

連続測定に基づく量子推定と量子フィードバック制御： レビューとオプトメカへの応用

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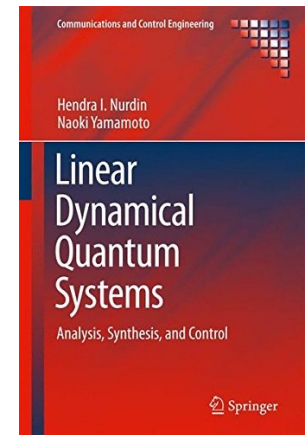
はじめに

情報が(基礎方程式の導出以外)やや古いです。また、スライド内の記法が統一されておりません。ただ、とくに実験技術の進展が素晴らしく、ここで紹介する内容は現在も活発に進展中です。スライド内のレビューに関する情報は主に以下から抜粋しております。

H. I. Nurdin and N. Yamamoto, Linear Dynamical Quantum Systems: Analysis, Synthesis, and Control, Springer (2017)

山本直樹, 連続時間カルマンフィルタと量子状態推定, 計測自動制御学会誌, 56-9, 662/667 (2017)

山本直樹, 量子フィードバック制御の数理, 数理科学, 585 (2012)



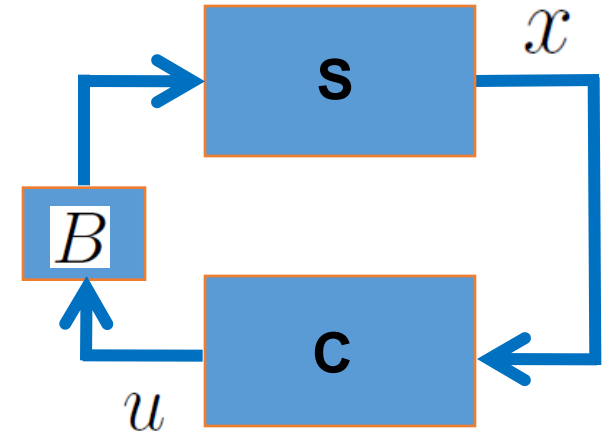
Control theory: Optimal linear feedback control

Linear system $\dot{x} = Ax + Bu$

Suppose x can be measured directly.

Control purpose :

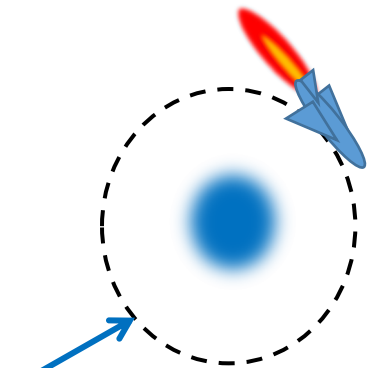
$$J_T = \frac{1}{2} \int_0^T (x_t^T P x_t + u_t^T Q u_t) dt \rightarrow \min.$$



FB control law that minimizes the above cost function is

$$\begin{cases} u_t^* = -B^T K_t x_t \\ \dot{K}_t = -K_t A - A^T K_t + K_t B B^T K_t - P \end{cases}$$

Target trajectory : $x = 0$

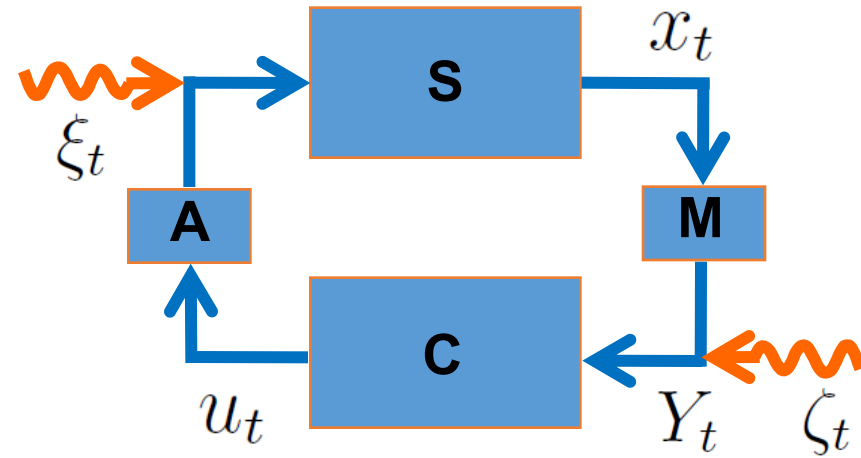


Control theory: Stochastic feedback control

Stochastic system

$$\begin{cases} \dot{x}_t = f(x_t) + g_1(x_t)u_t + g_2(x_t)\xi_t \\ Y_t = h(x_t) + \zeta_t \end{cases}$$

Before discussing how to control...



- **White noise approximation:** $\mathbb{E}(\xi_s \xi_t) = \delta(s - t)$
- Noise added in $[t, t + dt]$ = **Wiener increment** dW_t

Formally, $\xi_t = \frac{dW_t}{dt}$ and subjected to $\mathcal{N}(0, dt)$ hence $dW_t^2 = dt$

- Dynamics in $[t, t + dt]$ = **stochastic differential equation (SDE)**

$$\begin{cases} dx_t = f(x_t)dt + g_1(x_t)u_tdt + g_2(x_t)dW_t \\ dy_t = h(x_t)dt + dV_t \end{cases}$$

Control theory: Stochastic feedback control

Stochastic system

$$\begin{cases} dx_t = f(x_t)dt + g_1(x_t)u_tdt + g_2(x_t)dW_t \\ dy_t = h(x_t)dt + dV_t \end{cases}$$

Use the optimal estimate of x_t , i.e. the conditional expectation $\pi_t(x)$, and design an estimate-based FB control $u = F(\pi(x))$

Dynamics of $\pi_t(x)$ = filter equation:

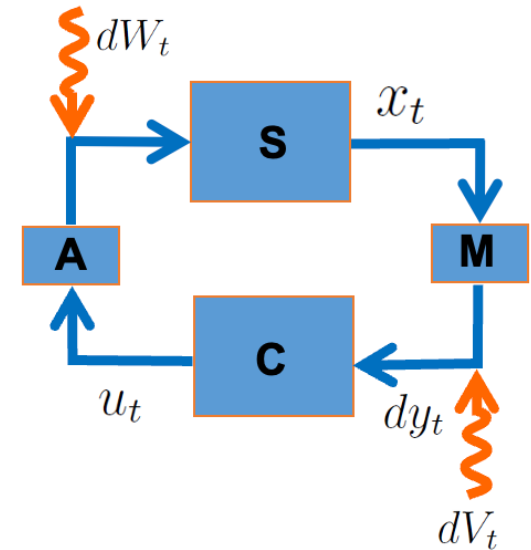
$$d\pi_t(x) = \pi_t(f(x))dt + [\pi_t(xh(x)) - \pi_t(x)\pi_t(h(x))][dy - \pi_t(h(x))dt]$$

Dynamics of the conditional probability is obtained via $\pi_t(x) = \int xp_t(x|\mathcal{Y}_t)dx$

$$dp_t(x) = \left[-\frac{\partial(p_tf)}{\partial x} + \frac{1}{2}\frac{\partial(p_tg^2)}{\partial x^2} \right]dt + p_t(x)[h(x) - \pi_t(h(x))][dy - \pi_t(h(x))dt]$$

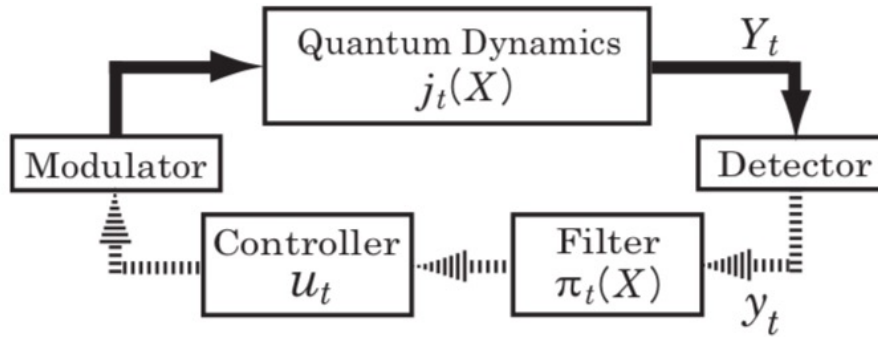
--- Kushner-Stratonovich equation (first term: Fokker-Planck eq.)

$$\text{Cost for optim control: } J_T = \frac{1}{2}\mathbb{E}\left[\int_0^T (x_t^2 + ru_t^2)dt\right] = \frac{1}{2}\mathbb{E}\left[\int_0^T (\pi_t(x^2) + ru_t^2)dt\right]$$



測定型量子フィードバック制御理論

創始者：S. Belavkin



システム方程式 (量子ランジュバン方程式) :

$$dj_t(X) = j_t(\mathcal{L}X)dt + j_t([L^*, X])d\mathcal{A}(t) + j_t([X, L])d\mathcal{A}(t)^*$$

$$dy_m(t) = j_t(L + L^*)dt + d\mathcal{A}(t) + d\mathcal{A}(t)^*$$

条件付き平均(最適推定値)のダイナミクス (量子フィルター方程式 or Belavkin Eq):

$$d\pi_t(X) = \pi_t(\mathcal{L}X)dt + (\pi_t(XL + L^*X) - \pi_t(X)\pi_t(L + L^*)) (dy_m(t) - \pi_t(L + L^*)dt)$$

条件付き量子状態のダイナミクス (確率マスター方程式) :

$$d\rho = \mathcal{L}^*\rho dt + (L\rho + \rho L^* - \text{Tr}(L\rho + \rho L^*)\rho) (dy_m(t) - \text{Tr}(L\rho + \rho L^*)dt)$$

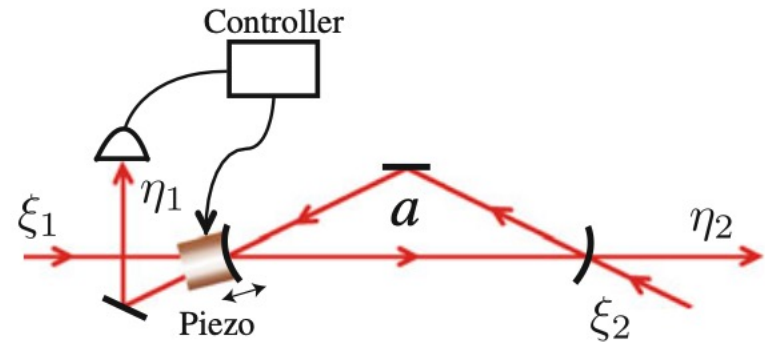
$$\pi_t(X) = \text{Tr}(X\rho_t)$$

システム方程式の例: optical cavity

$$H_{\text{sys}} = \omega_o a^* a \quad L_j = \sqrt{\kappa_j} \bar{a}$$

$$da = -\left(i\Delta + \frac{\kappa_1 + \kappa_2}{2}\right)adt - \sqrt{\kappa_1}d\mathcal{A}_1 - \sqrt{\kappa_2}d\mathcal{A}_2,$$

$$d\mathcal{Y}_1 = \sqrt{\kappa_1}adt + d\mathcal{A}_1, \quad d\mathcal{Y}_2 = \sqrt{\kappa_2}adt + d\mathcal{A}_2.$$



線形システム！ 1から2への周波数応答 = $|\Xi_{1 \rightarrow 2}[i\omega]|^2 = \frac{\kappa^2}{\omega^2 + \kappa^2} \sim \text{LPF}$ として機能

別の例: degenerate parametric oscillator

$$H_{\text{sys}} = i\epsilon (a^{*2} - a^2)/2, \quad L_j = \sqrt{\kappa_j} \bar{a}$$

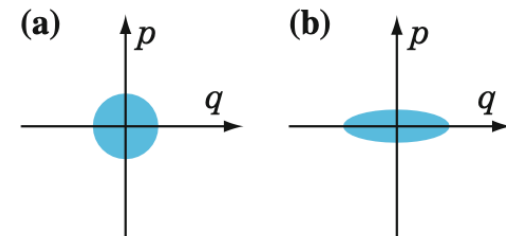
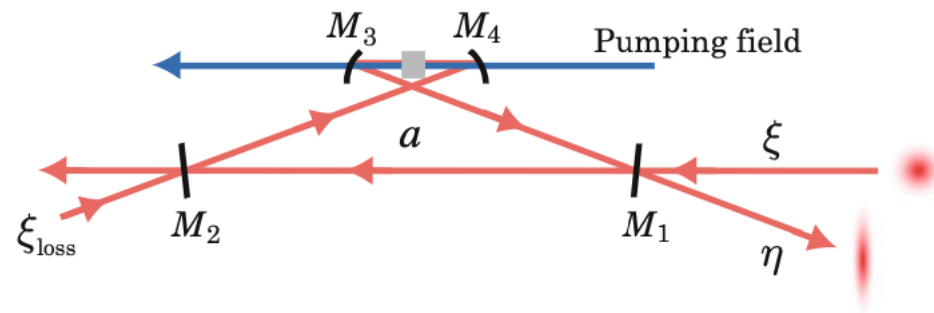
$$\frac{da}{dt} = -\frac{\gamma + \kappa}{2}a + \frac{\epsilon}{2}a^* - \sqrt{\gamma}\xi - \sqrt{\kappa}\xi_{\text{loss}}$$

$$\eta = \sqrt{\gamma}a + \xi$$

出力モードの実部(q) & 虚部(p) :

$$\langle |\eta^q[i\omega]|^2 \rangle = \frac{\omega^2 + (\gamma + \epsilon)^2/4}{\omega^2 + (\gamma - \epsilon)^2/4}, \quad \langle |\eta^p[i\omega]|^2 \rangle = \frac{\omega^2 + (\gamma - \epsilon)^2/4}{\omega^2 + (\gamma + \epsilon)^2/4}.$$

$$\langle |\eta^q[i\omega]|^2 \rangle \langle |\eta^p[i\omega]|^2 \rangle = 1$$



スクイズド光

Example: opto-mechanical system

$$H_{\text{har}} = m\omega_o^2 q_1^2/2 + p_1^2/2m \quad L = \sqrt{\kappa}a_2$$

$$\frac{d}{dt} \begin{bmatrix} q_1 \\ p_1 \\ q_2 \\ p_2 \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & 1/m & 0 & 0 \\ -m\omega^2 & 0 & \gamma & 0 \\ \hline 0 & 0 & -\kappa/2 & 0 \\ \gamma & 0 & 0 & -\kappa/2 \end{array} \right] \begin{bmatrix} q_1 \\ p_1 \\ q_2 \\ p_2 \end{bmatrix} - \sqrt{\kappa} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \xi^q \\ \xi^p \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} f$$

$$\begin{bmatrix} \eta^q \\ \eta^p \end{bmatrix} = \begin{bmatrix} \sqrt{\kappa} & 0 \\ 0 & \sqrt{\kappa} \end{bmatrix} \begin{bmatrix} q_2 \\ p_2 \end{bmatrix} + \begin{bmatrix} \xi^q \\ \xi^p \end{bmatrix}$$



Adiabatic elimination of the optical mode

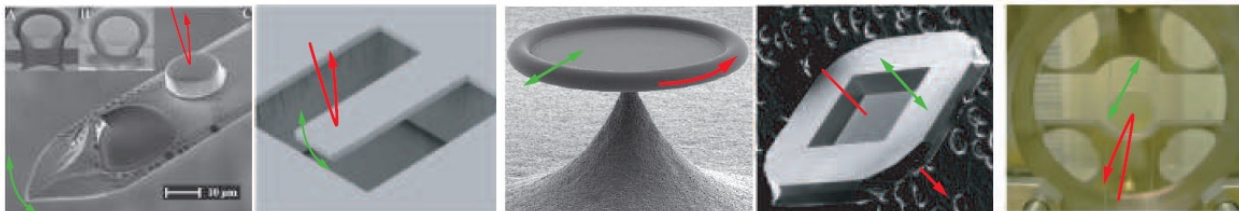
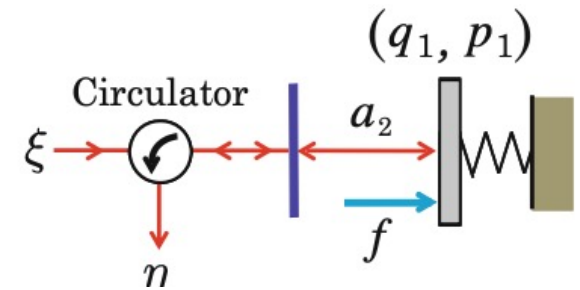
$$\frac{d}{dt} \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} = \begin{bmatrix} 0 & 1/m \\ -m\omega^2 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} - \sqrt{\lambda} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi^q + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f$$

$$y = [\sqrt{\lambda} \ 0] \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} - \xi^p$$

線形システム！

$$d\hat{x}_t = A\hat{x}_t dt + Fu_t dt + Bd\hat{W}_t$$

$$dy_t = C\hat{x}_t dt + Dd\hat{W}_t$$



量子フィードバック制御の例：
2準位原子のエネルギー状態を
励起状態に安定化

システム方程式：

$$dj_t(\sigma_x) = -\frac{\gamma}{2}j_t(\sigma_x)dt + u_tj_t(\sigma_z)dt + \sqrt{\gamma}j_t(\sigma_y)dZ_t,$$

$$dj_t(\sigma_y) = -\frac{\gamma}{2}j_t(\sigma_y)dt - \sqrt{\gamma}j_t(\sigma_x)dZ_t,$$

$$dj_t(\sigma_z) = -u_tj_t(\sigma_x)dt$$

フィルター方程式： $(x_t, z_t) = (\pi_t(\sigma_x), \pi_t(\sigma_z))$

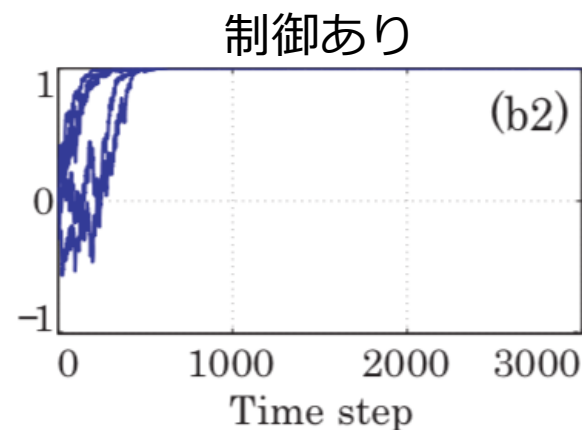
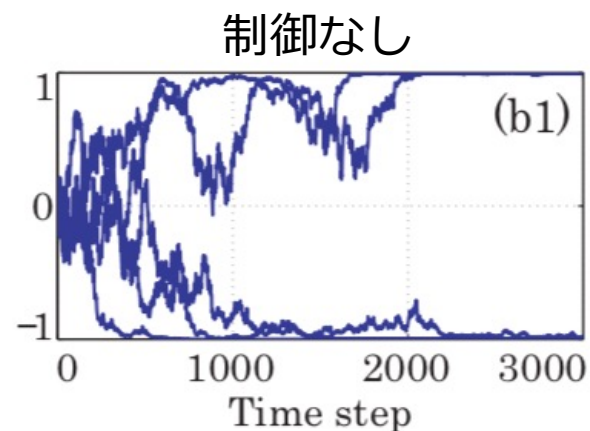
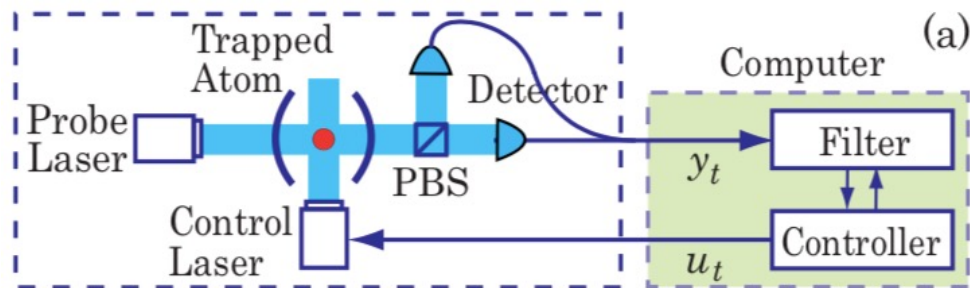
$$dx_t = -\frac{\gamma}{2}x_tdt + u_tz_tdt - \sqrt{\gamma}x_tz_tdw_t,$$

$$dz_t = -u_tx_tdt + \sqrt{\gamma}(1 - z_t^2)dw_t$$

リアプノフ関数： $V_t = \mathbb{E}[(z_t - 1)^2]$

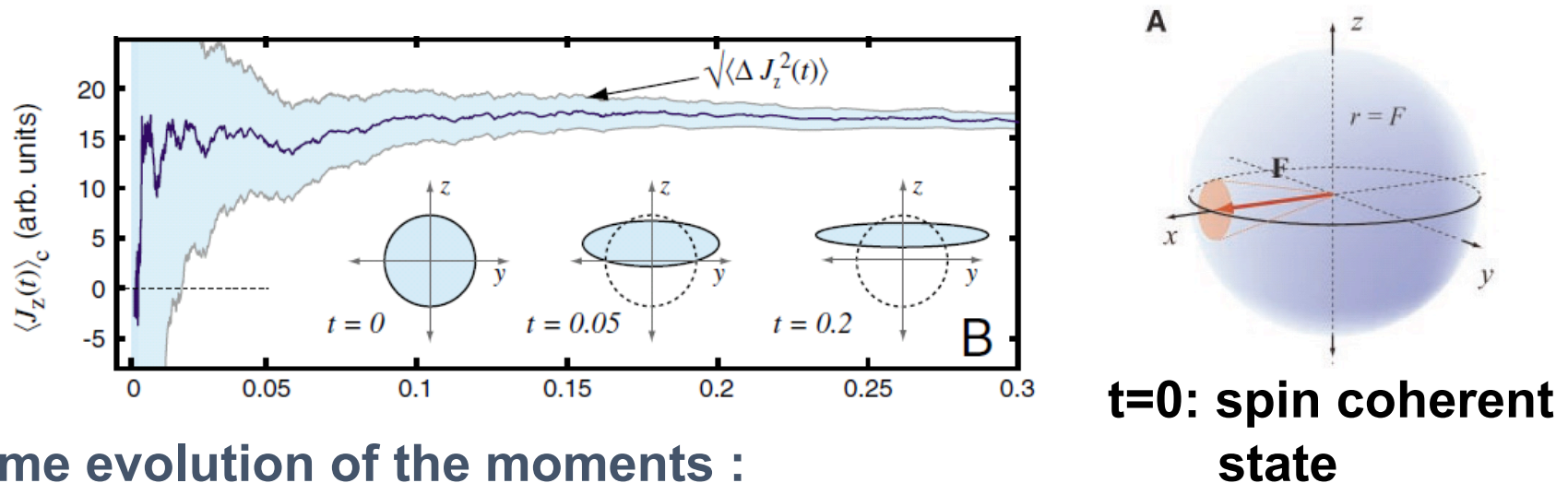
制御則： $u_t = -\gamma x_t$

効果： $dV/dt = -\gamma\mathbb{E}[x^2(1 - z)^2] \leq 0$



量子フィードバック制御の例: spin squeezing

(i) Probabilistic generation of spin squeezed state (Control : OFF)



Time evolution of the moments :

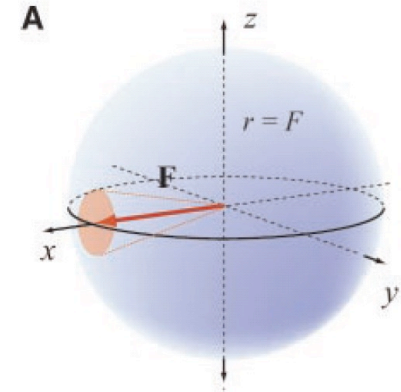
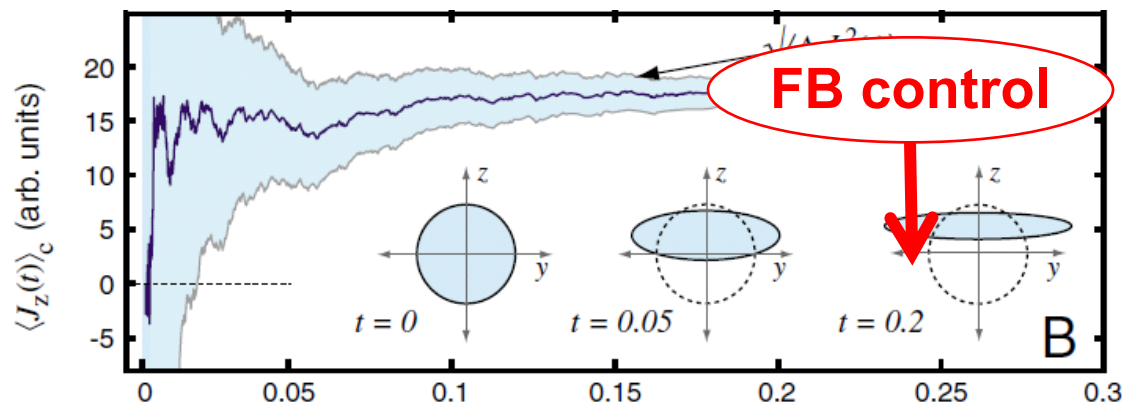
$$\begin{cases} d\pi_t(J_z) = 2\sqrt{M}\pi_t(\Delta J_z^2)[dy - 2\sqrt{M}\pi_t(J_z)dt] \\ d\pi_t(\Delta J_z^2) = -4M\pi_t(\Delta J_z^2)^2dt \end{cases}$$

The **1st moment fluctuates**, but the **2nd deterministically decreases**.

→ We get a spin squeezed state, but this vanishes when ensemble averaging.

量子フィードバック制御の例: spin squeezing

J. K. Stockton, GM Geremia, A. C. Doherty, and H. Mabuchi,
Robust quantum parameter estimation: coherent magnetometry
with feedback, PRA, 2003.

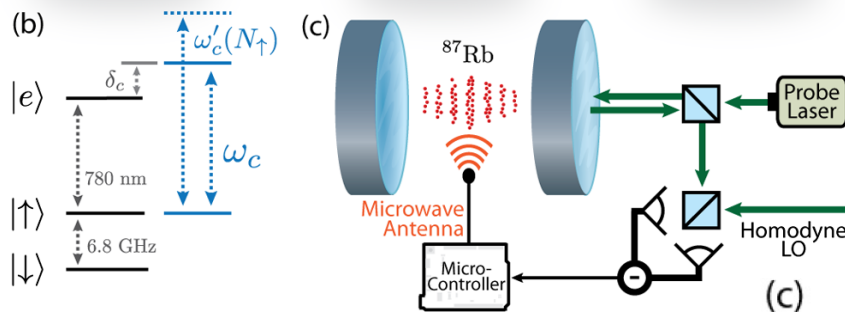
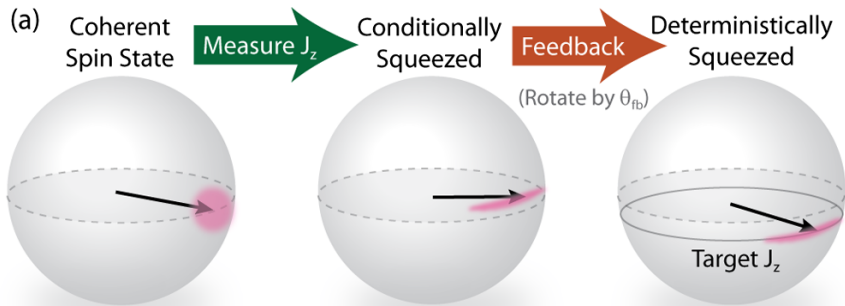


Dynamics (QSDE) $\left\{ \begin{array}{l} dj_t(J_x) = -\frac{M}{2} j_t(J_x) dt + u_t j_t(J_z) dt + \sqrt{M} j_t(J_y) (idB_t - idB_t^\dagger) \\ dj_t(J_y) = -\frac{M}{2} j_t(J_y) dt - \sqrt{M} j_t(J_x) (idB_t - idB_t^\dagger) \\ dj_t(J_z) = -u_t j_t(J_x) dt \end{array} \right.$

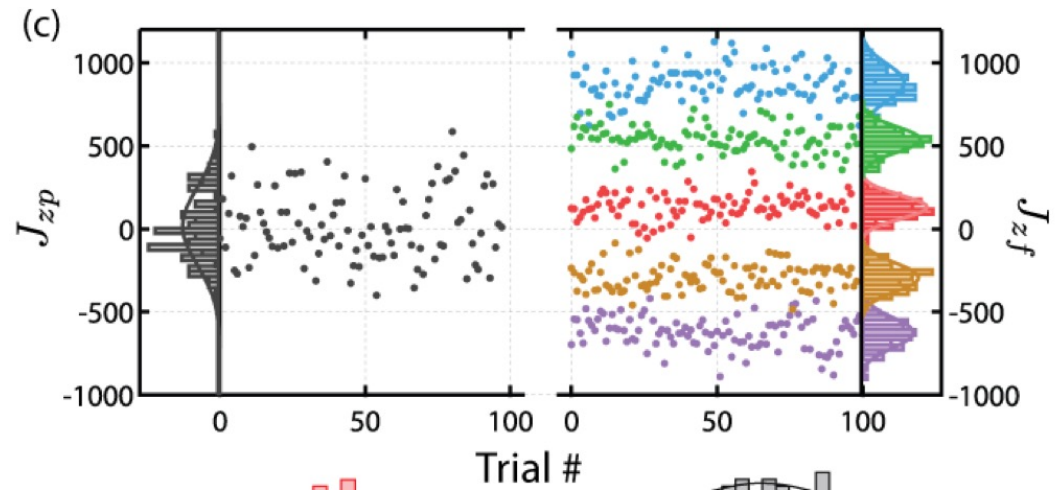
Goal: $J_T = \frac{1}{2} \mathbb{E} \left[\int_0^T (j_t(J_z)^2 + u_t^2) dt \right] = \frac{1}{2} \mathbb{E} \left[\int_0^T (\pi_t(J_z^2) + u_t^2) dt \right] \rightarrow \min.$

量子フィードバック制御の例: spin squeezing

Cox, et al. (NIST group), Deterministic squeezed states with collective measurement and feedback, PRL (2016)



Achieved 7.6 dB squeezing !



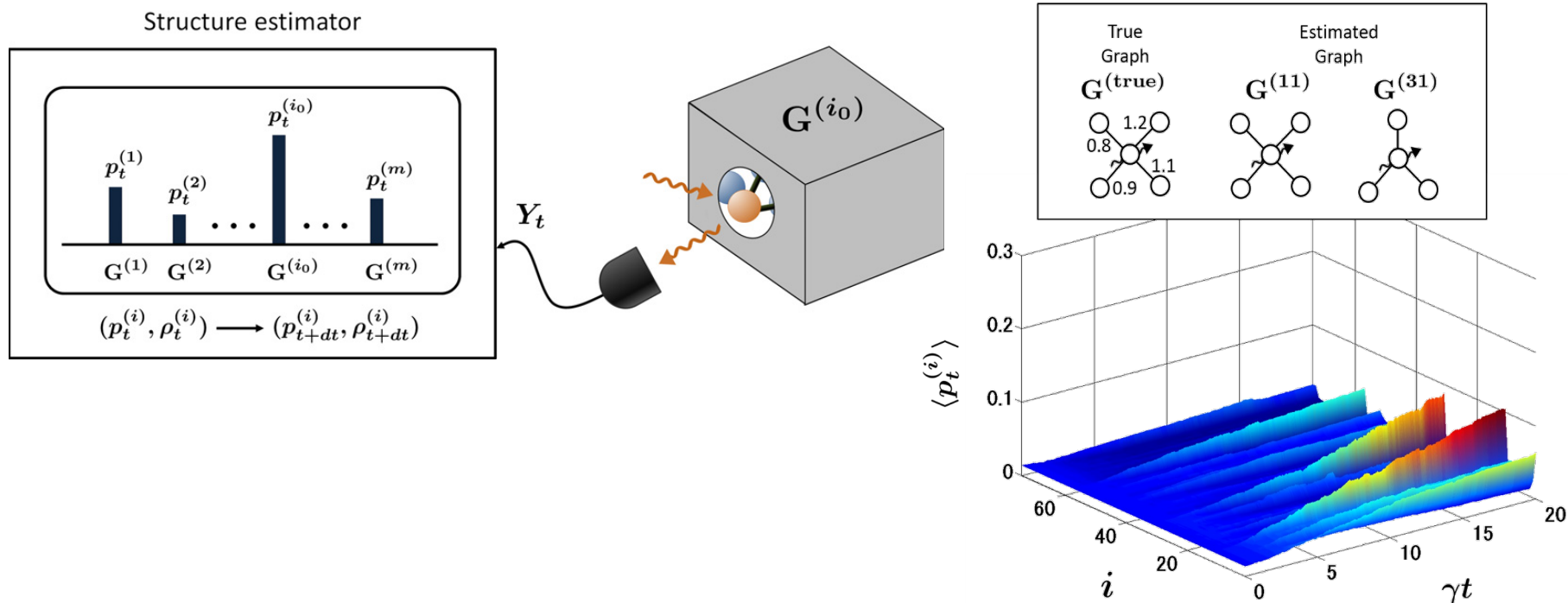
量子状態以外の情報を更新してもよい。

例：システムの構造に関する情報 $p_t^{(i)} = \mathbb{P}(\{G = G^{(i)}\} | \mathcal{Y}_t)$

Y. Kato and N. Yamamoto, Structure identification and state initialization of spin network with limited access, New Journal of Physics (2014)

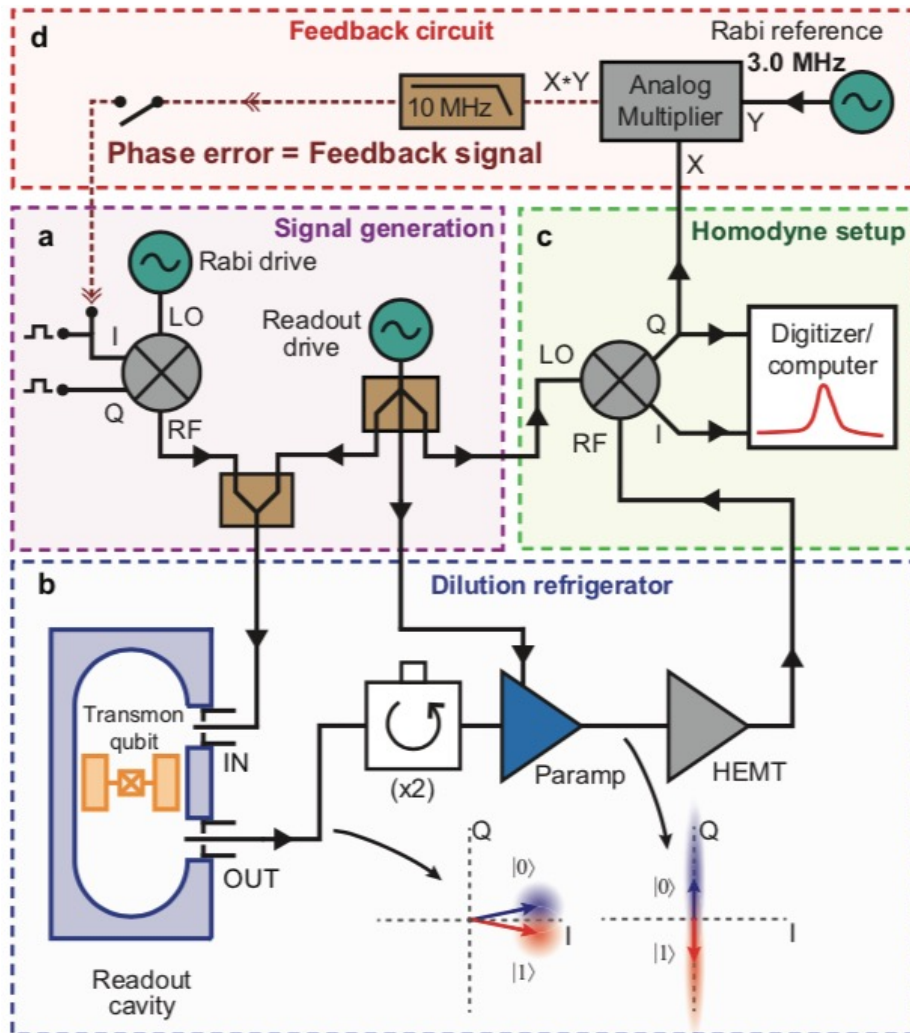
$$d\rho_t^{(i)} = -i[H^{(i)}, \rho_t^{(i)}] dt + \gamma \mathcal{D}[c]\rho_t^{(i)} dt + \sqrt{\gamma} \mathcal{H}[c]\rho_t^{(i)} (dY_t - 2\sqrt{\gamma} \text{Tr}(c\rho_t^{(i)}) dt)$$

$$dp_t^{(i)} = 2\sqrt{\gamma} \{ \text{Tr}(c\rho_t^{(i)}) - \text{Tr}(c\tilde{\rho}_t) \} p_t^{(i)} (dY_t - 2\sqrt{\gamma} \text{Tr}(c\tilde{\rho}_t) dt)$$

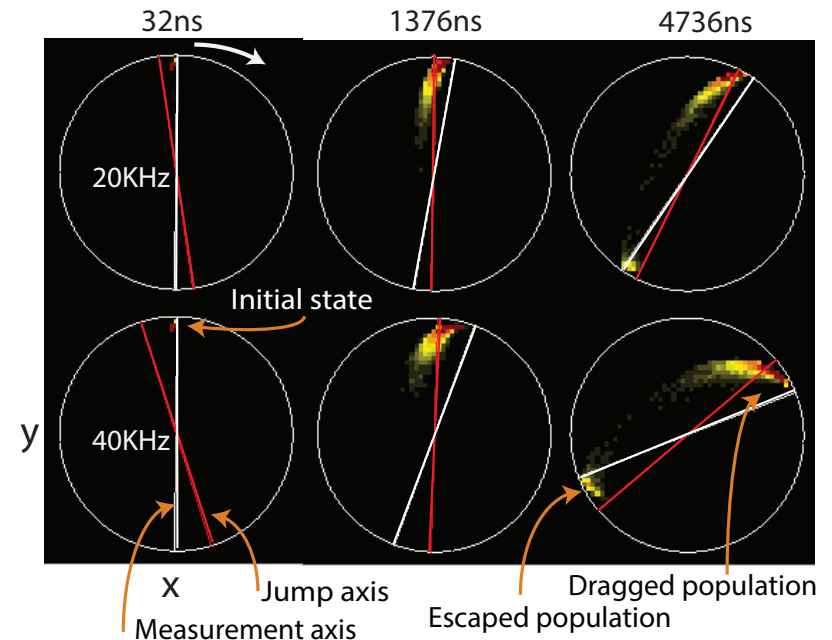


超伝導系での量子フィードバック制御実験

理論提案 : Tanaka and Yamamoto, Robust adaptive measurement scheme for qubit-state preparation, Phys. Rev. A, 2012.



実験実現 : Siddiqi Group (Berkeley), Incoherent qubit control using the quantum zeno effect, Phys. Rev. Lett., 2018.



Derivation

■ Interaction with the vacuum field

$$\begin{aligned} H &= i(cb_t^\dagger - c^\dagger b_t) \\ &= i(c \otimes b_t^\dagger - c^\dagger \otimes b_t) \end{aligned}$$

■ Quantum white noise

$$[b_s, b_t^\dagger] = \delta(s - t)$$

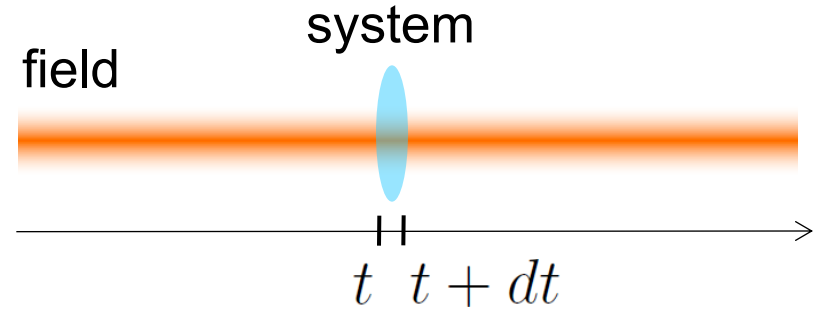
■ Quantum Wiener increment dB_t

Formally, $b_t = \frac{dB_t}{dt}$ and the output prob. of $dB_t + dB_t^\dagger$ is $\mathcal{N}(0, dt)$

$$dB_t^2 = dB_t^{\dagger 2} = dB_t^\dagger dB_t = 0 \quad dB_t dB_t^\dagger = dt$$

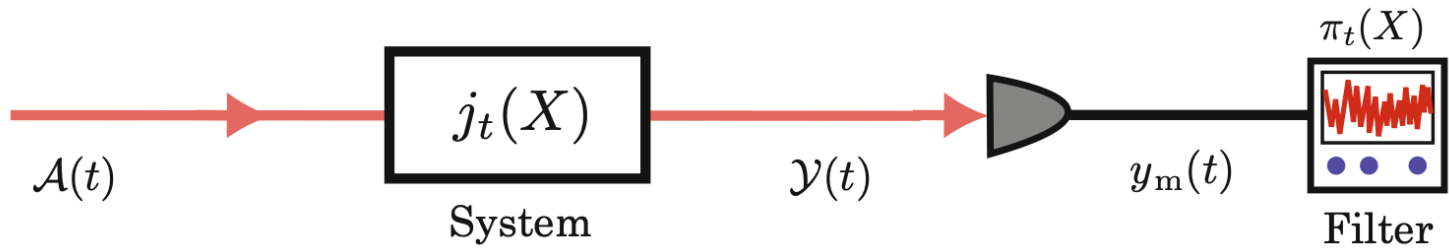
■ Interaction Hamiltonian in $[t, t + dt]$

$$\begin{aligned} H dt &= i(c dB_t^\dagger - c^\dagger dB_t) \\ &= i(c \otimes dB_t^\dagger - c^\dagger \otimes dB_t) \end{aligned}$$



when we measure this
via a homodyne detector





このハミルトニアンで駆動される(system + bathの)ユニタリダイナミクス :

$$dU(t) = \left((-iH - \frac{1}{2}L^*L)dt + Ld\mathcal{A}(t)^* - L^*d\mathcal{A}(t) \right) U(t), \quad U(0) = I,$$

システム物理量の時間発展 :

$$d\mathcal{A}(t)d\mathcal{A}(t)^* = dt$$

$$j_t(X) := U(t)^* X U(t)$$

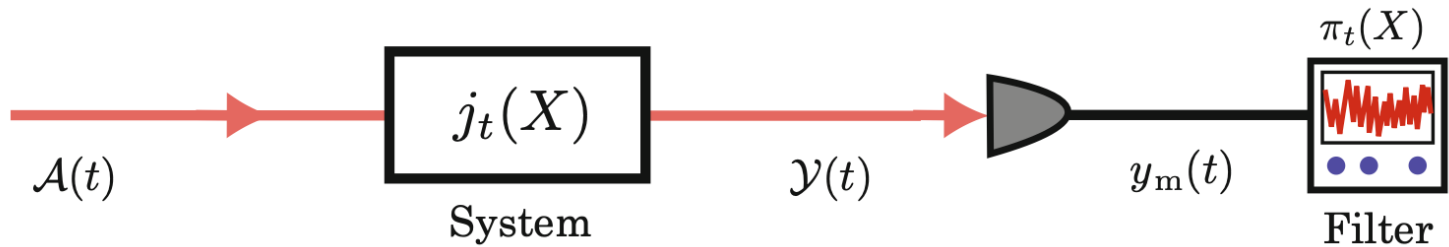
$$\longrightarrow dj_t(X) = j_t(\mathcal{L}X)dt + j_t([L^*, X])d\mathcal{A}(t) + j_t([X, L])d\mathcal{A}(t)^*$$

$$\mathcal{L}X := i[H, X] + L^*XL - \frac{1}{2}L^*LX - \frac{1}{2}XL^*L$$

Bathの物理量の時間発展 :

$$y_m(t) := \mathcal{Y}(t) + \mathcal{Y}(t)^* = U(t)^*(\mathcal{A}(t) + \mathcal{A}(t)^*)U(t)$$

$$\longrightarrow dy_m(t) = j_t(L + L^*)dt + d\mathcal{A}(t) + d\mathcal{A}(t)^*$$



つまり、システム方程式+出力方程式：

$$dj_t(X) = j_t(\mathcal{L}X)dt + j_t([L^*, X])d\mathcal{A}(t) + j_t([X, L])d\mathcal{A}(t)^*$$

$$dy_m(t) = j_t(L + L^*)dt + d\mathcal{A}(t) + d\mathcal{A}(t)^*$$

ポイント：

$$[y_m(s), y_m(t)] = 0, \quad \forall s, t. \quad \sim \text{出力方程式} = \text{時間連続測定過程！}$$

$$\longrightarrow \text{測定値の集合： } \mathcal{Y}_{m,t} = \text{vN}\{y_m(s), 0 \leq s \leq t\}$$

$$[y_m(s), j_t(X)] = 0, \quad \forall s \leq t \quad \sim \text{システム物理量とbath物理量が同時に} \\ \text{対角化可能} = \text{これらの同時分布が存在}$$

\longrightarrow 条件付き平均が定義可能：

$$\pi_t(X) = \arg \min_{Z \in \mathcal{Y}_{m,t}} \mathbb{P}[(j_t(X) - Z)^2] = \mathbb{P}(j_t(X) | \mathcal{Y}_{m,t})$$

$$d\pi_t(X) = \mathbb{H}_1 dt + \mathbb{H}_2 dy_m(t) \quad \text{とおく。} \quad \bar{h}(t) = \exp\left(\int_0^t h(s) dy_m(s) - \frac{1}{2} \int_0^t h(s)^2 ds\right)$$

$$\begin{aligned} d\mathbb{P}(\bar{h}(t)j_t(X)) &= \mathbb{P}\left(d\bar{h}(t)j_t(X) + \bar{h}(t)dj_t(X) + d\bar{h}(t)dj_t(X)\right) \\ &= \mathbb{P}\left(\bar{h}(t)h(t)j_t(L + L^*)dt \cdot j_t(X) + \bar{h}(t)j_t(\mathcal{L}X)dt + \bar{h}(t)h(t)d\mathcal{A}(t) \cdot j_t([X, L])d\mathcal{A}(t)^*\right) \\ &= \mathbb{P}\left(\bar{h}(t)h(t)j_t(LX + L^*X) + \bar{h}(t)j_t(\mathcal{L}X) + \bar{h}(t)h(t)j_t([X, L])\right)dt \\ &= \mathbb{P}\left(\bar{h}(t)h(t)j_t(XL + L^*X) + \bar{h}(t)j_t(\mathcal{L}X)\right)dt, \end{aligned}$$

$$\begin{aligned} \longrightarrow \quad \frac{d}{dt}\mathbb{P}(\bar{h}(t)j_t(X)) &= \mathbb{P}\left[\mathbb{P}\left(\bar{h}(t)h(t)j_t(XL + L^*X) + \bar{h}(t)j_t(\mathcal{L}X) \mid \mathcal{Y}_{m,t}\right)\right] \\ &= \mathbb{P}\left[\bar{h}(t)h(t)\mathbb{P}\left(j_t(XL + L^*X) \mid \mathcal{Y}_{m,t}\right) + \bar{h}(t)\mathbb{P}\left(j_t(\mathcal{L}X) \mid \mathcal{Y}_{m,t}\right)\right] \\ &= \mathbb{P}\left(\bar{h}(t)h(t)\pi_t(XL + L^*X) + \bar{h}(t)\pi_t(\mathcal{L}X)\right). \end{aligned}$$

$$\begin{aligned} d\mathbb{P}(\bar{h}(t)j_t(X)) &= d\mathbb{P}\left[\mathbb{P}\left(\bar{h}(t)j_t(X) \mid \mathcal{Y}_{m,t}\right)\right] = d\mathbb{P}\left(\bar{h}(t)\mathbb{P}(j_t(X) \mid \mathcal{Y}_{m,t})\right) \\ &= d\mathbb{P}\left(\bar{h}(t)\pi_t(X)\right) = \mathbb{P}\left(d\bar{h}(t)\pi_t(X) + \bar{h}(t)d\pi_t(X) + d\bar{h}(t)d\pi_t(X)\right) \\ &= \mathbb{P}\left[\bar{h}(t)h(t)\left(\pi_t(L + L^*)\pi_t(X) + \mathbb{H}_2\right) + \bar{h}(t)\left(\mathbb{H}_1 + \mathbb{H}_2\pi_t(L + L^*)\right)\right]dt \end{aligned}$$

$$\text{ゆゑに} \quad \mathbb{H}_1 + \mathbb{H}_2\pi_t(L + L^*) = \pi_t(\mathcal{L}X) \quad , \quad \pi_t(L + L^*)\pi_t(X) + \mathbb{H}_2 = \pi_t(XL + L^*X)$$

$$\longrightarrow \quad d\pi_t(X) = \pi_t(\mathcal{L}X)dt + (\pi_t(XL + L^*X) - \pi_t(X)\pi_t(L + L^*))\left(dy_m(t) - \pi_t(L + L^*)dt\right)$$

さらに、 $\iota(\pi_t(X)) = \text{Tr}(X\rho(t))$ より条件追き量子状態のダイナミクス：

$$d\rho = \mathcal{L}^*\rho dt + \left(L\rho + \rho L^* - \text{Tr}(L\rho + \rho L^*)\rho\right)\left(dy_{\text{m}}(t) - \text{Tr}(L\rho + \rho L^*)dt\right)$$

$$\mathcal{L}^*\rho := -\iota[H, \rho] + L\rho L^* - \frac{1}{2}L^*L\rho - \frac{1}{2}\rho L^*L$$

測定過程をすべて平均したもの：

$$\bar{\rho} := \mathbf{E}(\rho)$$

$$\frac{d\bar{\rho}}{dt} = \mathcal{L}^*\bar{\rho}dt = -\iota[H, \bar{\rho}] + L\bar{\rho}L^* - \frac{1}{2}L^*L\bar{\rho} - \frac{1}{2}\bar{\rho}L^*L \quad \sim \text{マスター方程式}$$

フィルター方程式は射影測定の時連続板を記述する

$$L = L^* \quad H = 0$$

$$\rightarrow dj_t(L) = j_t\left(L^*LL - \frac{1}{2}LL^*L - \frac{1}{2}L^*LL\right)dt + j_t([L^*, L])d\mathcal{A} + j_t([L, L])d\mathcal{A}^* = 0.$$

ゆえに $j_t(L) = j_0(L) = L$ つまり、システム物理量は時間変化しない。

一方、 L の推定値が従うフィルター方程式：

$$d\pi_t(L) = 2[\pi_t(L^2) - \pi_t(L)^2]d\nu = 2\langle (L - \langle L \rangle_c)^2 \rangle_c d\nu.$$

システム物理量 L は変化していないが、 L に関する情報が取れている
～非破壊測定

L の揺らぎ $\langle \Delta L^2 \rangle_c = \langle (L - \langle L \rangle_c)^2 \rangle_c$ はどうなる？ $\langle \cdot \rangle_c := \pi_t(\cdot)$

$$\frac{d}{dt} \mathbb{P}[\langle \Delta L^2 \rangle_c] = -4\mathbb{P}[\langle \Delta L^2 \rangle_c^2] \quad \text{ゆえに} \quad \mathbb{P}[\langle \Delta L^2 \rangle_c^2] \rightarrow 0$$

つまり、状態は L の固有状態の一つに収束する。射影測定！

量子カルマンフィルタ + LQGフィードバック制御

Entanglement-assisted quantum feedback control, Yamamoto and Mikami, Quantum Information Processing, 2017

量子機械系は線形モデルで良く近似でき、このとき、推定系はカルマンフィルタと全く同じになる！

システム = 線形量子系：

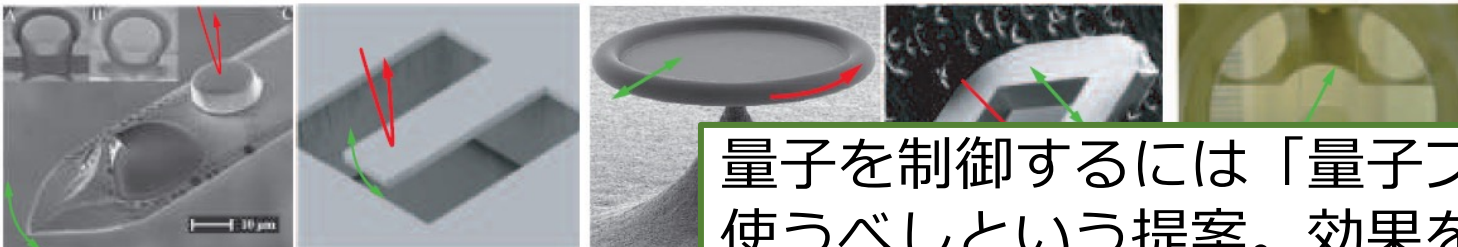
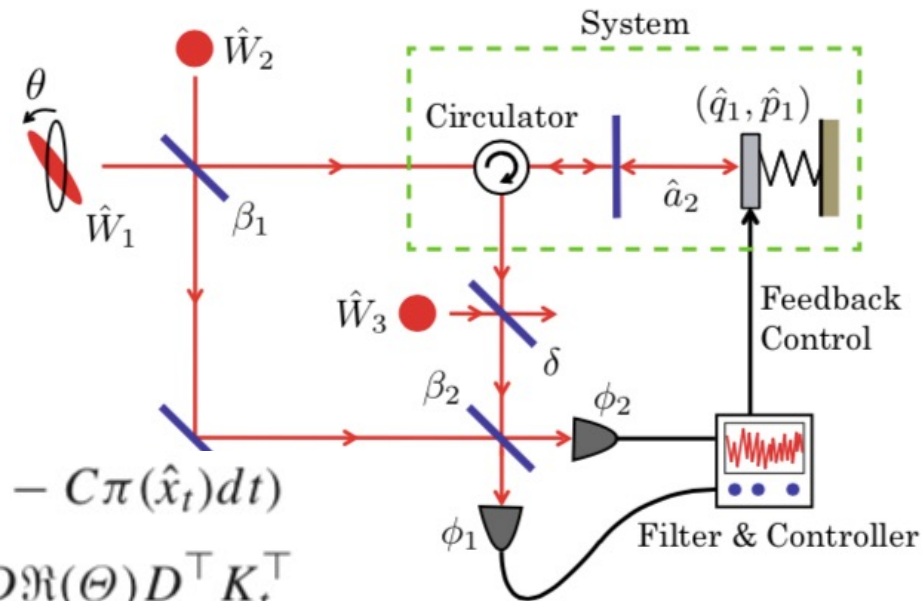
$$d\hat{x}_t = A\hat{x}_t dt + Fu_t dt + Bd\hat{W}_t$$

$$dy_t = C\hat{x}_t dt + Dd\hat{W}_t$$

推定系 = カルマンフィルタ：

$$d\pi(\hat{x}_t) = A\pi(\hat{x}_t)dt + Fu_t dt + K_t(dy_t - C\pi(\hat{x}_t)dt)$$

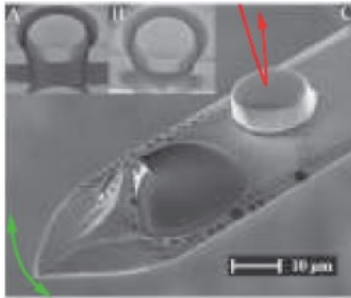
$$\dot{V}_t = AV_t + V_t A^\top + B\Re(\Theta)B^\top - K_t D\Re(\Theta)D^\top K_t^\top$$



量子を制御するには「量子プローブ」を使うべしという提案。効果を数値確認。

量子フィードバックできると何が嬉しいのか？

Conditional mechanical squeezing of a macroscopic pendulum near quantum regimes, Matsumoto and Yamamoto, arXiv



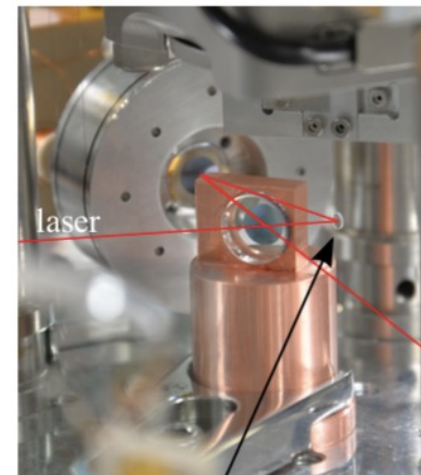
← みたいなナノデバイスではなく、数mgの巨視的デバイスを精密制御して、量子効果を発現させたい！

→ 重力の量子性、さらにはマクロ量子状態由来の重力の解明！

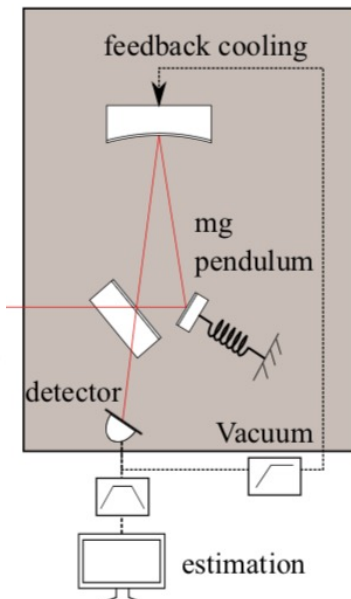
mg スケール振り子の入出力方程式：

$$\begin{aligned}\dot{q} &= \omega_m p, \\ \dot{p} &= -\omega_m q - \gamma_m p + \sqrt{2\gamma_m} p_{\text{in}} - \frac{4g_m}{\sqrt{\kappa}} x_{\text{in}} + \frac{8g_m\delta}{\sqrt{\kappa}} y_{\text{in}} \\ X &= -\frac{8g_m\delta\sqrt{\eta}}{\sqrt{\kappa}} q - \sqrt{\eta} x_{\text{in}} + 4\delta\sqrt{\eta} y_{\text{in}}.\end{aligned}$$

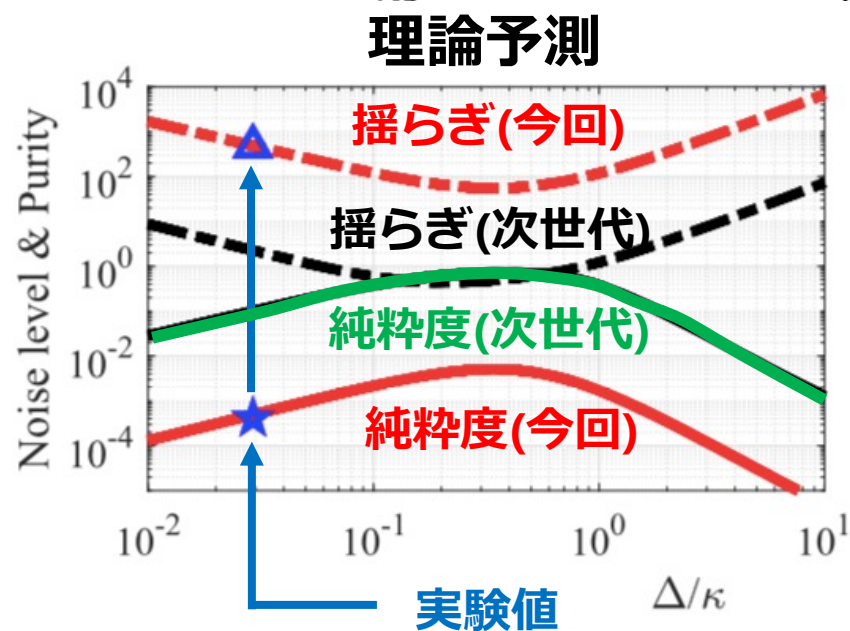
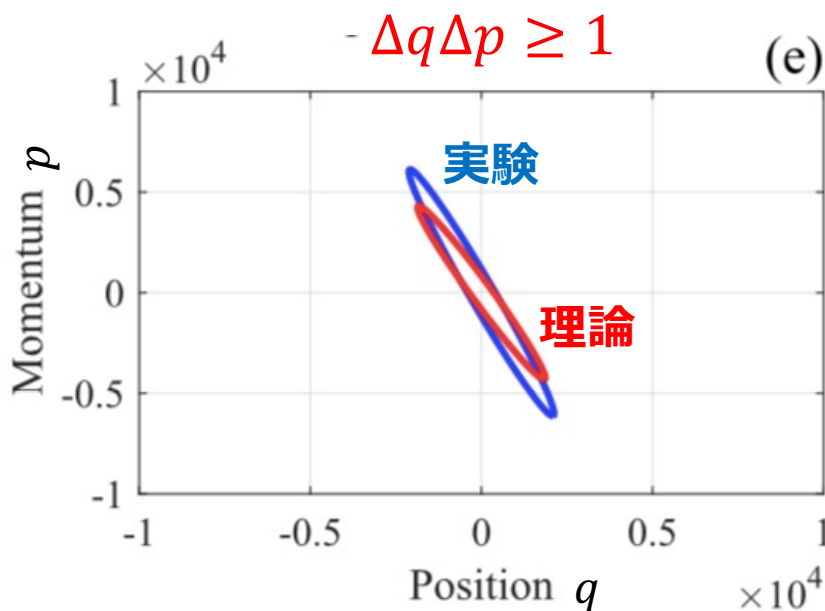
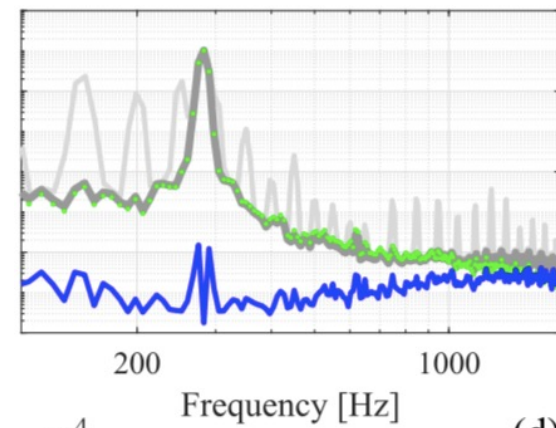
ウィーナーフィルタを用いて
条件付き共分散行列を計算、
理論と実験の整合性を検証



mg pendulum



パワースペクトルから、位置と運動量の条件付き共分散行列を計算。揺らぎの大きさおよび「量子純粋度」を算出。



作成済みの超低ロスの振子(次世代振子)を用いると、揺らぎの大きさ < 1 (量子スクイズド状態) が実現可能と予想！

ご清聴ありがとうございました。