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Probing axions and gravitons with magnon detectors

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Plan of my talk

- Motivation for graviton search
- Duality of axion and graviton
- Axion search with magnons
- Graviton search with magnons
- Summary

Motivation
for graviton search

Graviton (HFGWs) exists!

Can we detect a graviton?

The answer is negative.   Dyson 2013

The reason is simple.
GWs detected at LIGO have
a frequency 1kHz and an amplitude $h \approx 10^{-21}$.

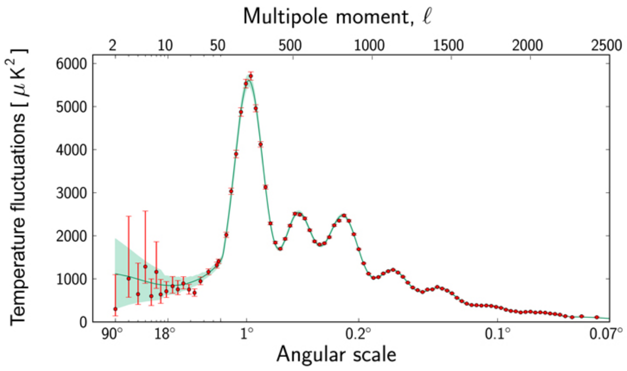
$$\rho \approx M_p^2 \omega^2 h^2 \qquad \rho_{\text{single}} \approx \omega^4 \quad \Rightarrow \quad n_{graviton} \approx \frac{M_p^2 h^2}{\omega^2} \approx 10^{38}$$

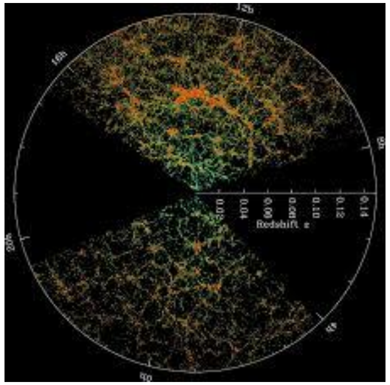
Hence, it is difficult to resolve a single graviton.

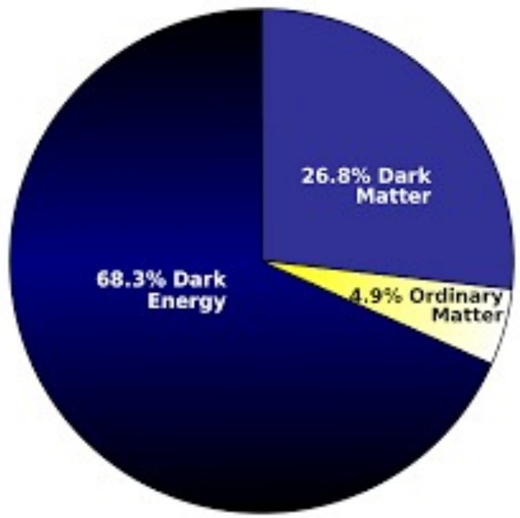
It might be possible to detect a graviton by inventing detectors
for high frequency gravitational waves (HFGWs).
With this expectation, I do not distinguish between HFGWs and gravitons.

Remarkably, as we will see,
the detection of gravitons is useful for the dark matter search.

Dark matter exists!







What is it?

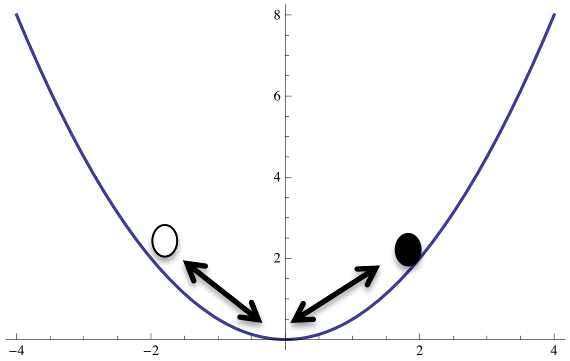
A coherently oscillating axion field in the mass range

$$10^{-20}\mathrm{eV} \leq m \leq 1\mathrm{eV}$$

can be a candidate for the dark matter.

Axion dark matter

$$V(a) = \frac{1}{2} m a^2$$



Axion is coherently oscillating

$$a = a_0 \cos mt$$

Energy density is given by

$$\rho_{DM} = \frac{1}{2}\dot{a}^2 + \frac{1}{2}m^2 a^2 \approx \frac{1}{2}m^2 a_0^2$$

Pressure is given by

$$p_{DM} = \frac{1}{2}\dot{a}^2 - \frac{1}{2}m^2 a^2 \approx -\frac{1}{2}m^2 a_0^2 \cos(2mt)$$

On cosmological time scale, the pressure vanishes on average.
Thus, the axion can mimic the cold dark matter on large scales.

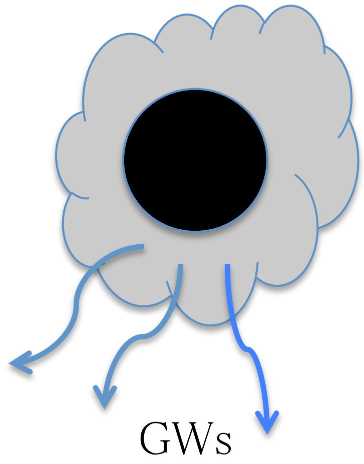
$$\left(\frac{m}{10^{-6}\mathrm{eV}}\right) \approx \left(\frac{f}{10^{9}\,\mathrm{Hz}}\right)$$

$$H_0 \approx 10^{-33}\mathrm{eV}$$

Axion superradiance instability

Super−radiance instability of rotating BHs occurs
when the Compton wavelength of an axion
and the gravitational radius of a BH becomes comparable.

$$\left(\frac{M}{M_\odot}\right) \approx \left(\frac{R_S}{3\text{km}}\right) \approx \left(\frac{m}{10^{-10}\text{eV}}\right)^{-1}$$



The instability induces the rapid growth of axion cloud
which significantly reduces the spin of the BH and produces GWs.

Thus, the finding of a rapidly rotating BHs implies
that there is no axion with the mass whose Compton wavelength
corresponds to the gravitational radius of the BH.

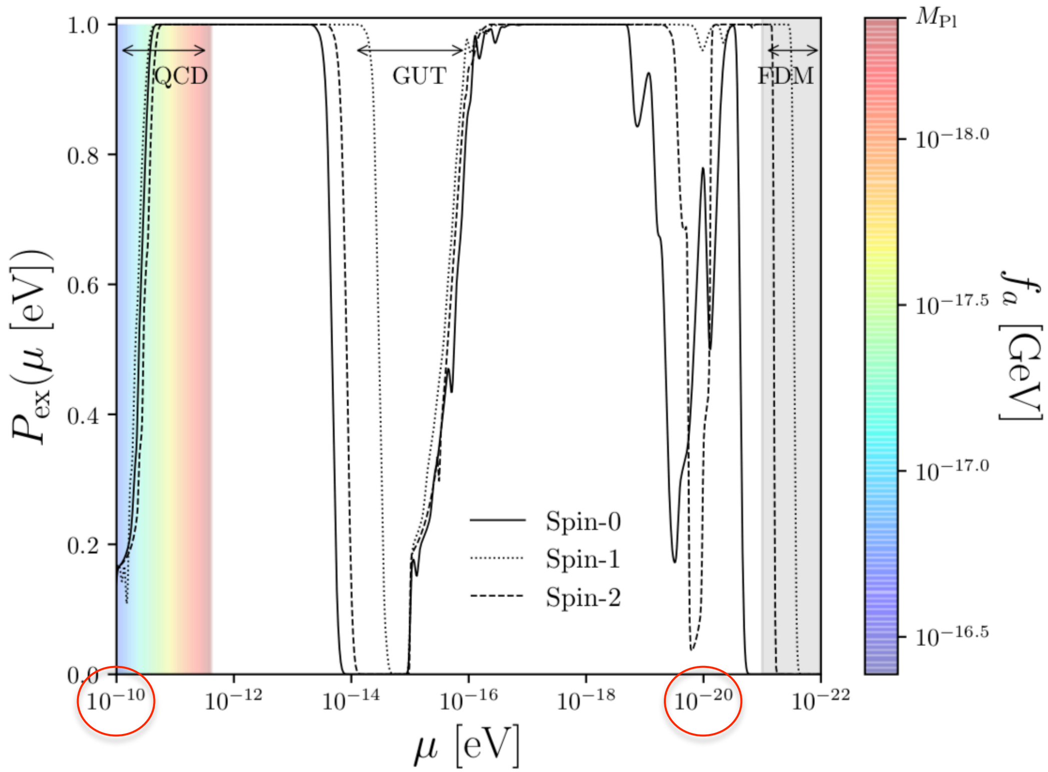
Hence, the rapidly rotating astrophysical BHs in the mass range

$$1 M_\odot \leq M \leq 10^{10} M_\odot$$

can constrain the axion with the mass

$$10^{-20}\,\text{eV} \leq \text{m} \leq 10^{-10}\,\text{eV}$$

Constraints on axion from BHs



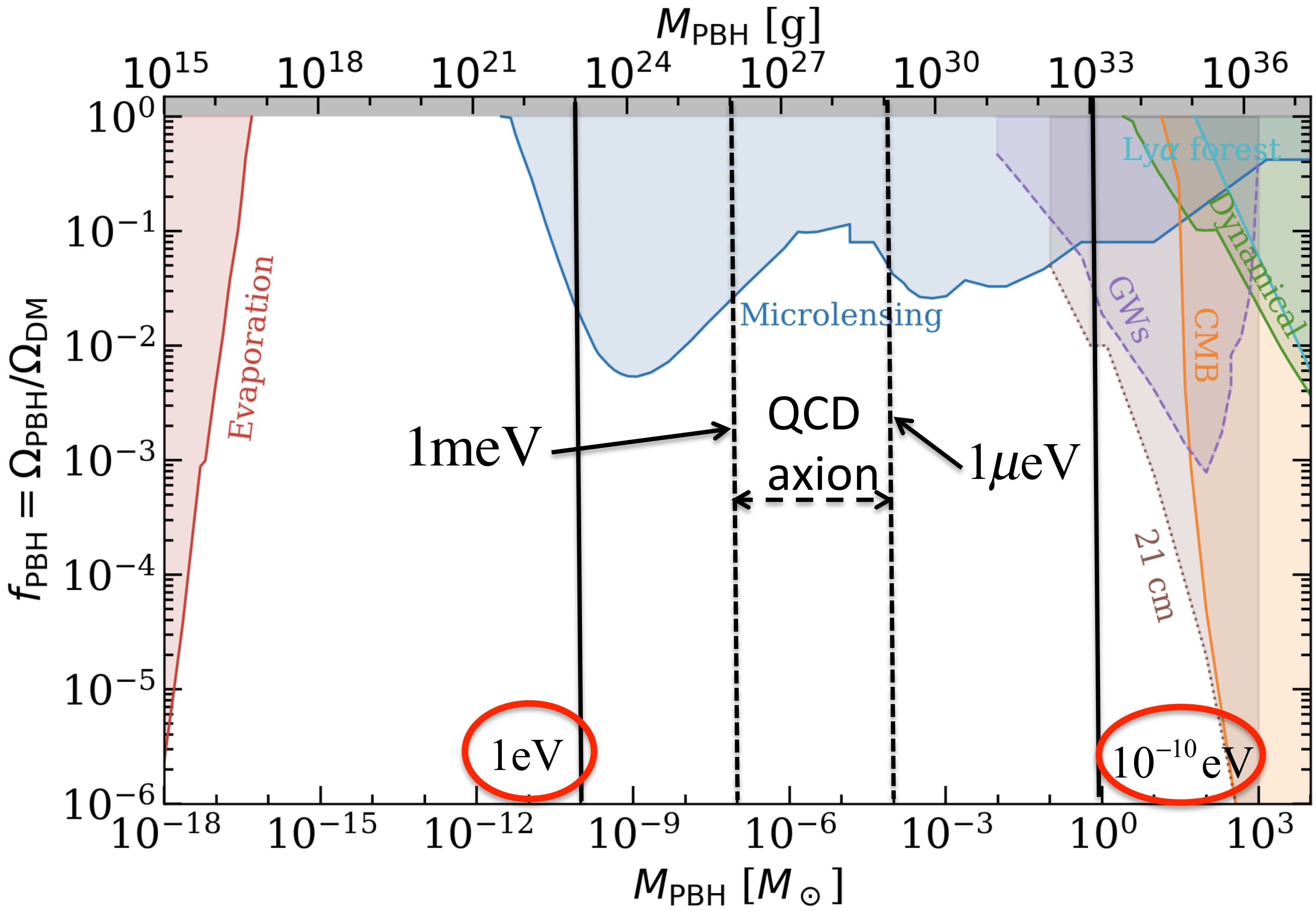
Stott 2020

Thus, we can now focuss on the mass range $10^{-10}\mathrm{eV} \le \mathrm{m} \le 1\mathrm{eV}$

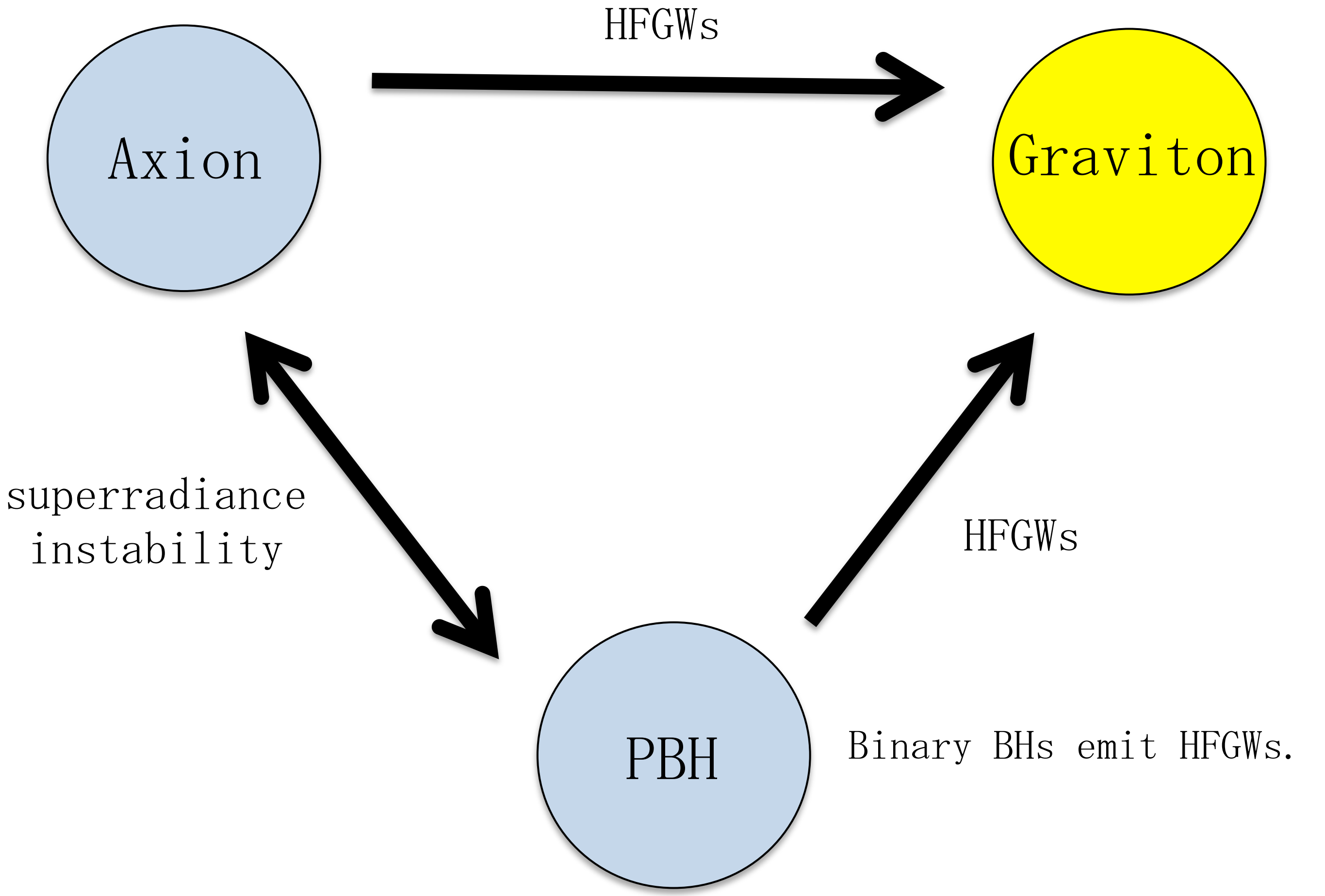
If we find a BH with a mass smaller than the solar mass, we can further constrain the axions.

In fact, such primordial BHs have been discussed as the dark matter.

Constraints on PBHs

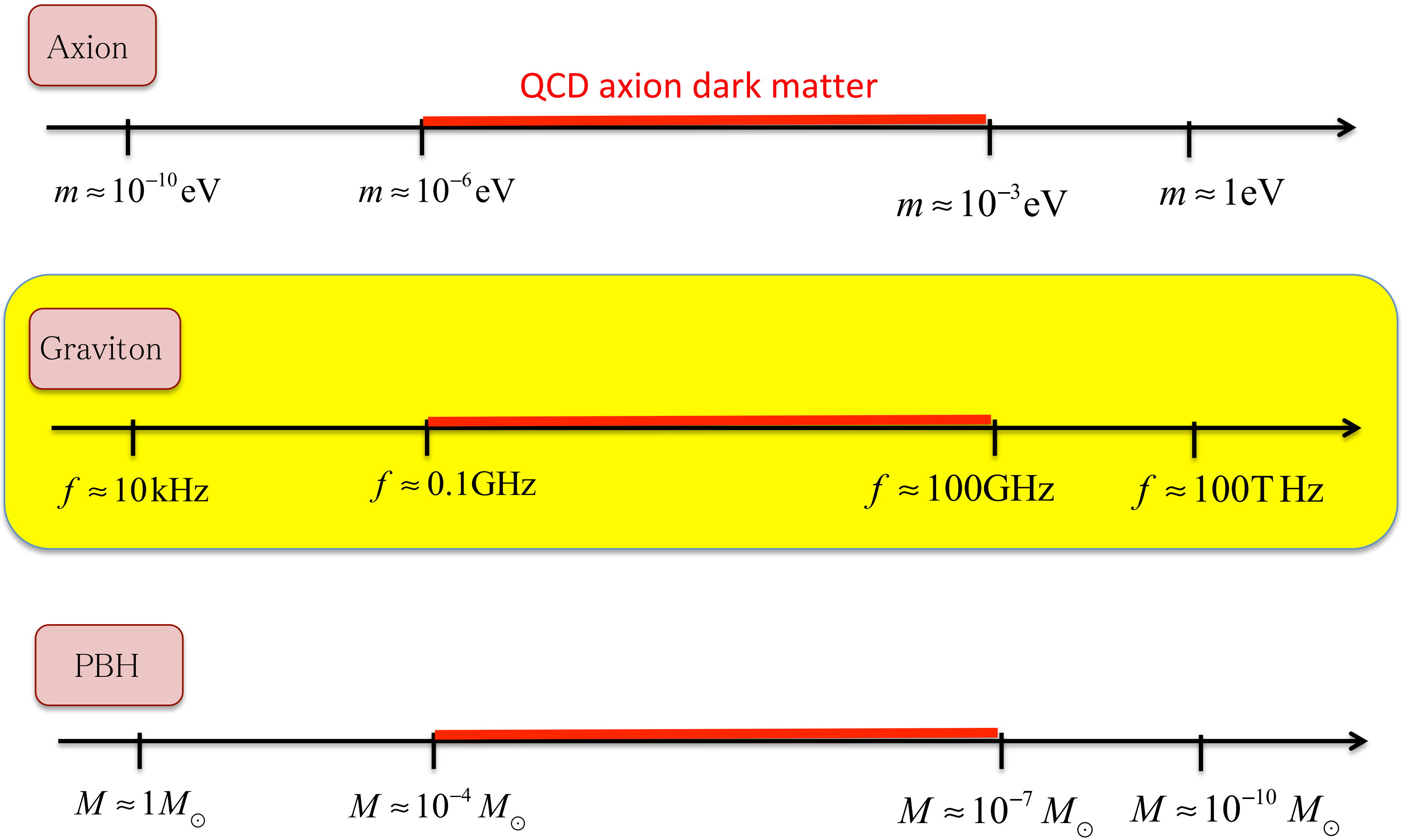


Exploring axion and PBH with graviton



Gravitons (HFGWs) can probe
the two dark matter candidates, axion and PBH.

Axion, graviton, and PBH



Duality of axion and graviton

Duality of axion and graviton experiments

Both axions and graviton are coherently oscillating.

graviton		axion
$h_{ij} \propto \cos(\mathbf{k}\cdot\mathbf{x} - \omega t), \quad \omega = \lvert\mathbf{k}\rvert$	⟺	$a = a_0 \cos mt$

$$S = \frac{M_p^2}{2}\int\sqrt{-g}\,R \quad \Longleftrightarrow \quad S = -\frac{1}{2}\int\sqrt{-g}\left[\left(\partial_\mu a\right)^2 + m^2 a^2\right]$$

Remarkably, there exists a duality between axion and graviton experiments.

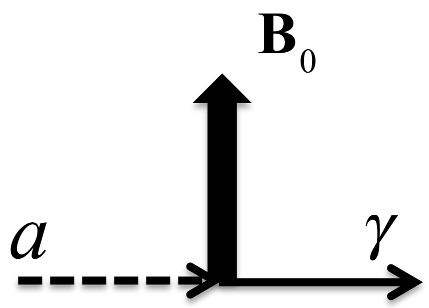
Detectors useful for axion search
is also useful for graviton search

Detectors useful for axion search
is also useful for graviton search

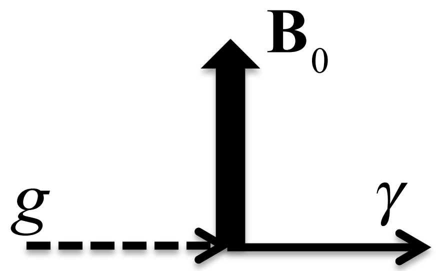
Axion - photon - graviton conversion

$$S=\int\sqrt{-g}\left[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}+\frac{1}{4}a\,\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\right]$$

$L\supset a\,\mathbf{E}\cdot\mathbf{B}_0$ Sikivie 1983



$L\supset h_{ij}\,E^iB_0^j$ Gertsenshtein 1962



The photon could be other excitations such as phonons, magnons,… .

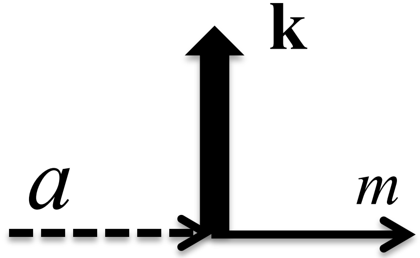
A useful application of duality is
to use existing data from axion experiments
for giving constraints on gravitons.

An application of duality

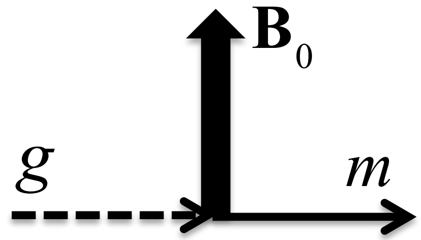
Here, we take magnons and show how to use duality to probe gravitons.

Axion - magnon - graviton conversion

$L \supset S \cdot \nabla a$ Barbieri et al. 1989



$L \supset h_{ij} S^i B_0^j$ Ito et al. 2020



A. Ito, T. Ikeda, K. Miuchi and J. Soda,
``Probing GHz gravitational waves with graviton-magnon resonance,''
Eur. Phys. J. C80, no.3, 179 (2020) [arXiv:1903.04843 [gr-qc]].

Axion search with magnons

T. Ikeda, A. Ito, K. Miuchi, J. Soda, H. Kurashige and Y. Shikano, ``Axion search with quantum nondemolition detection of magnons,'' [arXiv:2102.08764 [hep-ex]].

Spin waves

Dirac equation $\quad i\gamma^{\mu}\left(\partial_{\mu} - ieA_{\mu}\right)\psi = m\psi$

Non-relativistic approximation

$$H \simeq \frac{\left(\mathbf{p} - e\mathbf{A}\right)^2}{2m_e} - 2\mu_B \hat{S}\cdot\mathbf{B}$$

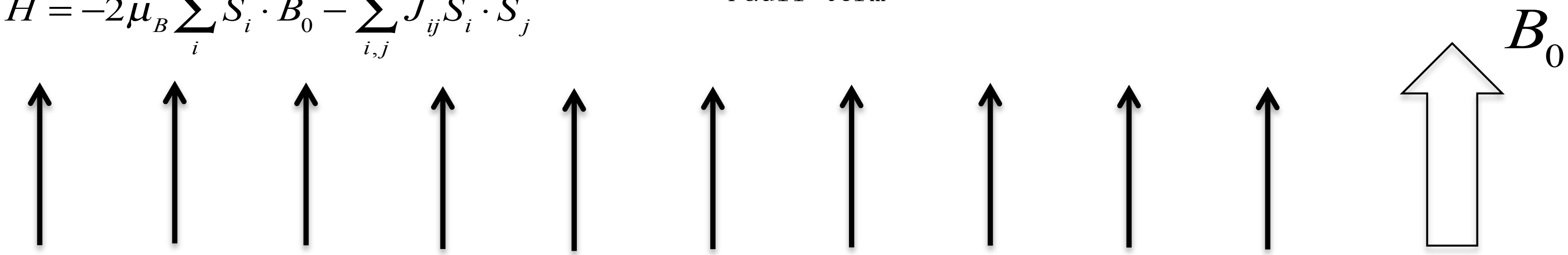
Pauli term

Bohr magneton

$$\mu_B = \frac{e}{2m_e}$$

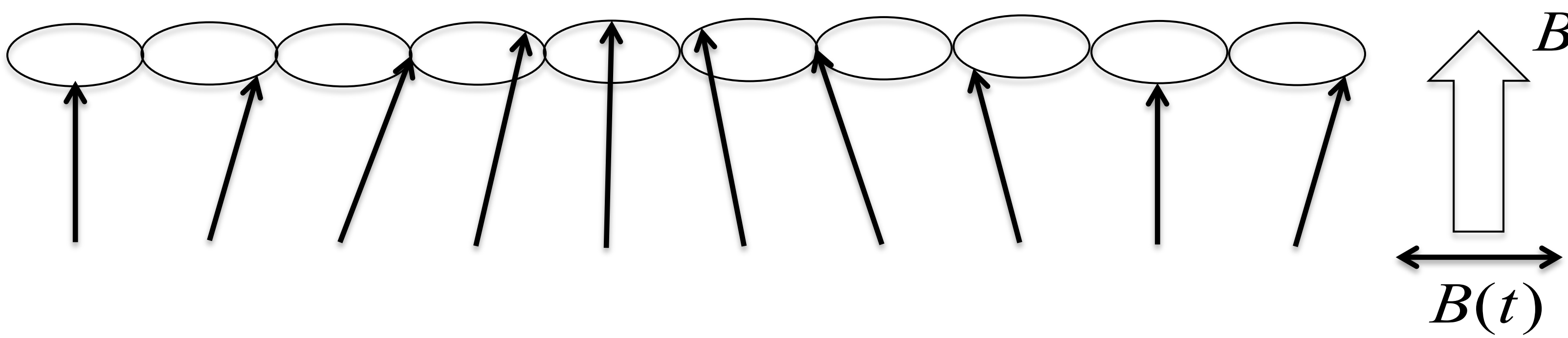
$$\mathbf{S} = \frac{1}{2}\sigma$$

$$H = -2\mu_B \sum_i \hat{S}_i \cdot \vec{B}_0 - \sum_{i,j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$



Usually, microwaves are used for generating spin waves with an external magnetic field applied.

$$H = -2\mu_B \sum_i \hat{S}_i \cdot \left(\vec{B}_0 + B(t)\right) - \sum_{i,j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$



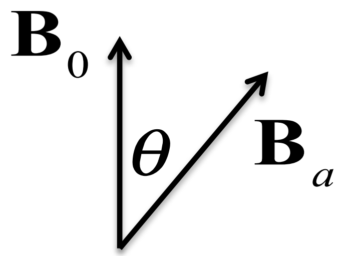
Magnons are quantum excitations of spin waves.

Axion – magnon interaction

Interaction term $$L_{\text{int}} = -ig_{aee} a\bar{\psi}\gamma_5\psi = \frac{g_{aee}}{2m}\partial_\mu a\bar{\psi}\gamma^\mu\gamma_5\psi$$

In a nonrelativistic approximation,

$$H \simeq \frac{(\mathbf{p}-e\mathbf{A})^2}{2m_e} - 2\mu_B\hat{S}\cdot\mathbf{B}_0 - 2\mu_B\hat{S}\cdot\mathbf{B}_a(t) \qquad B_a(t) = \frac{g_{aee}}{e}\nabla a(t)$$



The coherently oscillating axion can plays a roll of microwaves.

$$\vec{B}_a(t) = \left(\frac{1}{2}B_a\sin\theta\left(e^{-i\omega_a t} + e^{i\omega_a t}\right), 0, 0\right) \qquad f_a = \frac{\omega_a}{2\pi} = \frac{m_a c^2}{h} = 0.24\left(\frac{m_a}{1.0\mu\text{eV}}\right)\text{GHz}$$

We consider only Kittel mode(k=0)

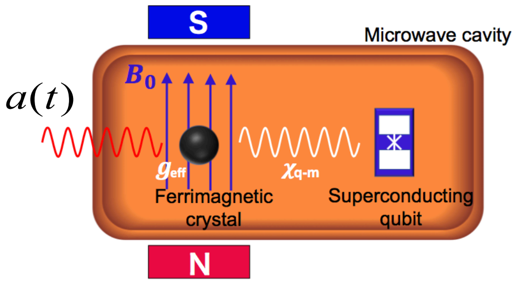
$$H_{m-q} = \hbar\omega c^\dagger c + g_{eff}\left(c^\dagger e^{-i\omega_a t} + ce^{i\omega_a t}\right)$$

Case of YIG $N \approx 10^{22}$

$$\hbar\omega = 2\mu_B B_0 \qquad g_{eff} = \frac{1}{2}\mu_B B_a\sin\theta\sqrt{N}$$

Magnon limits on axions

Read out by Q-bit



$$B_a = \frac{g_{aee}}{e} \nabla a$$

$$\simeq 4.4 \times 10^{-8} g_{aee} \left(\frac{\rho_{DM}}{0.45 \mathrm{GeV/cm}^3} \right)^{1/2} \left(\frac{v}{300 \mathrm{km/s}} \right) [\mathrm{T}]$$

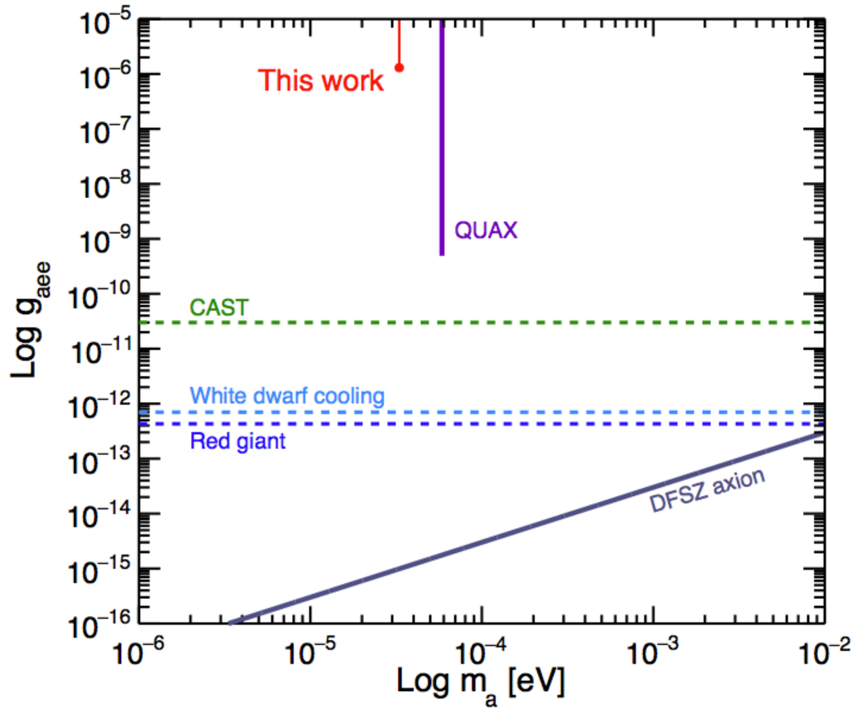
magnon limits on the axion electron coupling constant

$$B_a < 4.1 \times 10^{-14} [\mathrm{T}]$$

$$\longrightarrow \quad g_{aee} < 1.3 \times 10^{-6}$$

Axion is coherently oscillating

$$a(t) \approx \frac{\sqrt{2\rho_{DM}}}{m_a} \cos m_a t$$



Ikeda et al. 2018

Graviton search with magnons

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Eur. Phys. J. C80, no.3, 179 (2020) [arXiv:1903.04843 [gr-qc]].

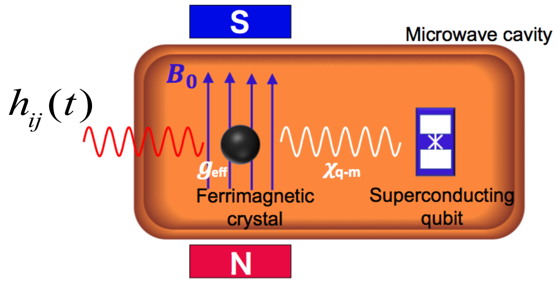
A. Ito and J. Soda,
``A formalism for magnon gravitational wave detectors,''
Eur. Phys. J. C80, no.6, 545 (2020) [arXiv:2004.04646 [gr-qc]].

Graviton – magnon interaction

GWs (TTgauge)

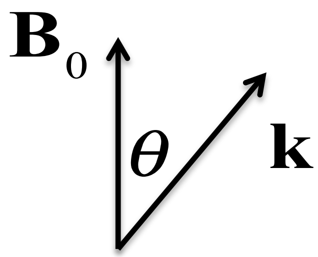
$$h_{ij}(t) = h^{+}(t)e_{ij}^{+} + h^{\times}(t)e_{ij}^{\times}$$

$$h^{+,\times}(t) = \frac{h^{+,\times}}{2}\left(e^{-i\omega_h t} + e^{i\omega_h t}\right)$$

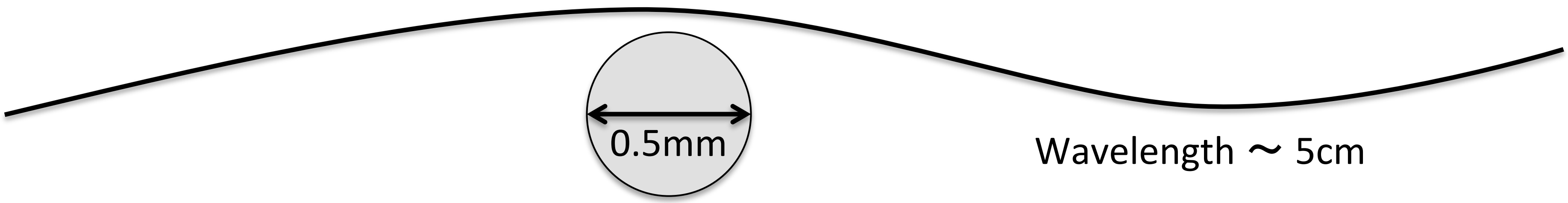


$$H = -2\mu_B \sum_i \hat{S}_i \cdot \vec{B}_0 - \mu_B \sum_i S_i^a h_{az}(t) B_0 - \sum_{i,j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$

$$H_{m-q} = \hbar\omega c^{\dagger}c + g_{eff}\left(c^{\dagger}e^{-i\omega_h t} + c e^{i\omega_h t}\right)$$



$$g_{eff} = \frac{1}{4\sqrt{2}}\mu_B B_0 \sin\theta \sqrt{N}\left[\cos^2\theta\left(h^{(+)}\right)^2 + \left(h^{(\times)}\right)^2\right]^{1/2}$$



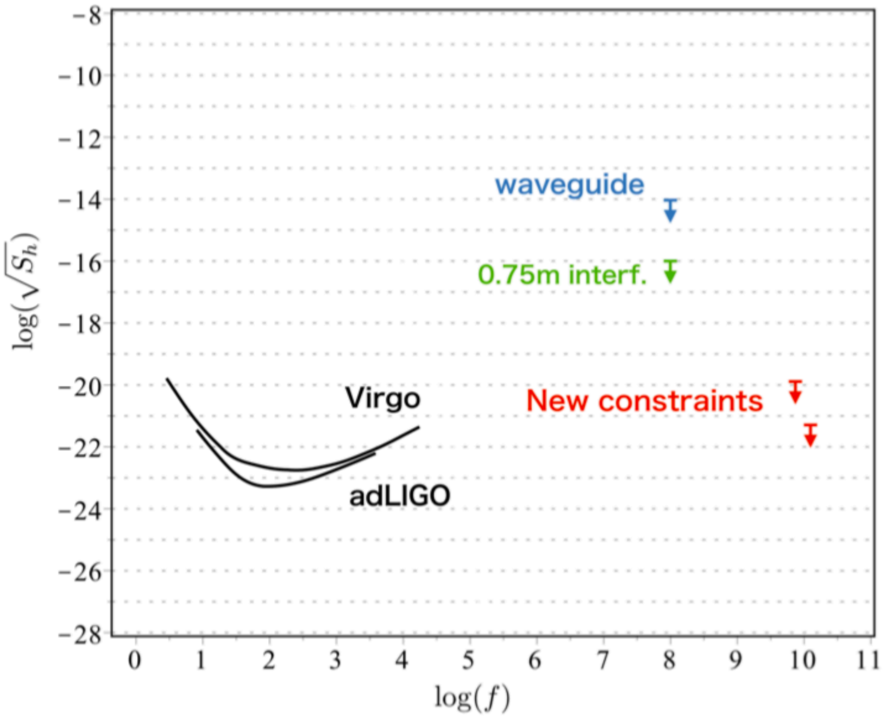
Search for GHz graviton

Ito et al. 2019
Ito & Soda 2020

Limits on axions

$$g_{eff} < \begin{cases} 3.5\times10^{-12}\,\mathrm{eV} & \text{QUAX, Crescini et al. 2018} \\ 3.1\times10^{-11}\,\mathrm{eV} & \text{Flower et al. 2018} \end{cases}$$

$$g_{eff} = \frac{1}{4\sqrt{2}}\mu_B B_z \sin\theta\sqrt{N}\left[\cos^2\theta\left(h^{(+)}\right)^2 + \left(h^{(\times)}\right)^2\right]^{1/2}$$



$$\sqrt{S_h} \approx h < \begin{cases} 7.6\times10^{-22}\,\mathrm{Hz}^{-1/2} & \text{at } 14\ \mathrm{GHz} \\ 1.2\times10^{-20}\,\mathrm{Hz}^{-1/2} & \text{at } 8.2\ \mathrm{GHz} \end{cases}$$

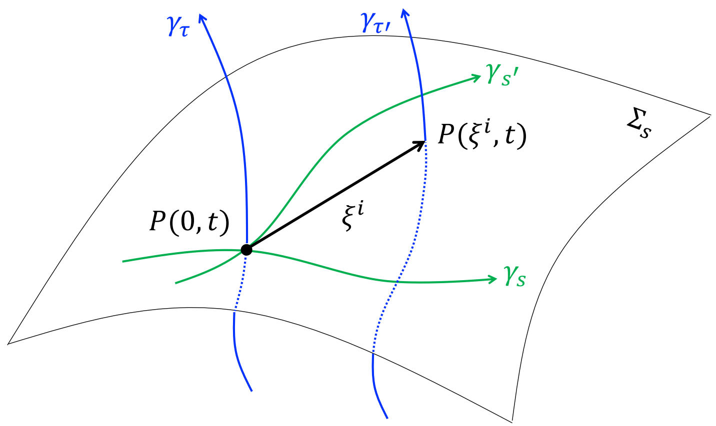
Proper detector frames

Dirac eq. in curved spacetime

$$i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu - \Gamma_\mu - ieA_\mu \right) \psi = m\psi$$

Proper detector frame

$$\begin{cases} g_{00} = -1 - 2a_i - R_{0i0j} x^i x^j \\ g_{0i} = -\omega_k \varepsilon_{0ijk} x^j - \frac{2}{3} R_{0jik} x^j x^k \\ g_{ij} = \delta_{ij} - \frac{1}{3} R_{ikjl} x^k x^l \end{cases}$$



In a nonrelativistic approximation,

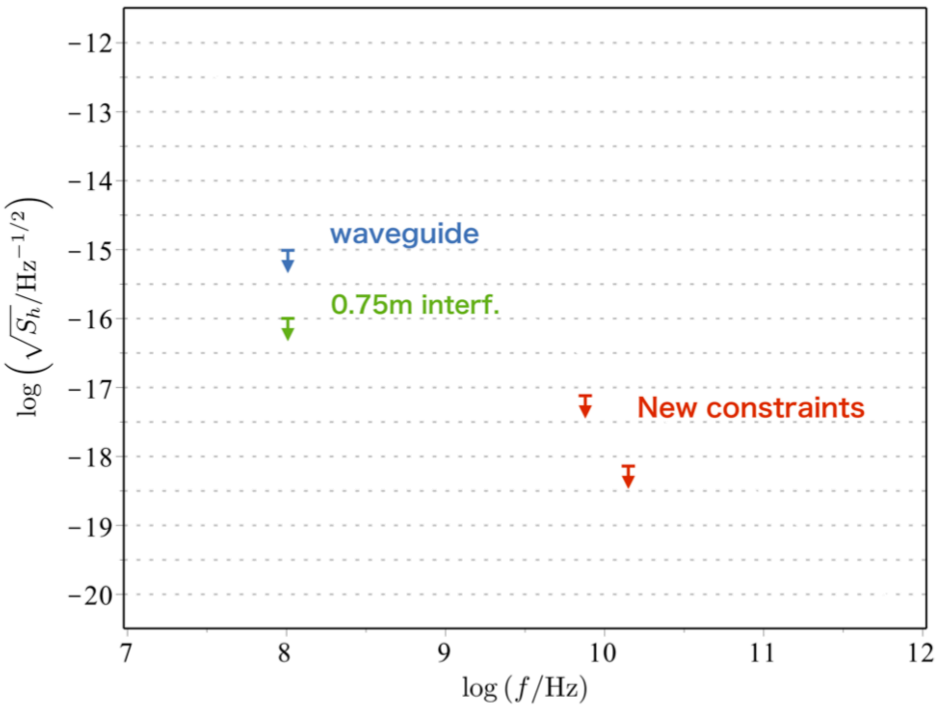
$$H = m\left(1 + a_i x^i + \frac{1}{2} R_{0k0l} x^k x^l \right) - eA_0 - \omega_i S^i + \frac{i}{6} R_{0iki} x^k$$

$$-\frac{e}{m} S^i B^j \left[\left(1 + a_k x^k + \frac{1}{2} R_{0k0l} x^k x^l + \frac{1}{6} R_{mkml} x^k x^l \right) \delta_{ij} + \frac{1}{3} R_{jkil} x^k x^l \right]$$

$$+ \cdots$$

$$R_{0k0l} = -\frac{1}{2} \frac{\partial^2}{\partial t^2} h_{kl}$$

Magnon limits on GHz graviton



$$\left(\frac{\ell}{\lambda}\right)^2$$ suppression factor

By constructing a detector with $\ell \approx \lambda$

we can improve the sensitivity significantly.

Summary

- We need HFGW detectors for exploring the dark matter physics.
- Axion and graviton experiments have a kind of duality.
- It is known that Magnon is useful for axion search.
- From the duality, magnon should be useful for graviton search.
- Utilizing the available data of axion search, we have given magnon limits on GHz graviton.
- If we develop a detector specific to graviton,
  we can expect more sensitivity and
  extend observable frequency ranges.