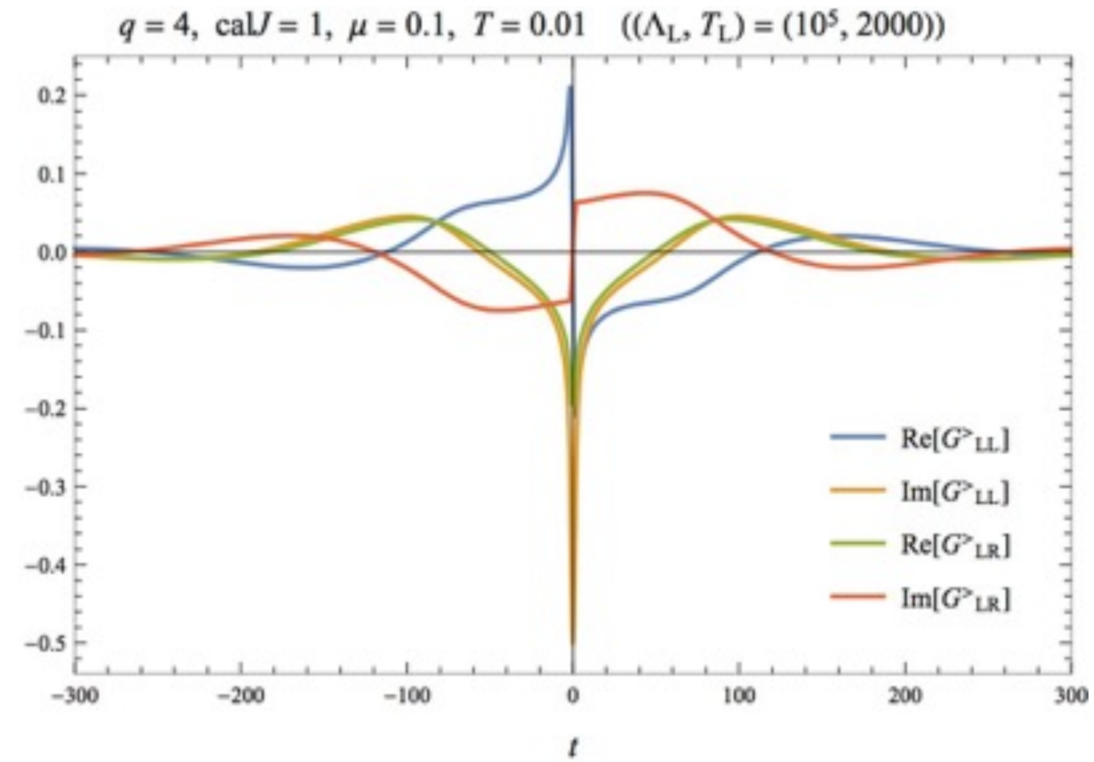
Real Time

Low temperature



oscillating behavior

High temperature



exponential decay

Green's function in conformal limit at 0 temperature

- Imaginary time correlator in ground state

$$G_c(\tau) = c_\Delta \left(\frac{\alpha(\mu)}{\mathcal{J} \sinh(\alpha(\mu)\tau)} \right)^{2\Delta}, \; \alpha(\mu) = \frac{\mathcal{J}}{2} \left[\frac{\Gamma(2-2\Delta)\Gamma(\Delta)^2}{\Gamma(2\Delta+1)\Gamma(1-\Delta)^2} \frac{1}{(2c_\Delta)^{(q-2)}} \right]^{\frac{1}{2(1-2\Delta)}} \underline{\left(\frac{\mu}{\mathcal{J}}\right)^{\frac{1}{1-2\Delta}}}$$

$$G_{\mathrm{off},c}(\tau) = 2i\alpha(\mu)\mu^{-1} \frac{\Gamma(1-\Delta)^2}{\Gamma(\Delta)^2} e^{-2\alpha(\mu)\Delta|\tau|} {}_2F_1(2\Delta, 2\Delta; 1; e^{-2\alpha(\mu)|\tau|})$$

- analytic expression for Euclidean correlators → continue to real time

$$G^>(u) = G_E(iu+\epsilon), \quad G^<(u) = G_E(iu-\epsilon)$$

- Real time correlator in ground state

$$G^>(u_1, u_2) = \langle \psi_i(u_1)\psi_i(u_2)\rangle = e^{-i\pi\Delta} c_\Delta \left(\frac{\alpha}{\mathcal{J}\sin(\alpha(u_1-u_2)-i\epsilon))} \right)^{2\Delta}$$

which oscillates, the period is set by the mass gap

singularity at $u_2 - u_1 = 0, \pm\frac{2\pi}{\alpha}, \pm\frac{4\pi}{\alpha}$ $\cdots$ is regulated in the full (UV complete) answer.

Dynamics without deformation: Quantum Quench

- Real time evolution under SYK Hamiltonian

Consider time dependent Hamiltonian $H_{\mathrm{def}}(u) = H_{SYK} + \mu\theta(-u)H_M$

$\rightarrow$ Solve the Schwarzian mechanics, $L = -\dfrac{N\alpha_S}{\mathcal{J}} \int \{f(u), u\}$

w/ $\begin{cases} \text{Energy} \quad \langle G_{\boldsymbol{s}}(\mu)|H_{SYK}|G_{\boldsymbol{s}}(\mu)\rangle \\ \text{Initial condition is set from} \quad f(u) = \tan(\alpha(\mu)u) \quad \text{for } u < 0 \end{cases}$

The solution is

$$f(u) = \frac{2\alpha_S}{\varepsilon(\Delta)}\frac{\pi}{\beta\mathcal{J}}\tanh\left(\frac{\pi}{\beta}u\right) \qquad \frac{2\pi^2\alpha_S}{\beta^2\mathcal{J}} = \alpha(\mu)\frac{\Gamma(2\Delta)\Gamma(1-\Delta)^2\Gamma(1-4\Delta)}{\Gamma(\Delta)^3\Gamma(1-2\Delta)}$$

- diagonal correlator: $G^{>}(u_1, u_2) = e^{-i\pi\Delta}\left(\dfrac{\pi}{\beta\mathcal{J}\sinh\frac{\pi}{\beta}(u_1 - u_2)}\right)^{2\Delta}$

exactly the same with thermal one!

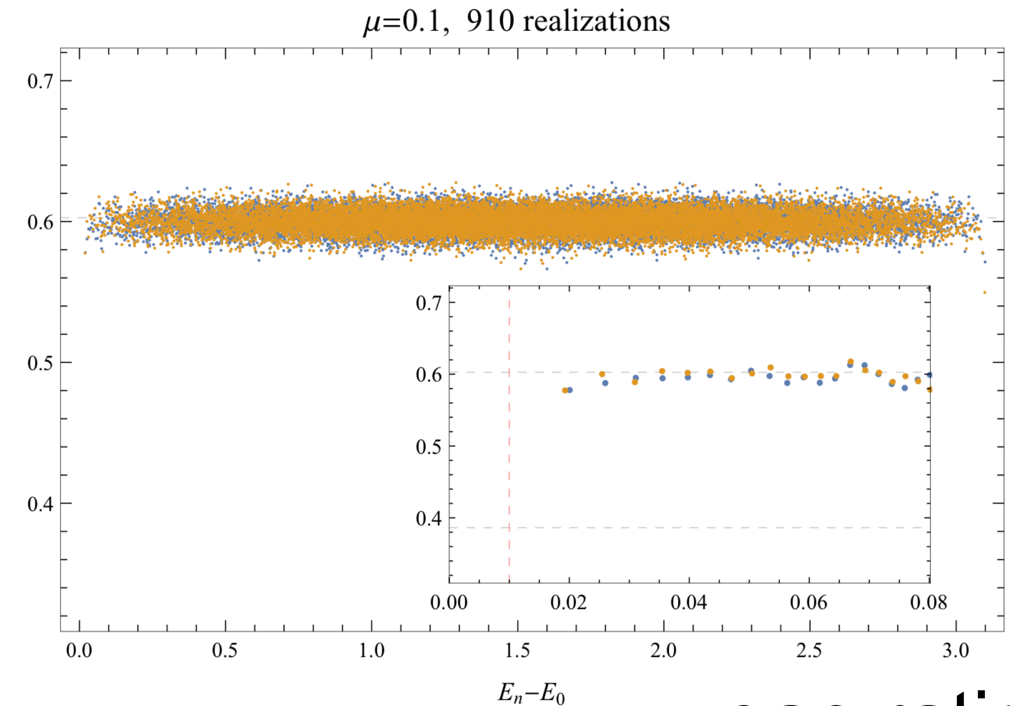
- Spin exp val: $\langle S_k(u)\rangle = \langle S_k(0)\rangle\left(\dfrac{1}{1 + (\frac{2\alpha_S}{\varepsilon(\Delta)}\frac{\pi}{\beta\mathcal{J}})^2\tanh^2(\frac{\pi}{\beta}u)}\right)\left(\dfrac{1}{\cosh\frac{\pi}{\beta}u}\right)^{4\Delta}$

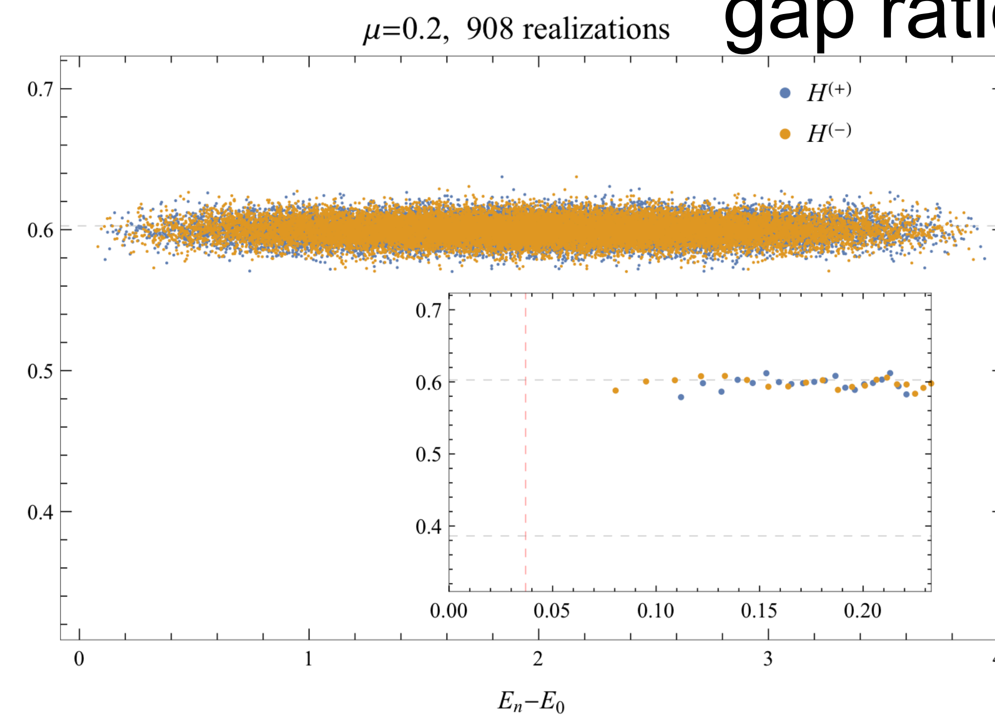
Excited state properties of mass deformed theory

To see the quantum chaotic property

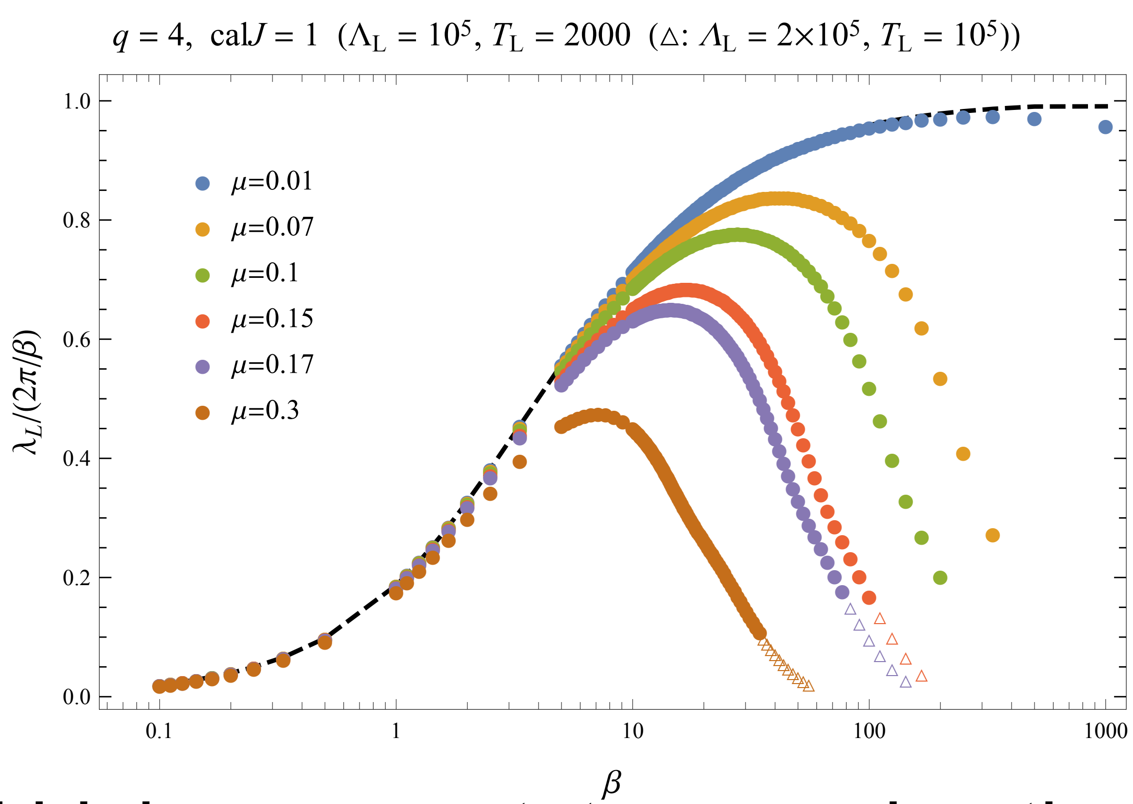
- the level statistics
  (random matrix property)
  by exact diagonalization for $N = 30$
  (match with GUE)

- Lyapunov exponents at large N





gap ratio

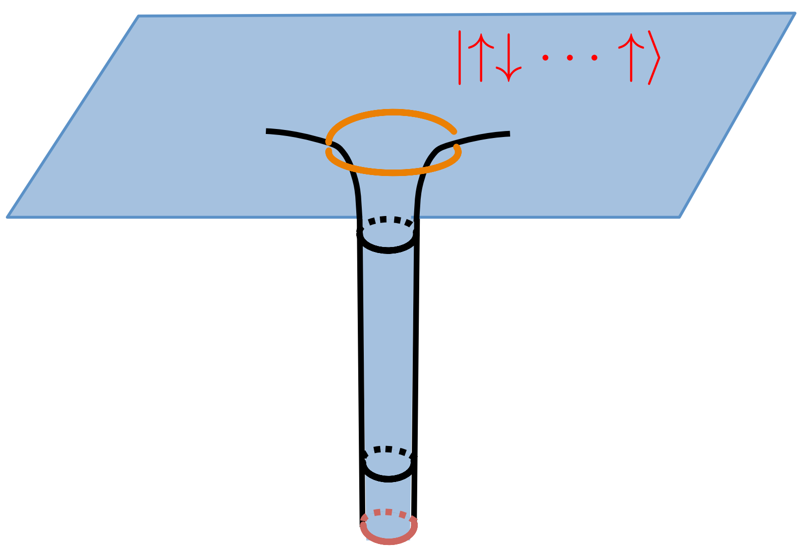


and high energy states are chaotic = same with SYK

Gravity interpretation

Low excitation = Cap off

Couple quantum field
To quantum computer



$|\uparrow\downarrow\cdots\uparrow\rangle$

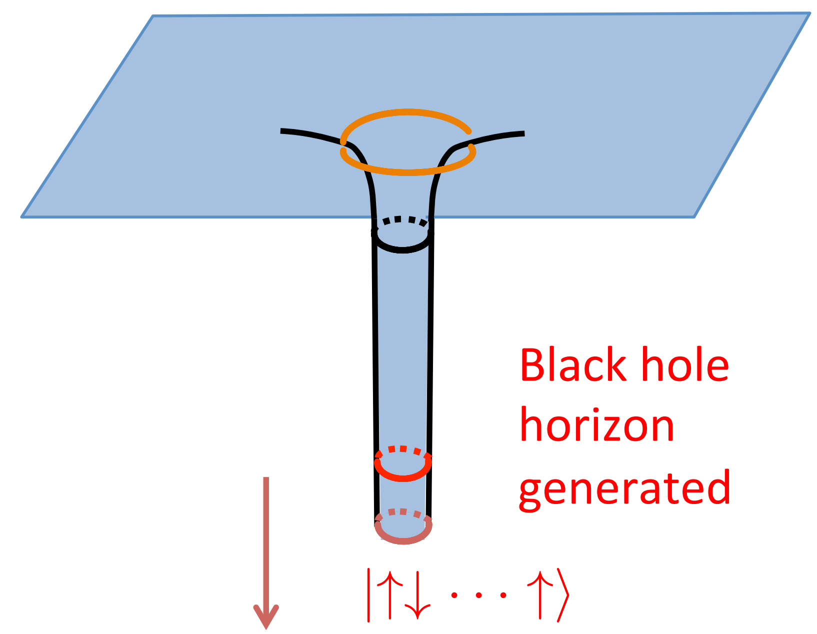
Brane with
Quantum information

like holographic
superconductor/insulator

[Hartnoll-Herzog-Horowitz 08]
[Nishioka-Takayanagi-Ryu 09]

Quantum Quench
Large excitation = BH

Lose control over
Quantum computer side



Black hole
horizon
generated

$|\uparrow\downarrow\cdots\uparrow\rangle$

Brane with information
falling

like holographic
Non Fermi Liquid

[Herzog-Kovtun-Sachdev-Son 07]
[Faulkner-Liu-McGreevy-Vegh 09]

Success/Failure of teleportation $\leftrightarrow$ phase of matter + dynamics

Falling brane represent information scrambling

Importance of state dependent deformation

To escape interiors, we should know

$$\begin{cases} \text{correct pair } \psi_{2k-1} \text{ and } \psi_{2k} \\ \text{correct sign } s_k \end{cases}$$

and correlate $|G_{\underline{\boldsymbol{s}}}(\mu)\rangle$ and $H_{\rm def} = H_{SYK} + i\mu \sum_{k=1}^{N/2} \underline{s_k \psi_{2k-1}\psi_{2k}}$

Sign flip → excitation of $\Delta E = \mu |\langle S_k \rangle|$

many excitations → evolved by chaotic part of Hamiltonian $H_{\rm def}$

→ black hole formation = failure of teleportation

Teleportation at late time

- Can we escape interiors of scrambled state $e^{-iH_{SYK}T}|G_{\boldsymbol{s}}(\mu)\rangle$?

  $\rightarrow$ Simple deformation $H_{\rm def}$ does not work.

- deformation including "time reversal" $H'_{\rm def} = H_{SYK} + \mu \sum s_k e^{-iH_{SYK}T} S_k e^{iH_{SYK}T}$

  can escape interiors, but $e^{-iH_{SYK}T} S_k e^{iH_{SYK}T}$ is a complex operator.

- "Size" of operator grows [Robart-Stanford-Streicher 18] [Qi-Streicher 19]

  Each expansion coefficients $e^{-iH_{SYK}T} S_k e^{iH_{SYK}T} = \sum_{i=1}^{N} \sum a_{i_1,\cdots,i_l}(T)\psi_{i_1}\cdots\psi_{i_l}$

  is very small, highly depends on each realization $J_{i_1\cdots i_q}$, similar to SFF

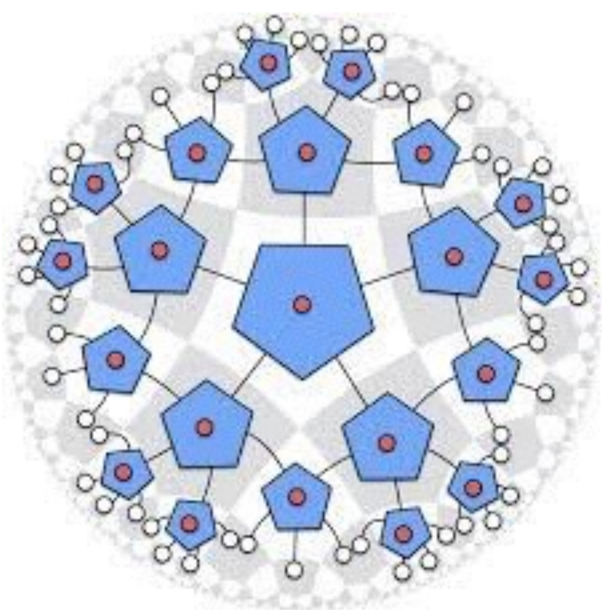
  [CGHPSSSST 16] [TN 19]

Compare with Toric code

	Toric code	Deformed SYK
code	degenerate ground states	Low excitation (global AdS) [cf: Almheiri-Dong-Harlow- 14]
error	Anyon excitation (deconfined) [cf: Fradkin-Shenker 79]	Black Hole formation
operator	Wilson loops	Bulk operator

SYK side is close to holographic (HAPPY) code

[Harlow-Pastawski-Preskill-Yoshida, 15]



①Introduction

- **connections between Quantum Gravity, cond-mat and QI**
- **Review of the SYK model**

②Model for SYK teleportation

③SYK Lindbladian

direction: QI + QG → QI + cond-mat

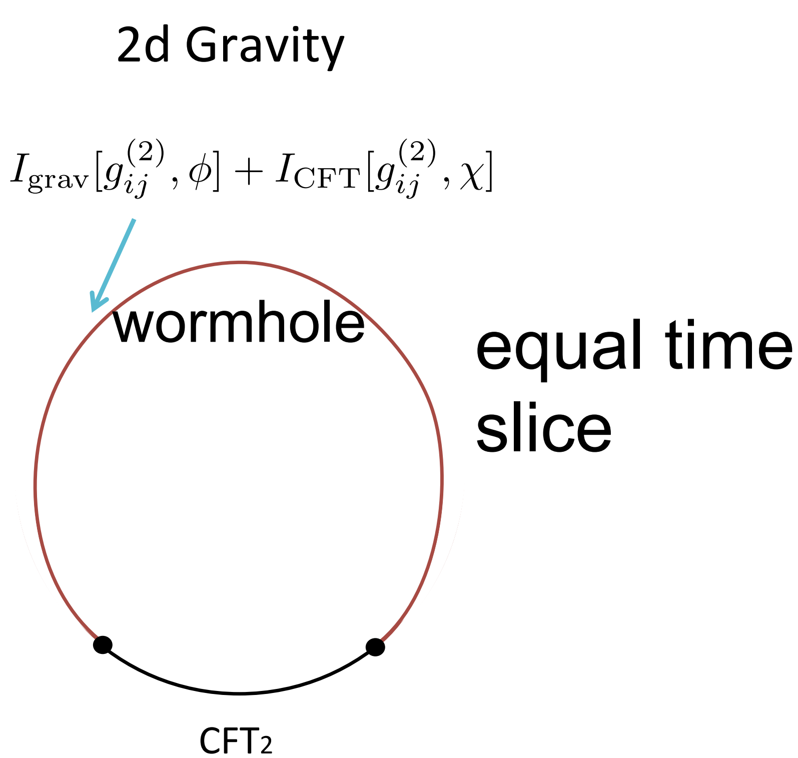
based on

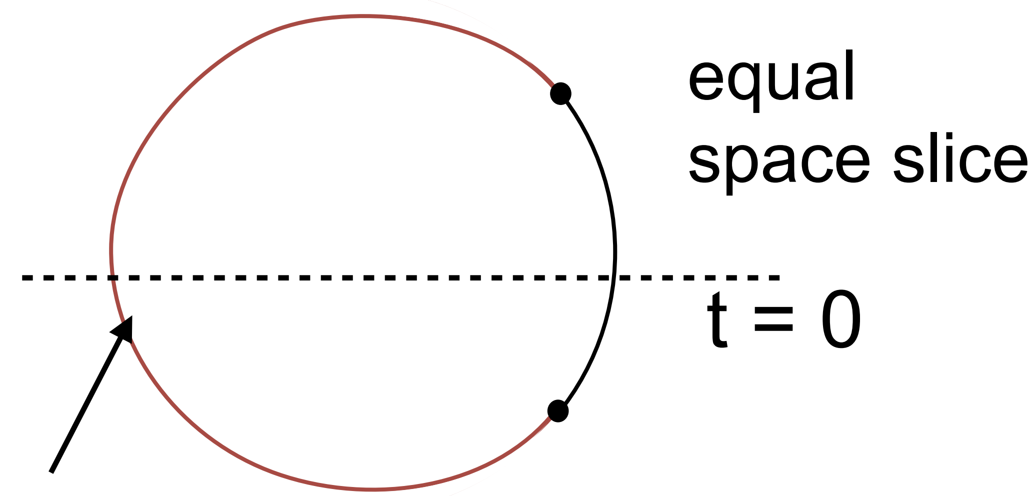
A.Kulkarni (Princeton U)- TN - S.Ryu (Princeton U), arXiv 2021.13489

3.Lindbrad SYK

Motivation from Quantum Gravity

- There is an argument that we should also allow bra-ket wormholes when we consider the density matrix of the universe [Page, 86] [cf: Anous-Kruthoff-Mahajan, 20]
- wick rotation of traversable wormholes gives such examples in gravity counterpart (dialton gravity + 2d CFT)





wormhole connects bra and ket
[Chen-Gorbenko-Maldacena, 20]

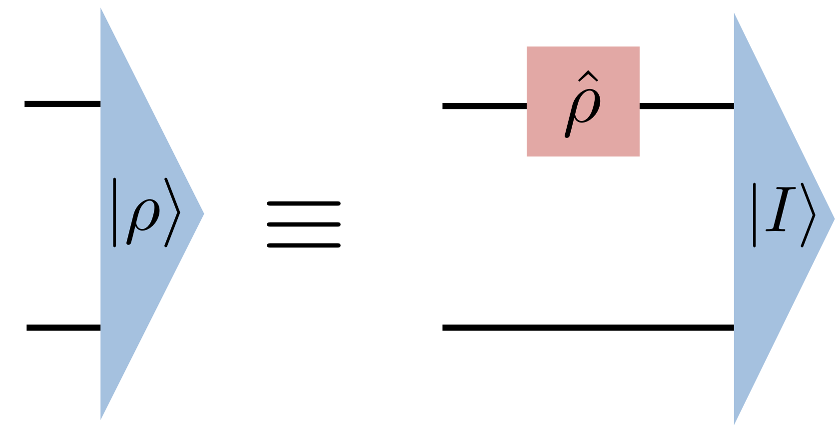
Question: condensed matter application? SYK counterpart?

Let us consider bra-ket coupling in the SYK: Lindbradian SYK models

Schwinger-Keldysh formalism for Lindbladian:

First, we use the state-operator map

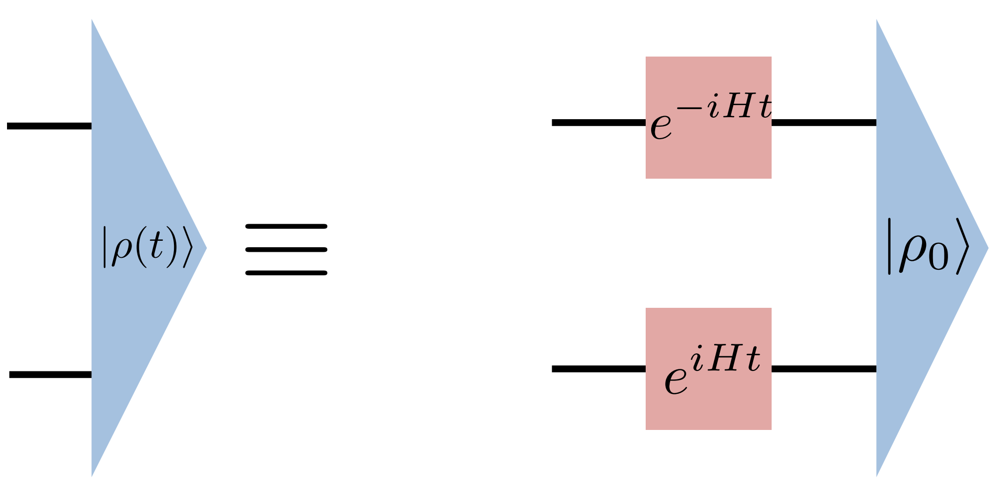
(Choi–Jamiołkowski isomorphism)



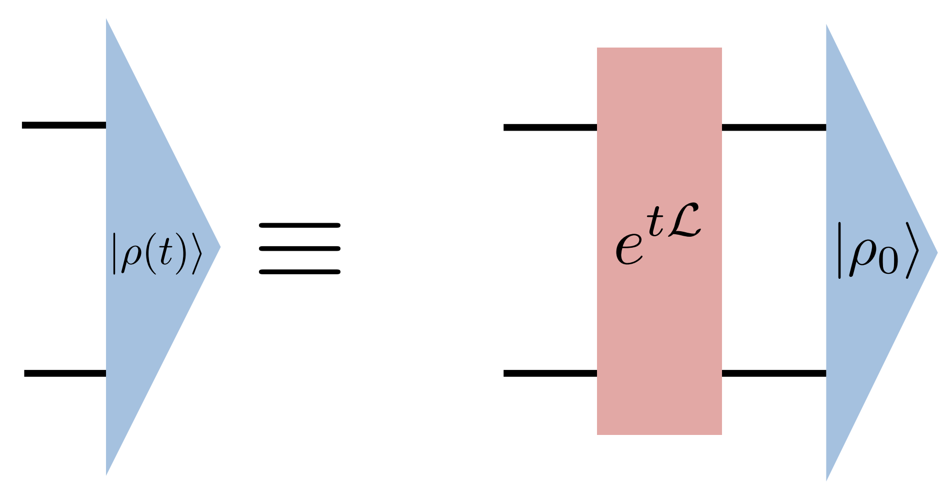
Lindbrad equation:

$$\frac{d}{dt}\rho(t) = -i[H_{SYK}, \rho(t)] + \sum_k (L_k \rho(t) L_k^\dagger) - \frac{1}{2}\sum_k (L_k^\dagger L_k)\rho(t) - \frac{1}{2}\rho(t)\sum_k (L_k^\dagger L_k)$$
$$\equiv \mathcal{L}(\rho(t))$$

Hamiltonian evolution



Lindbradian evolution



Models for SYK Lindbladian: [A.Kulkarni, TN S.Ryu, 21]

(1)An example of Non Random Jump operators: $L_i = \sqrt{\mu}\psi^i$

After channel-state mapping, we get operator similar [Maldacena-Qi 18] model :

$$\mathcal{L} = -iH^{+}_{SYK} + i(-1)^{\frac{q}{2}} H^{-}_{SYK} - i\mu \sum_i \psi^i_+ \psi^i_- - \mu \frac{N}{2} \mathbb{I}_+ \otimes \mathbb{I}_-$$

(2)An example of Random Jump operators: $L^a = \sum_{i<j} K^a_{ij} \psi^i \psi^j$

$$\langle K^a_{ij} \rangle = 0 \qquad \langle |K^a_{ij}|^2 \rangle = \frac{K^2}{N^2} \qquad a = 1 \sim M$$

$$\mathcal{L} = -iH^{+}_{SYK} + iH^{-}_{SYK} + \sum_a \sum_{i<j} \sum_{k<l} K^a_{ij} \bar{K}^a_{ij} \left(\psi^i_+ \psi^j_+ \psi^k_- \psi^l_- + \frac{1}{2} \psi^k_+ \psi^l_+ \psi^i_+ \psi^j_+ + \frac{1}{2} \psi^k_- \psi^l_- \psi^i_- \psi^j_- \right)$$

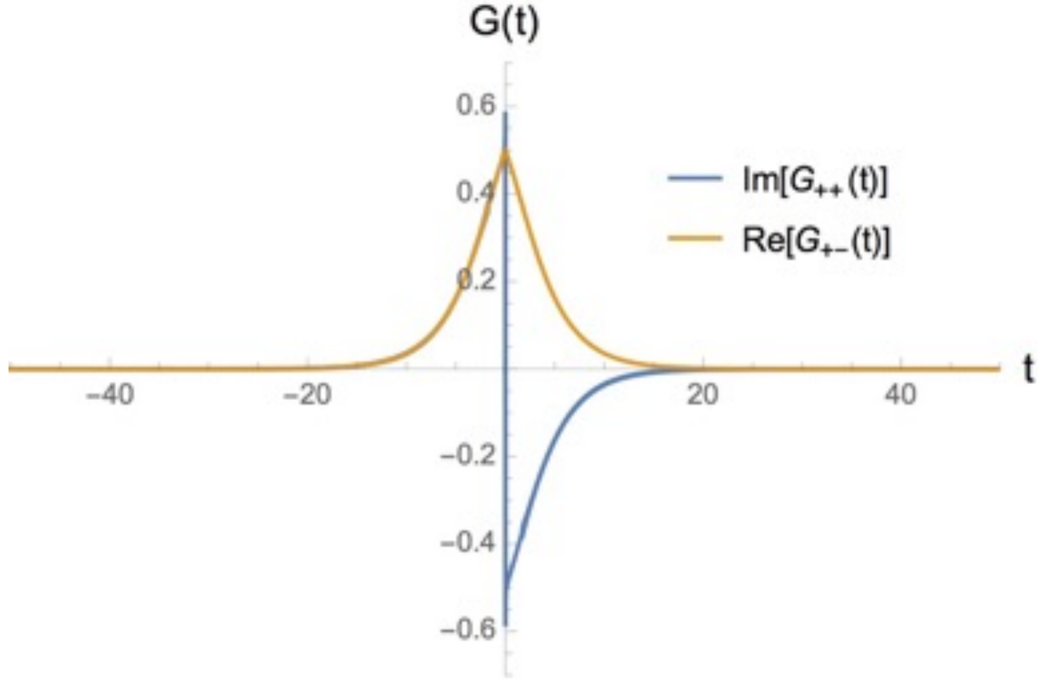
Similar to coupled SYK models in [Kim-Klebanov-Tarnopolsky-Zhao,19]

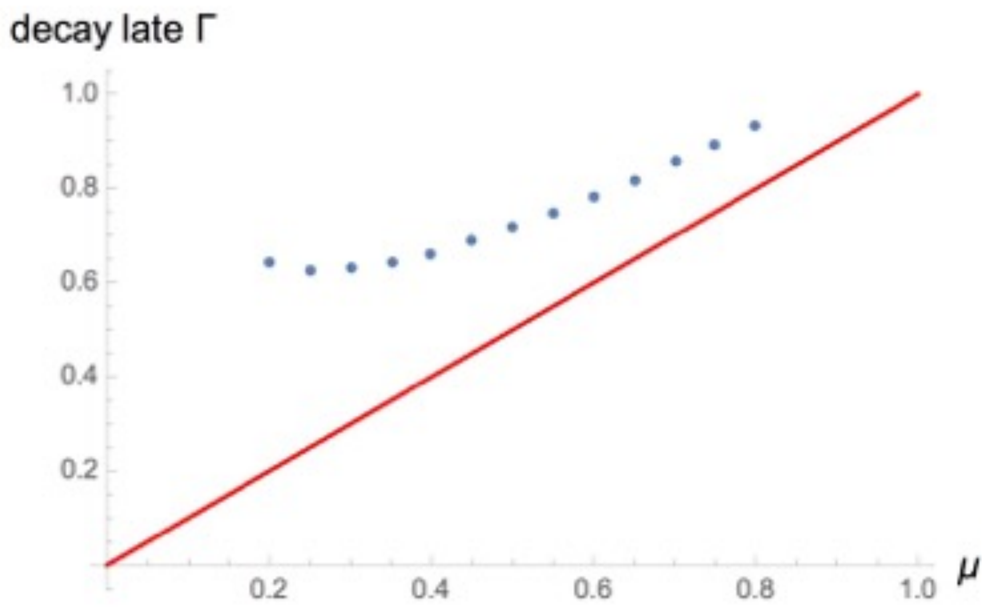
Also similar to Wishart SYK, [Iyoda-Katsura-Sagawa,18] SUSY SYK, [Fu-Gaiotto-Maldacena-Sachdev,16]

- Both models are solvable, strongly coupled Lindbladian at large N

Stationary state Green's function:

At large N limit, the Schwinger-Dyson equation is tractable





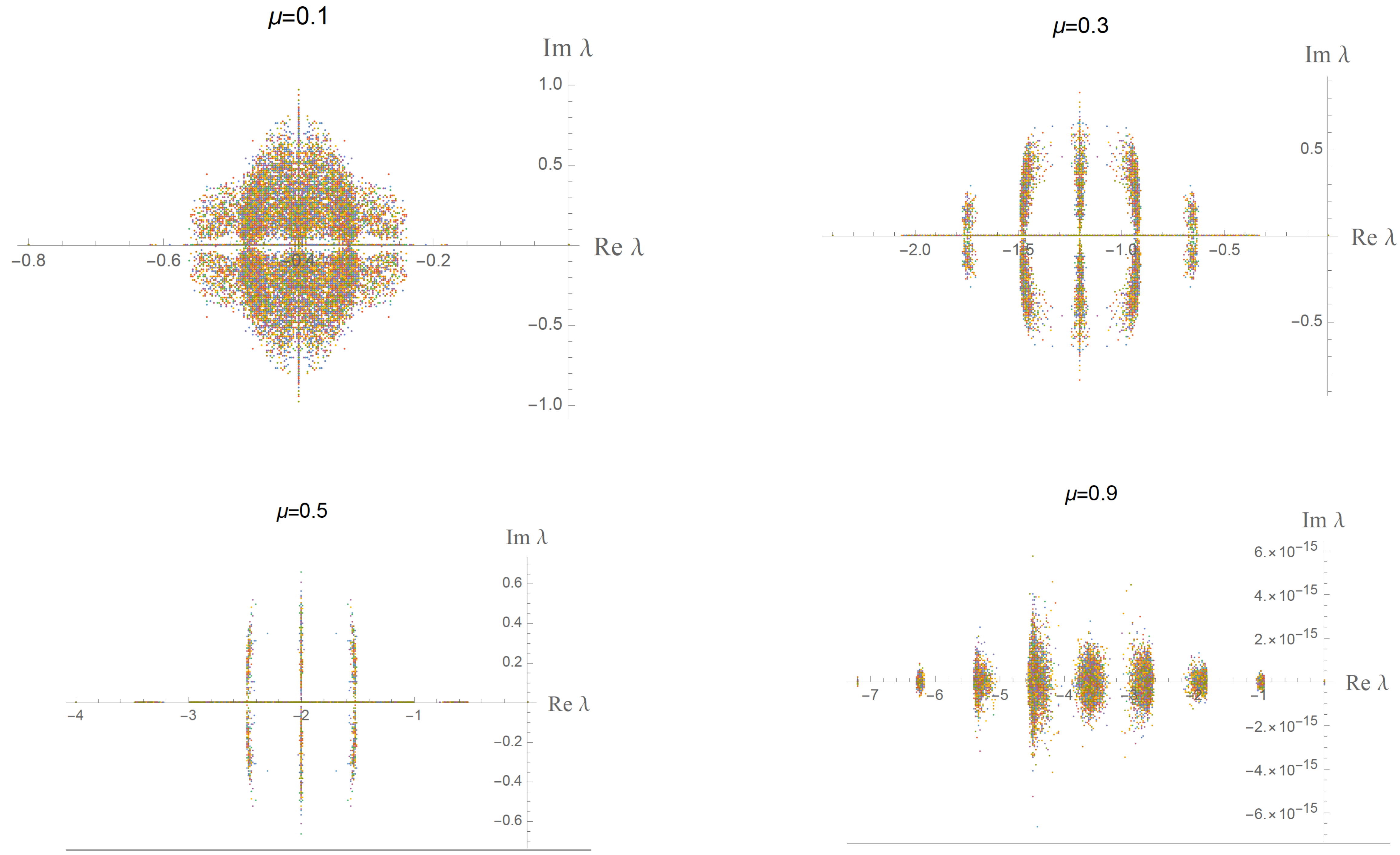
At late time, Green's functions show exponential decay

At large q limit with $q\mu = \hat{\mu}$ fixed, the system analytically solvable

$$G_{++}(t) = -\frac{i}{2}\left(1 + \frac{1}{q} g_{++}(t)\right) \qquad e^{g_{++}(t)} = \frac{1}{\mathcal{J}^2 \cosh^2(\alpha|t| + \gamma)}$$

$$\alpha = \mathcal{J}\sqrt{\left(\frac{\hat{\mu}}{2\mathcal{J}}\right)^2 + 1} \qquad \gamma = \operatorname{arcsinh}\left(\frac{\hat{\mu}}{2\mathcal{J}}\right) \qquad \Gamma = \frac{2\alpha}{q}$$

Lindbladian spectrum: Non Random

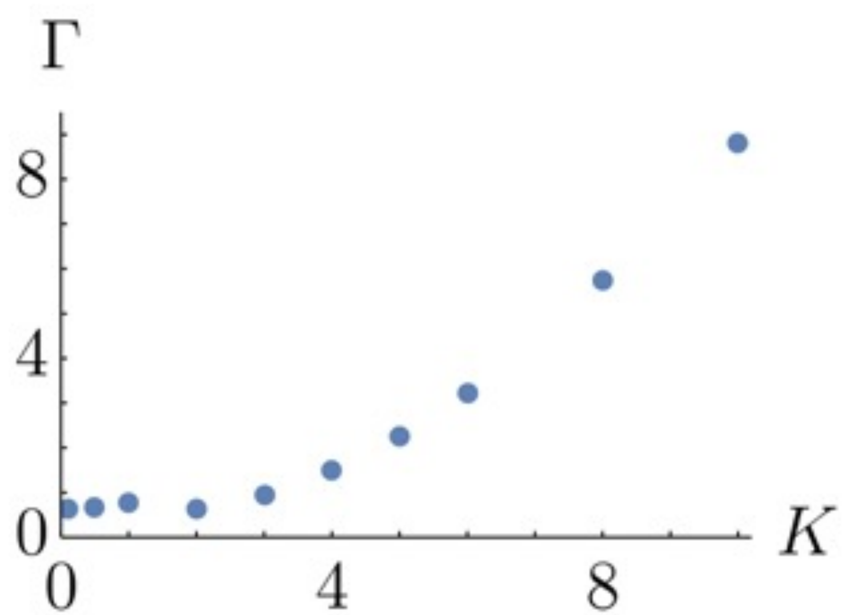


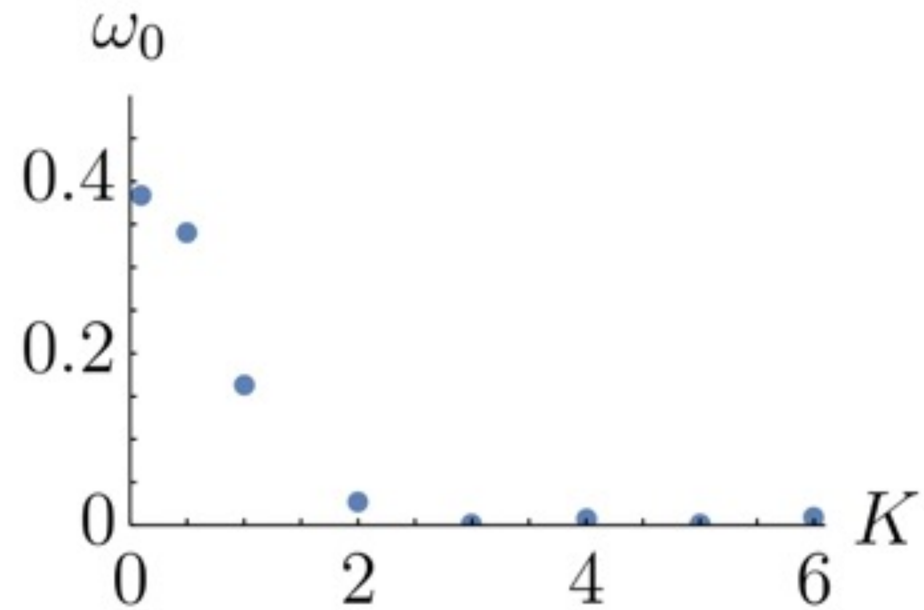
Real-Complex transition? [cf: Hamazaki-Kawabata-Ueda, 18]

Stationary state Green's function: Random

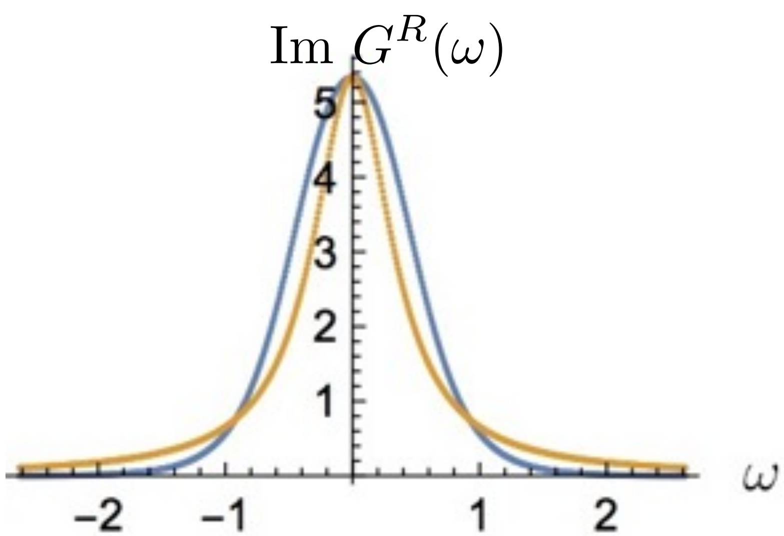
Solvable at large N and M, fixed M/N

$$G^R(t) \sim A e^{-\Gamma t} \sin(\omega_0 t + \phi)$$

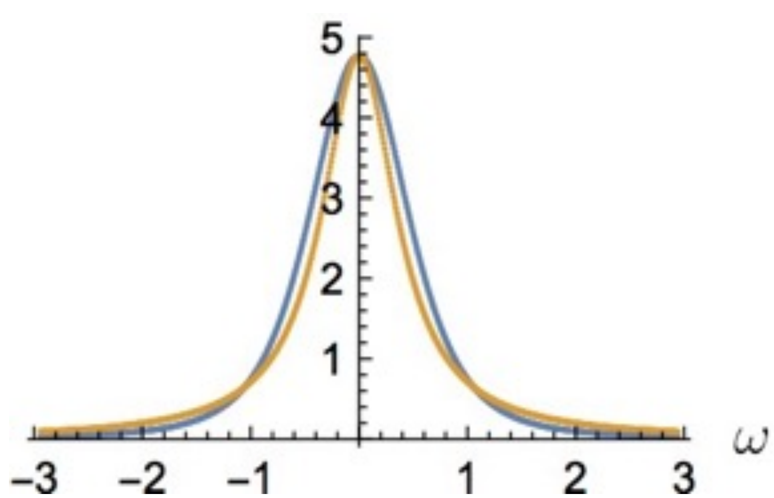




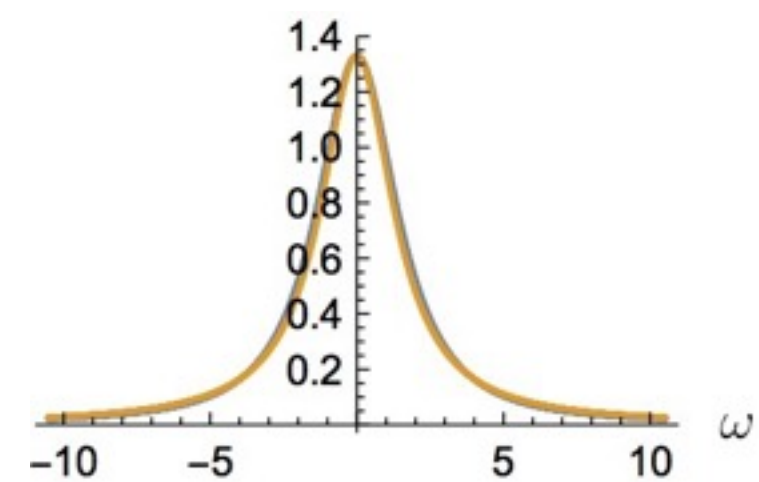
K = 0.1



K = 1

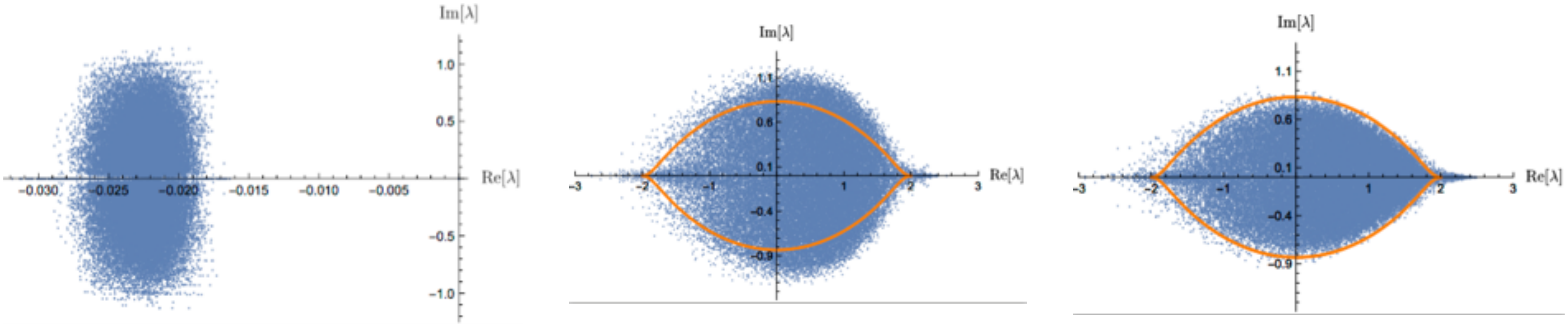


K = 10



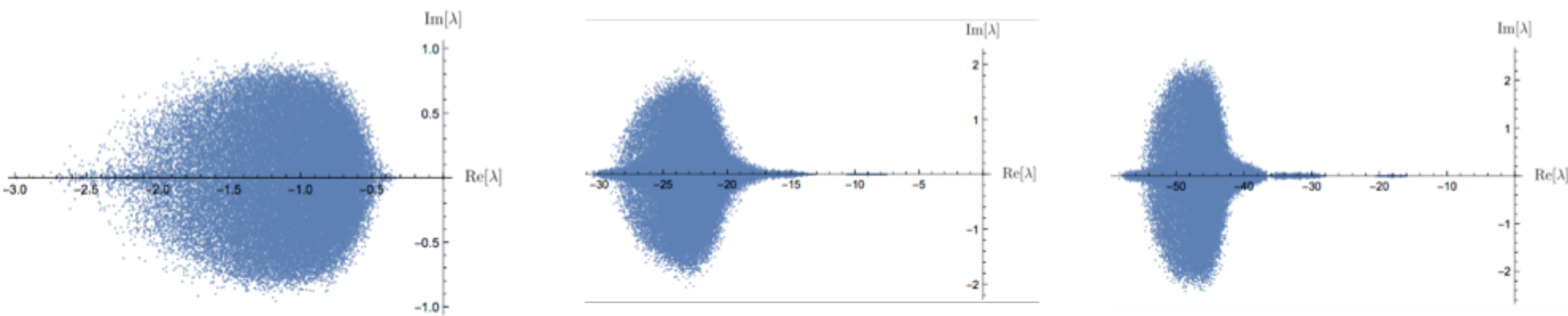
Lindbladian spectrum: Random

- Fixed M (number of jumps), increase K (strength of single jump)



Lemon shape at large K

- Fixed K, increase M



Formation of “island” at large M

Some comments:

- SYK Lindbradian ↔ couples SYK model
- stationary SYK Lindbladian ↔ Ground state of coupled SYK = wormhole
- Solvable at large N, we can read off the "gap" = decay rate
- At large q analytically solvable, a part of spectrum are also available
- Dissipation drives system to infinite temperature,

  In particular, we can not use Schwarzian theory (quantum gravity modes)

  (similar problem in de Sitter continuation of SYK/BH [Maldacena-Turiaci-Yang, 19])

Conclusion

・Brief review of interplay of cond-mat, quantum gravity and quantum info

・Review of Entanglement and wormholes

・Introduction to SYK model and its quantum gravity sector (Schwarzian)

・Analysis of mass deformed SYK model,used in the context of teleportation

variational approximation by projected states (“continuous tensor networks”)

correlation functions and dynamics

success/failure of teleportation and phases of matters

・Analytic continuation coupled SYK motivated by wormhole counterpart

Braket coupling in Schwinger Keldysh = Lindbladian dynamics

Non Random Jumps = analytic continuation of coupled SYK

Models are solvable at large N

Numerical study of the Lindbladian spectrum

Future problems

- Real time dynamic in deformed SYK Hamiltonian

- Further study of SYK Lindbladian and their Non-equilibrium phases

  [work in progress]

- More on solvable Lindbladian (random t-J, complex SYK etc…)

- (Quantum) Spin glass phase and black holes

  [cf:Anous-Haehl, 21]

- Non unitarity evolution in CFT, string theory etc…

  [cf: Dijkgraaf-Heidnreich-Jefferson-Vafa, 16]

- More application of Cond-mat → quantum gravity

  [cf:talk in Strings by Xiao-Gang Wen 21]

- More application of quantum gravity →quantum information

  [cf:Hayden-Penington 20]