String Theory and Blackholes - I, II

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General Relativity + Quantum Mechanics

2 pillars of modern physics

GR → describes well large scale structure of the universe.
tested to 1/10000 in expts that measure the gravitational red shift. Hulse-Taylor pulsar....

QM → basic theory of the microscopic world, describes atomic, molecular, nuclear and HEP phenomena from atomic dimensions $\sim 1\text{Å} \sim 10^{-8}$ cm to $\gtrsim 10^{-16}$ cm (Precision tests of electroweak theory).

But

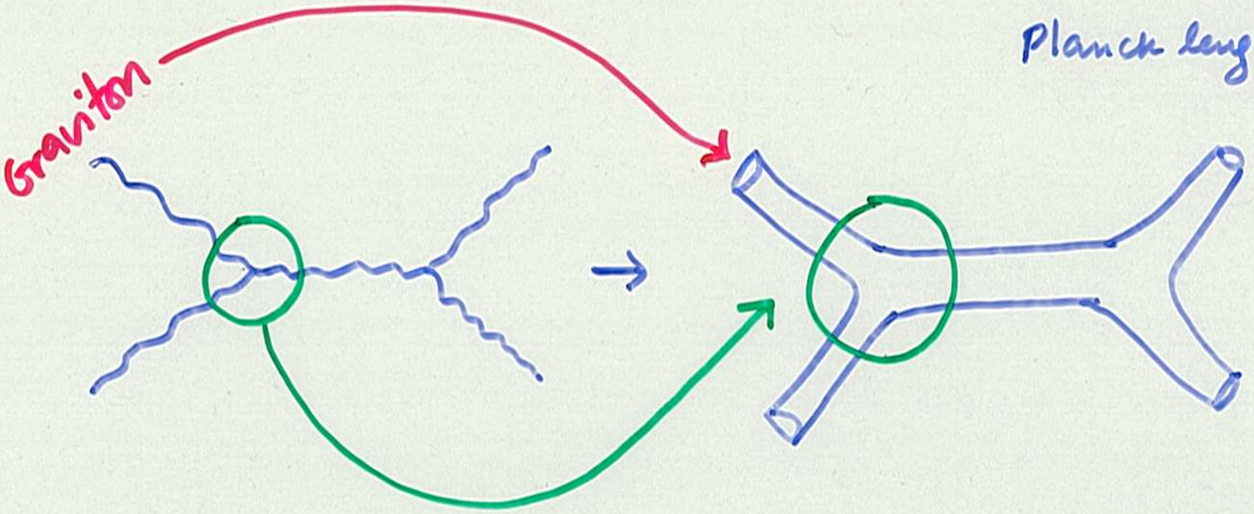
QM applied to GR

leads to fundamental difficulties.

(A) Short distance infinities

- Short distance or high energy behavior of GR is not renormalizable
- strength of the interaction grows at high energy $\propto \frac{E\, l_P}{\hbar c}$ $\quad l_P = \left(\frac{G_N}{c^3}\hbar\right)^{1/2}$

Planck length



fuzz $\sim l_s$ (string length)

$G_N \sim g_{str}^2\, l_s^2$

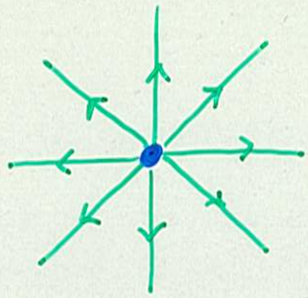
Interaction is smeared over a region of size l_s, the string scale.

l_P is a derived concept.

(B) Quantum Mechanics + Blackholes $\Rightarrow$ unitarity problem

Blackholes ... $\boxed{\text{classical}}$

Electromagnetism :



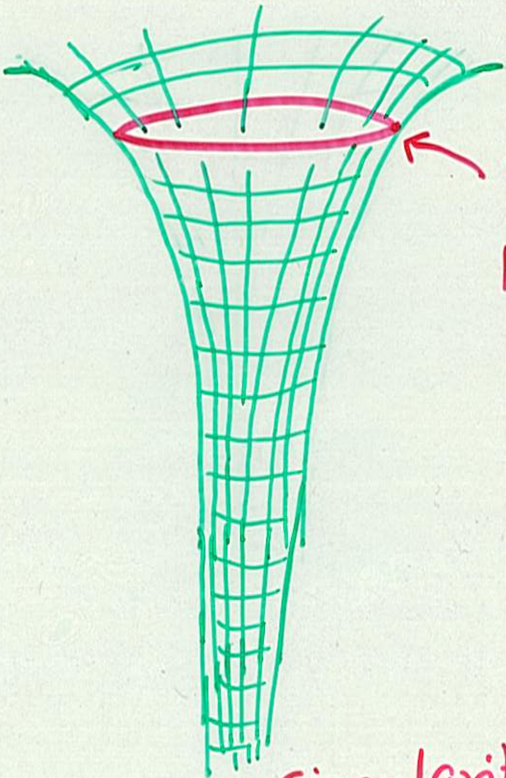
Coulomb :

$$A_0 = \frac{e}{r}, \quad A_i = 0$$

$$\vec{E} = \vec{\nabla} A_0 = \frac{e}{r^2}$$

General Relativity : Schwarzschild :

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 c^2 + \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)} + r^2 d\Omega^2$$



$r_h = \frac{2GM}{c^2}$

horizon

$A_h = 4\pi r_h^2$

singularity

- $r \gg \frac{2GM}{c^2}$

$$\frac{c^2}{2}(g_{00} - 1) = \phi = -\frac{GM}{r}$$

Newtonian gravity !

- $r \leq \frac{2GM}{c^2}$

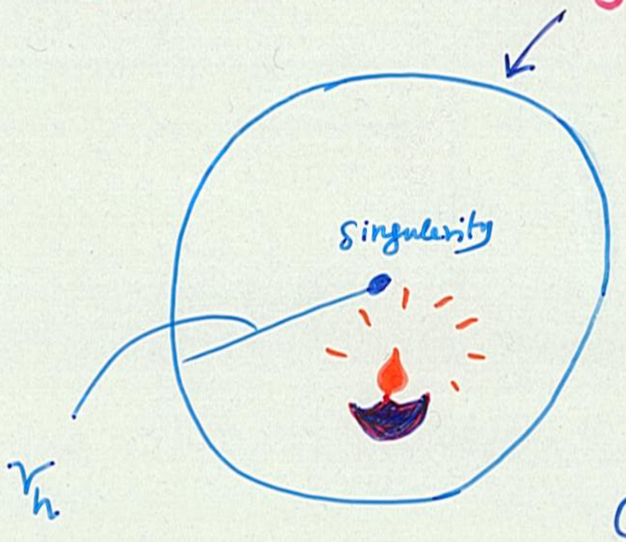
Blackhole

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

$$\kappa^2 = G$$

Black hole - Horizon

BH Thermodynamics



Anything that falls inside (including light) dissapears.

Light cannot exit from behind the horizon of a black hole

Bekenstein noted that the existence of a bh violates the second law of thermodynamics.

S  $\neq 0$ outside bh

S  $= 0$ inside bh

$$S_{final} < S_{initial} \quad !$$

<u>unless</u> the bh itself has entropy

$$S_{bh} = a\, A_h \;,\quad A_h = 4\pi r_h^2 = 16\pi\, G^2 M^2$$

$$\left[\; S_{M+\delta M} > S_M \;\right] \Rightarrow \frac{\partial S_{bh}}{\partial M} = \frac{1}{kT} \neq 0$$

Hawking Radiation : (Enter Quantum Mechanics)

In quantum theory blackholes are blackbodies — they radiate.

If there is absorption then there has to be emission: $\langle i | H_{int} | f \rangle = \langle f | H_{int} | i \rangle^*$

Large mass blackhole :

$$T = \frac{\hbar c}{8\pi M G} \approx 10^{-8} K \quad (M \gtrsim M_\odot)$$

1st law of thermodynamics $\Rightarrow dM = T dS$

$$S = \frac{A_h}{4 l_p^2} \qquad l_p^2 = \frac{\hbar G}{c^3} \approx 10^{-66} cm^2$$

Bekenstein-Hawking formula contains all 3 : $\hbar, c, G$.

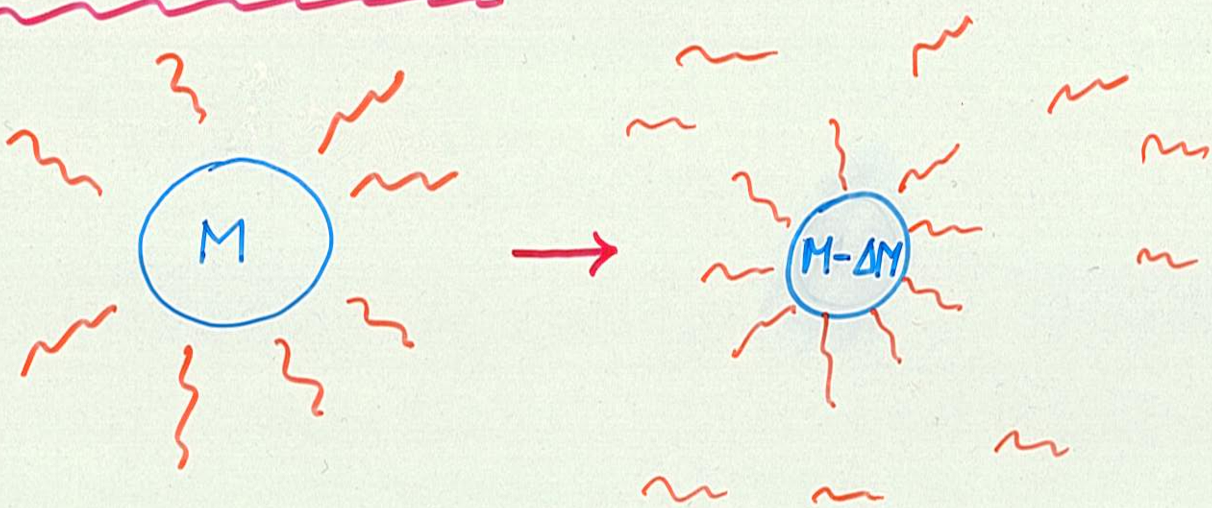
Hawking radiation rate :

$$\Gamma(\vec{k}) = \frac{d\vec{k}}{(2\pi)^d} \frac{\sigma_{abs}(k)}{(e^{k/T} - 1)}$$

absorption cross section

decay rate of blackhole

- **Information Puzzle:**



EVAPORATION

Hawking radiation is THERMAL

Pure state $\longrightarrow$ Mixed state.

$$\rho = \sum_i |i\rangle\langle i| \quad \longrightarrow \quad \rho = \sum_i e^{-\frac{E_i}{T}} |i\rangle\langle i|$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\rho^2 = \rho \quad \longrightarrow \quad \rho^2 \neq \rho$$

not allowed
by
Quantum Mechanics!

This is called the information paradox.

- **Bh singularity** $\quad |R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}| \sim \frac{1}{M^2 r^6}$

A short tour of String theory

The basic idea is very simple:

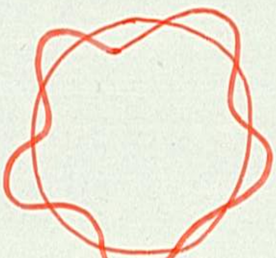
LAWS OF MOTION ARE FORMULATED IN TERMS OF EXTENDED OBJECTS

Historically:

1 - dim. extended object "a string".

2 types

closed



Vibrational modes are interpreted as 'particle states'

open

Massless spectrum

graviton, ...

↓

General Relativity, $G_N \propto g_s^2 l_s^8$

gauge field .....

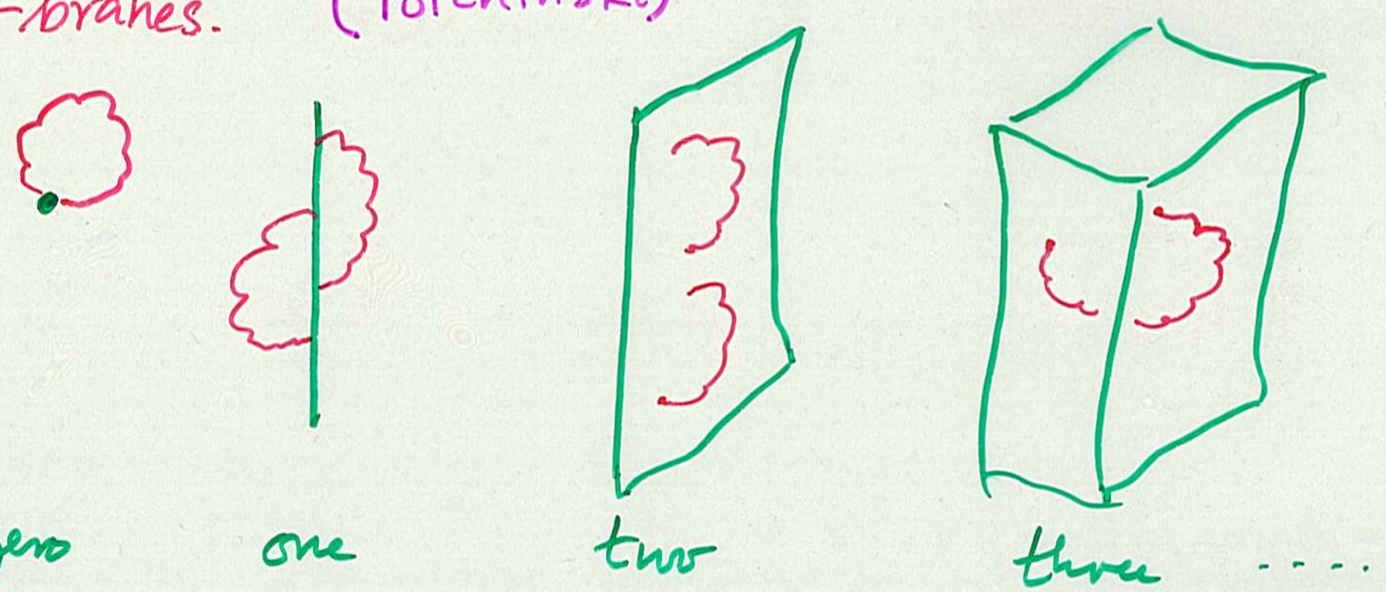
↓

non-Abelian gauge theory, $g_{YM}^2 \propto g_s$

These facts are perhaps the most important reasons why we believe in String theory

There is more ..... D - Branes

In the closed superstring there are higher tensor gauge fields $C_{\mu\nu\rho\ldots}$ which are sourced by other extended objects ... called D-branes. (Polchinski)

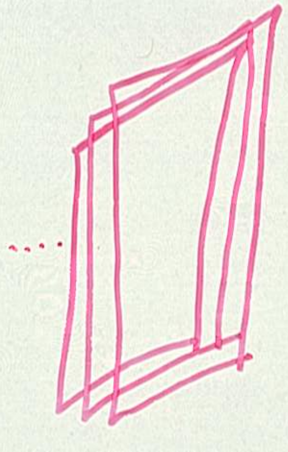


Collective excitations are described by open strings (gauge fields ....)

$$M \propto \frac{1}{g_s} \Rightarrow V_{gravity} = \frac{g_s^2}{g_s} \sim g_s$$

Weak gravitational field as $g_s \to 0$,

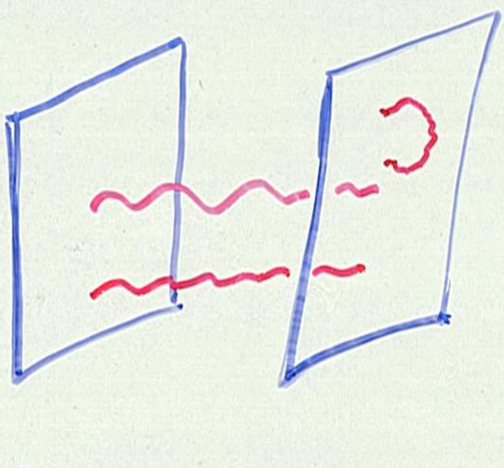
but putting a large number 'N' on top of each other $\Rightarrow V_{grav.} \sim (N g_s)$!!



'N' of them produce a sizable field $\sim (g_s N)$

It is these objects which are the 'new degrees' of freedom ('New' because of duality symmetries of string theory) ....

How do D-branes interact?



by exchange of open strings.

D-branes $\sim$ 'quarks'

open strings $\sim$ gluons

Bound states of a large number of D-branes can model a blackhole if $N g_s \gg 1$

Solution in a Model System

Type IIB superstring theory also has another set of 'higher' gauge fields:

$$C^{(2)} = C^{(2)}_{\mu\nu}\, dx^\mu \wedge dx^\nu$$ and the field

strength

$$F^{(3)} = dC^{(2)} = F^{(3)}_{\mu\nu\lambda}\, dx^\mu \wedge dx^\nu \wedge dx^\lambda$$

also $d = 10$.

Low energy (long wavelength) effective lagrangian:

$$S \sim \frac{1}{G_N^{(10)}} \int d^{10}x\, \sqrt{-g}\left(R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12} e^{\phi} (F^{(3)})^2 \right)$$

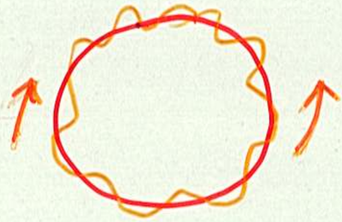
main point is that there is a (nearly susy) black hole sol. in 4+1 dim. while the other 5 dimensions are the torii $S^1 \times T^4$.

$$\underbrace{1\ 2\ 3\ 4\ 0}_{BH}\ \ \underset{S^1}{\overset{5}{\uparrow}}\ \ \underbrace{6\ 7\ 8\ 9}_{T4}$$

This black hole has a mass, 2 charges corresponding to $\underset{Q_5}{F^{(3)}}$ and $\underset{Q_1}{{}^*F^{(3)}}$

The sol. also has left and right moving gravitational waves on the circle along x_5.

$$N_L = N + n \,, \quad N_R = n \,, \quad n \ll N$$



x_5

The black hole has a finite temperature and entropy, and it Hawking radiates.

The main message here is that the low energy effective lagrangian of IIB string theory has a black hole characterized by 4 charges : $Q_1, Q_5, N+n, N$.

It has all the characteristics of a Schwarzschild bh with charges, except 1) its specific heat is +ve.

$$\left(\text{for schwarzschild} : \; M \propto \frac{1}{T} \;\; \therefore \; \frac{\partial M}{\partial T} < 0 \right)$$

2) Asymptotically the space time is

$$AdS_3 \times S^3 \times T^4$$

The D1-D5 system

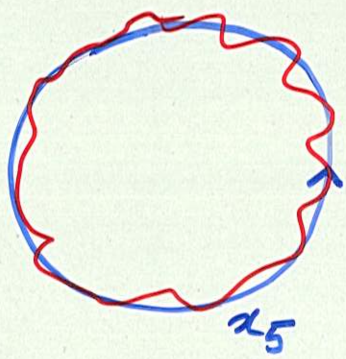
Q_1 D1 branes on S^1 along x^5

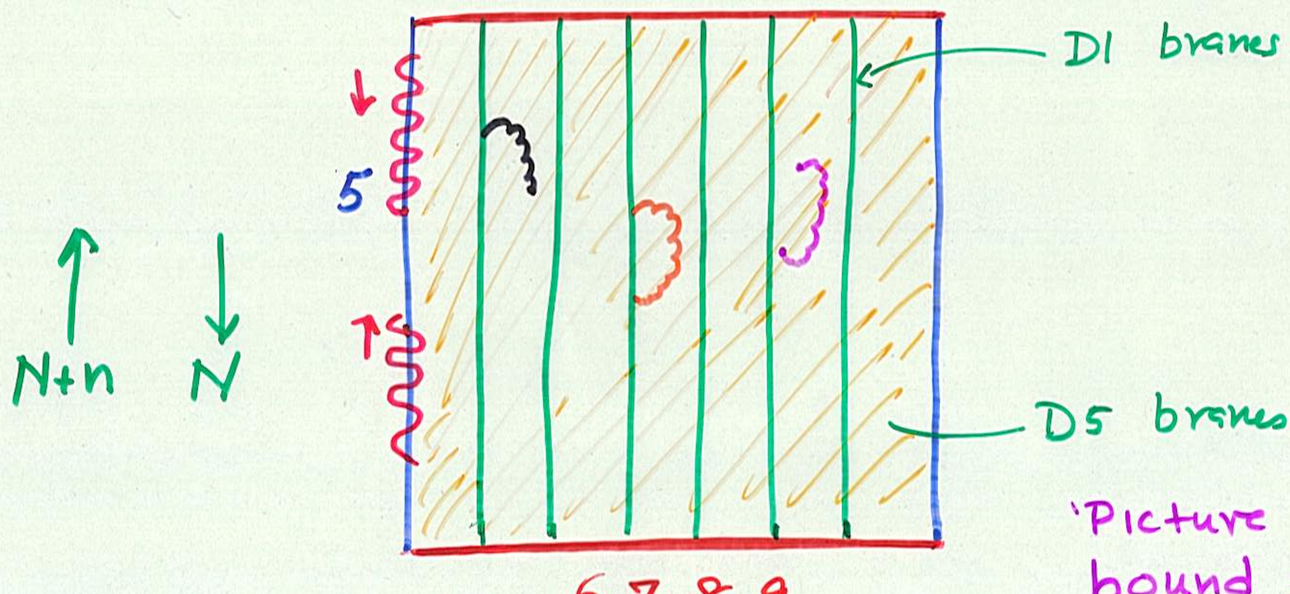
Q_5 D5 branes on $S^1 \times T^4$ $(x^5, x^6, x^7, x^8, x^9)$

$$1 \quad 2 \quad 3 \quad 4 \quad 0 \quad \underbrace{\overset{\text{D1}}{\overset{\downarrow}{5}} \quad 6 \quad 7 \quad 8 \quad 9}_{\text{D5}}$$

choose $R \gg l_s = \sqrt{\alpha'}$

Introduce left + right moving momentum modes on S^5 : $N+n$ + N





'Picture of bound state'.

The constituent model is a complicated interacting system of D1 + D5 branes.

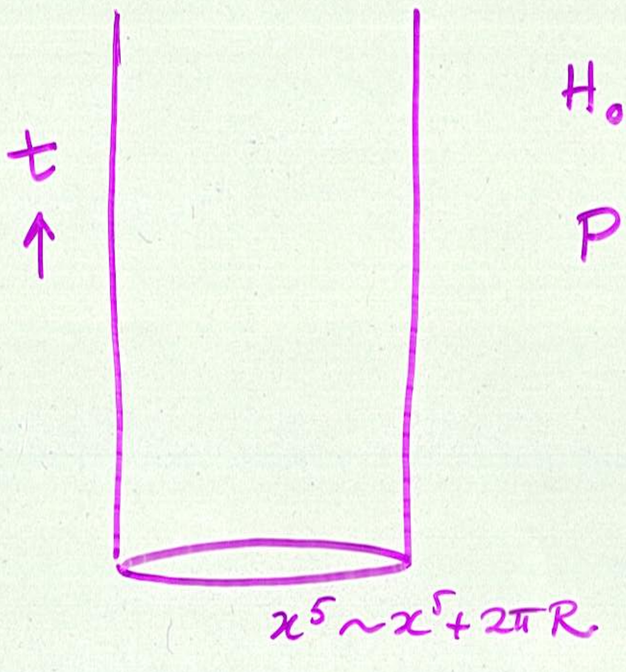
$\Downarrow$

low energy dynamics (collective excitations of the bd. state)

described by a supersymmetric conformal field theory in 1+1 dim.

Central charge $C = 6Q_1Q_5 = 4Q_1Q_5 + \frac{4}{2}Q_1Q_5$

$4Q_1Q_5$ is the number of independent open strings between D1 + D5



$$H_0 = \frac{N}{R} + \frac{2n}{R} \equiv -i\frac{\partial}{\partial t}$$

$$P = \frac{N}{R} \equiv -i\frac{\partial}{\partial x^5}$$

Brane dynamics ... contd.

Effective field theory of low lying modes is a superconformal field theory in 2-dim.

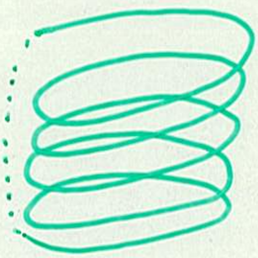
$(x_5, t) \equiv (\sigma, t)$.

The field $x_A^i(\sigma, t)$ takes values in the symmetric product $\underbrace{T^4 \times T^4 \times \cdots \times T^4}_{Q_1 Q_5}$

$Q_1 Q_5$ times.

Maximum entropy state corresponds to longest cycle of the permutation group $S(Q_1 Q_5)$.

$$\tilde{X}_A^i\,(\sigma + 2\pi(A-1)) \equiv x_A^i\,(\sigma)$$



At each winding you go up by 1 thread and identify first + last threads.

$$\tilde{X}^i(\sigma, t) = \frac{1}{\sqrt{4\pi}} \sum_{n>0} \left(\frac{a_n^i}{\sqrt{n}}\, e^{in(\sigma - t)/Q_1 Q_5} + \tilde{a}_n^i\, e^{-in(\sigma + t)/Q_1 Q_5} \right) + \text{c.c.}$$

$\sigma \to \sigma + 2\pi \quad \Rightarrow \quad a_n^i \longrightarrow e^{in/Q_1 Q_5} a_n^i$, etc.

Blackhole state:

$$|BH\rangle = \prod_{n=0}^{\infty} \prod_{i=1}^{4} C(n,i) \left(a_n^{i\,+}\right)^{N^i_{n,L}} \left(\tilde{a}_n^{i\,+}\right)^{N^i_{n,R}} |0\rangle$$

$$a_n^i |0\rangle = \tilde{a}_n^i |0\rangle = 0 \quad , \quad C(n,i) \ni \langle BH | BH \rangle = 1$$

$|BH\rangle$ is bose symmetric if

$$\frac{n}{Q_1 Q_5} = \text{integer} \equiv \tilde{n}$$

$$\tilde{N}_L = \sum_{n,i} n N^i_{nL} = Q_1 Q_5 \sum_{\tilde{n},i} \tilde{n} N^i_{\tilde{n}L} = Q_1 Q_5 N_L$$

Sim. for $\tilde{N}_R = Q_1 Q_5 N_R$.

Hardy-Ramanujan method can evaluate the number of states for fixed $\tilde{N}_L$ + $\tilde{N}_R$:

$$\Rightarrow \quad \Omega = e^{2\pi\left(\sqrt{\tilde{N}_L} + \sqrt{\tilde{N}_R}\right)} \quad , \tilde{N}_{L,R} \gg 1$$

$$\Rightarrow S = \ln \Omega = 2\pi \sqrt{Q_1 Q_5} \left(\sqrt{N_L} + \sqrt{N_R}\right) \; !$$

a new insight to S-V result

Hence: the number of states with fixed value of $H = \frac{N}{R} + \frac{2n}{R}$ and $P = \frac{N}{R}$

$$\Omega \sim e^{2\pi\sqrt{Q_1 Q_5}\left(\sqrt{N+n} + \sqrt{n}\right)}$$

for $Q_1, Q_5, N \gg 1$.

$$\Rightarrow S = \ln \Omega = 2\pi\sqrt{Q_1 Q_5}\left(\sqrt{N+n} + \sqrt{n}\right)$$

Strominger-Vafa (agrees with GR!)

$$\Rightarrow \frac{1}{T_H} = \beta_H = \frac{\partial S}{\partial\left(\frac{2n}{R}\right)} = \frac{R}{2}\frac{\partial S}{\partial n} = \pi R\sqrt{\frac{Q_1 Q_5}{n}}$$

(agrees with GR!).

note: $N \gg n$

$$S = 2\pi\sqrt{Q_1 Q_5 N} + 2\pi\sqrt{Q_1 Q_5 n}$$

using $T_H^2 \sim n$

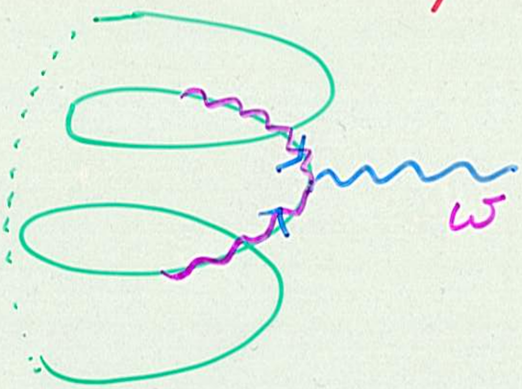
$\Delta S \sim T_H$ characteristic of 1-dim. gas.

Hawking radiation:

Interaction Lagrangian:

$$\mu \int d\sigma dt \; h^{89}(\sigma,t) \sum_{A=1}^{Q_1Q_5} \partial X^8 \bar{\partial} X^9$$

determined to be $\mu = 1$



E , P

$\left(\frac{\omega}{2}, -\frac{\omega}{2}\right)$ → ω

$\left(\frac{\omega}{2}, \frac{\omega}{2}\right)$

One can calculate microstate → microstate

S-matrix:

$$S_{fi} = \frac{\sqrt{2}\,\kappa_5\, \omega_1 \omega_2 \tilde{R} \; \delta_{n_1,n_2} \; 2\pi\delta(\omega-\omega_1-\omega_2)}{\sqrt{\omega_1 \tilde{R}\; \omega_2 \tilde{R}\; \omega V_4}}$$

$$\times \sqrt{N^8_{L,n_1}} \sqrt{N^9_{R,n_2}} \qquad , \tilde{R} = RQ_1Q_5$$

Quantum Statistical Mechanics:

$$P_{Abs} = \frac{1}{\Omega_I} \sum_{if} |S_{fi}|^2$$

$$P_{emis} = \frac{1}{\Omega_F} \sum_{if} |S_{fi}|^2$$

$$P_{Abs} - P_{emis} \equiv \sigma_{abs}(\omega) \longleftarrow$$ classical absorption cross section.

$$\| $$

$$2\pi^2 r_1^2 r_5^2 \frac{\pi R}{2} \frac{\rho(T_H)}{\rho(2T_L)\rho(2T_R)}$$

$\downarrow$

exactly matches the GR calculation of Hawking's formula !

$$\rho(x) = (e^{R/x} - 1)$$

$$T_{L,R} = \frac{\sqrt{N_{L,R}}}{\pi R Q_1 Q_5}$$

Detailed account of this program (including our contribution) in

Justin Raj David, Gautam Mandal + SRW
Phys. Rpts. Vol. 369, #6, 2002.

Collaborators: A. Dhar, F. Hassan, G. Mandal, Justin David in various combinations