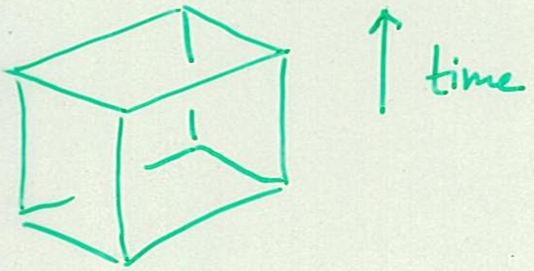
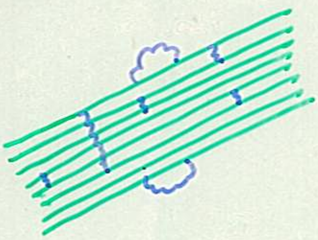
Holography: AdS / CFT correspondence

• A D3 brane sweeps out a 3+1 dim. space-time



• A stack of coincident D3 branes interact by the exchange of open strings



N

D3 branes

[massless mode is a non-Abelian gauge field with N^2 components] $(A_\mu)_{ij}$

Conformal invariant point in N D3 brane dynamics is described by

U(N) gauge theory in 3+1 dim. with

$\mathcal{N}=4$ supersymmetries

Conformal sym. SO(2,4)
R-sym SO(6).

Dual (closed string) description:

A stack of N coincident D3 branes in 9+1 dim. are a source of a 'geometry' and a flux field.

Flux field is a 5-form F_5,

$$\int_{S^5} F_5 = N$$

Geometry is $AdS_5 \times S^5$ matching the conformal invariance of the D3 brane dynamics

Isometries of $AdS_5 \times S^5$: $SO(2,4) \times SO(6)$

AdS/CFT correspondence:

Type IIB string theory in $AdS_5 \times S^5$ is dual to $SU(N)$ Yang-Mills theory in 3+1 dim. with $N=4$ supersymmetry.

$$\left(\frac{R}{l_s}\right)^4 = g_{YM}^2 N \quad , \quad g_{YM}^2 = g_s$$

AdS_5 (Anti-de Sitter space in 5-dim.)

$$R_{ij} = \Lambda g_{ij} \,, \quad \Lambda = -\frac{4}{b^2} < 0 \qquad \text{'} b \equiv R \text{'}$$

$$ds^2 = -V(r)\,dt^2 + V^{-1}(r)\,dr^2 + r^2 d\Omega^2_{(S^3)}$$

$$V(r) = 1 + \frac{r^2}{b^2}$$

- Time like boundary at $r = \infty$

$$ds^2 \xrightarrow[r \gg b]{} r^2\left(-b^{-2}dt^2 + d\Omega^2\right)$$

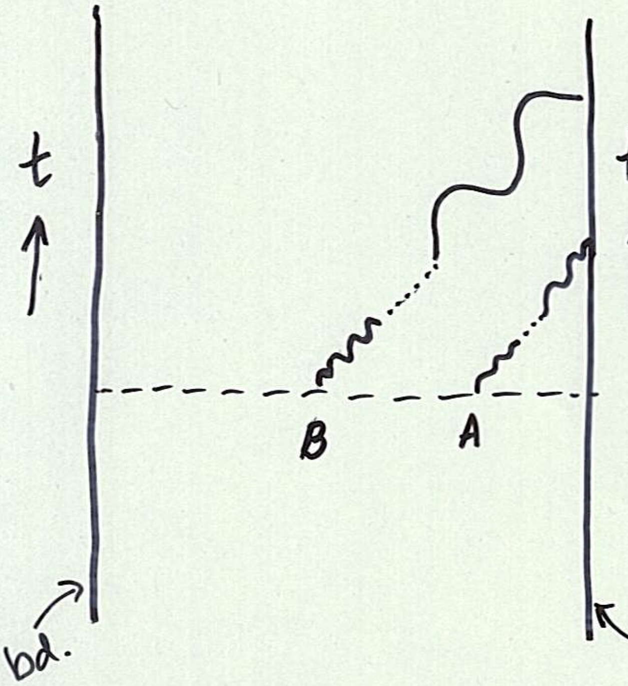
Gauge theory spacetime

conformally equivalent to 4-dim. Minkowski space time.

- Gravitational potential:

$$\sqrt{G^{00}} = \sqrt{1 + \frac{r^2}{b^2}} \approx \frac{r}{b}$$

increases for $r \gg b$ $\Rightarrow$ red shift



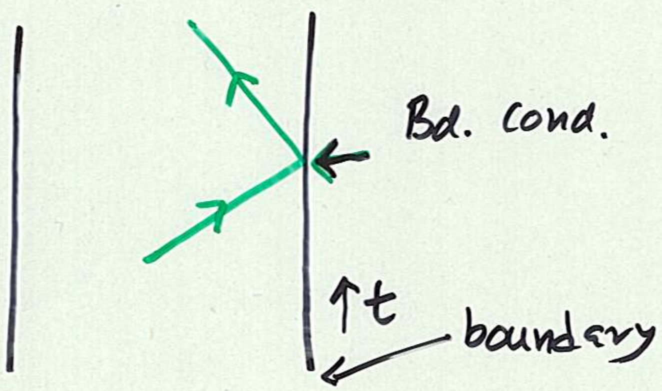
photons $\lambda_B > \lambda_A$

Information about 'B' spread out over a larger radius, than for 'A' UV/IR

extra dim. $\Leftrightarrow$ scale on the boundary

In this space time one needs to specify boundary conditions at $r = \infty$.

Time dependent bd. conditions can cause emission and absorption of signals at the boundary to which the bulk theory responds.



(Witten)

$$\phi(r,t,x) = \int dx' \, K(r,t;x,x') \, \phi(r=\infty, t, x')$$

bulk field

boundary condition

This is the holographic correspondence.

e.g. if $\phi = T_{\mu\nu}$ (Energy momentum tensor)

then we get components of $g_{\mu\nu}(r,t,x)$

AdS$_5$ Blackholes (review)

$$ds^2 = -V(r)\,dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega_3^2$$

$$V(r) = \frac{r^2}{R^2} + 1 - \frac{\omega_4 M}{r^2} \quad , \quad \omega_4 = \frac{16\pi}{3\Omega_3}\cdot G_N$$

($M=0$ is AdS$_5$)

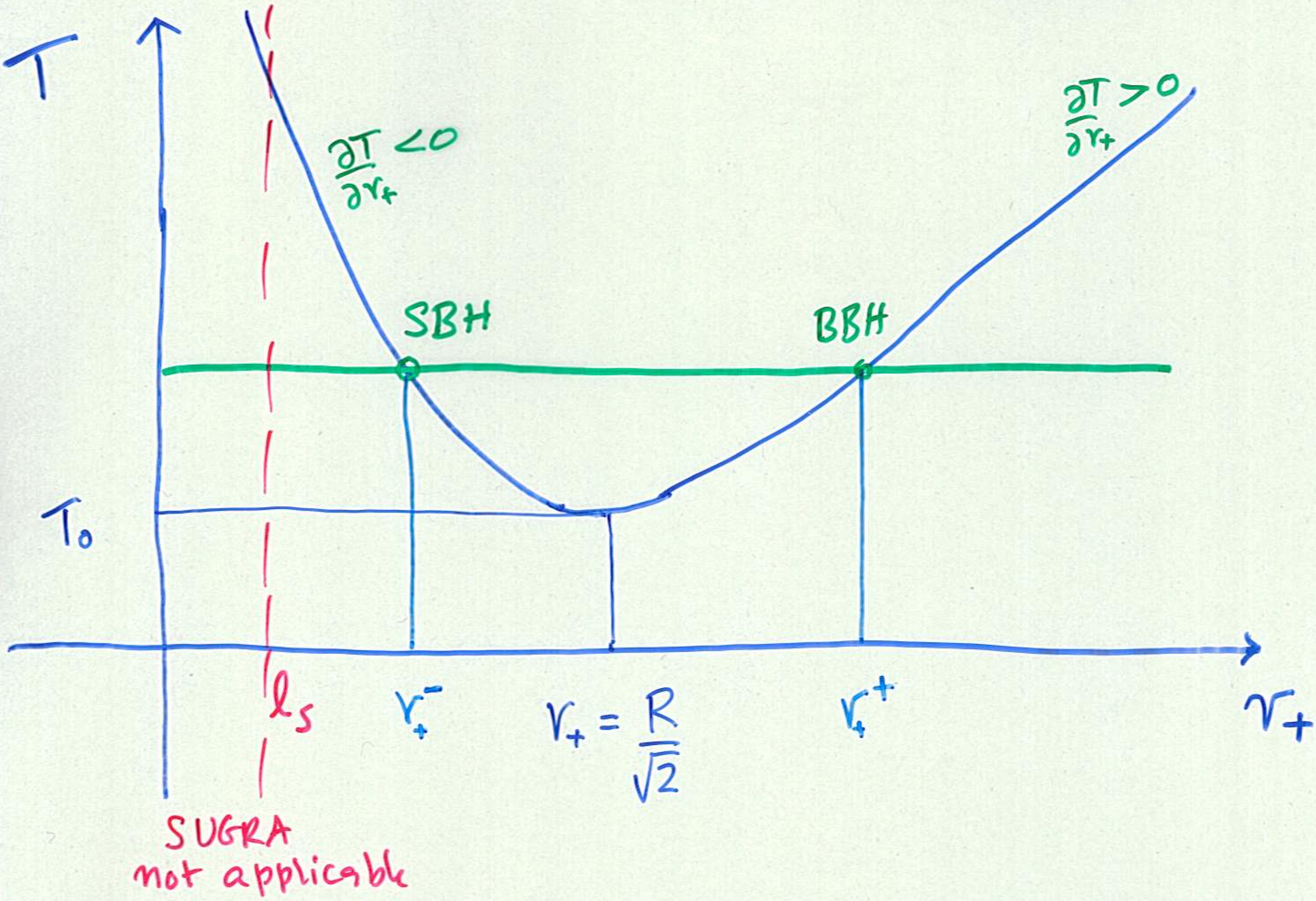
Horizon:

$$V(r)=0 \Rightarrow \frac{r_+^2}{R^2} + 1 - \frac{\omega_4 M}{r_+^2} = 0$$

$$\Rightarrow M = \frac{r_+^2}{\omega_4}\left(1 + \frac{r_+^2}{R^2}\right)$$

Euclidean $t \to it$ (no conical singularity):

$$\Rightarrow \frac{1}{T} = \beta = \frac{2\pi R^2 r_+}{2r_+^2 + R^2}$$



$$T_0 = \frac{\sqrt{2}}{\pi}\frac{1}{R} \qquad (T_0 R \sim 1) \qquad R \gg l_s$$

stringlength

$$\left(\frac{r_+^{\pm}}{R}\right)_{\pm} = \frac{1}{\sqrt{2}}\left(\frac{T}{T_0} \pm \sqrt{\left(\frac{T}{T_0}\right)^2 - 1}\right)$$

$$M_{\pm} = \frac{R^2}{\omega_4}\left(\frac{r_+^{\pm}}{R}\right)_{\pm}^2\left(1 + \left(\frac{r_+^{\pm}}{R}\right)_{\pm}^2\right)$$

Euclidean Action:

$$I = \frac{r_+^3}{G_N} \frac{\omega_3}{8} \frac{(R^2 - r_+^2)}{2R^2 + r_+^2}$$

BBH : $I_+ \equiv I(r_+^+)$

SBH : $I_- \equiv I(r_+^-)$

I_+ can be +ve, 0, -ve

$I_- > 0 \qquad (r_+^- < R)$

At $\quad r_+^+ = R \;,\; I_+ = 0 \quad$ (Hawking-Page)

$\quad r_+^+ > R \;,\; I_+ < 0 \quad (I_+ < I_{AdS} = 0)$

$$T_{HP} = \sqrt{\frac{9}{8}}\, T_0$$

At $T = T_{HP} = \sqrt{\frac{9}{8}}\, T_0 \sim \frac{1}{R}$

there is a first order phase transition.

$\Delta F = M$ for large M.

Stability properties:

AdS_5 + BBH are local minima

SBH is local maximum

(one -ve eigenvalue)

$\searrow$

Gregory-Laflamme

of SUGRA action.

(Euclidean)

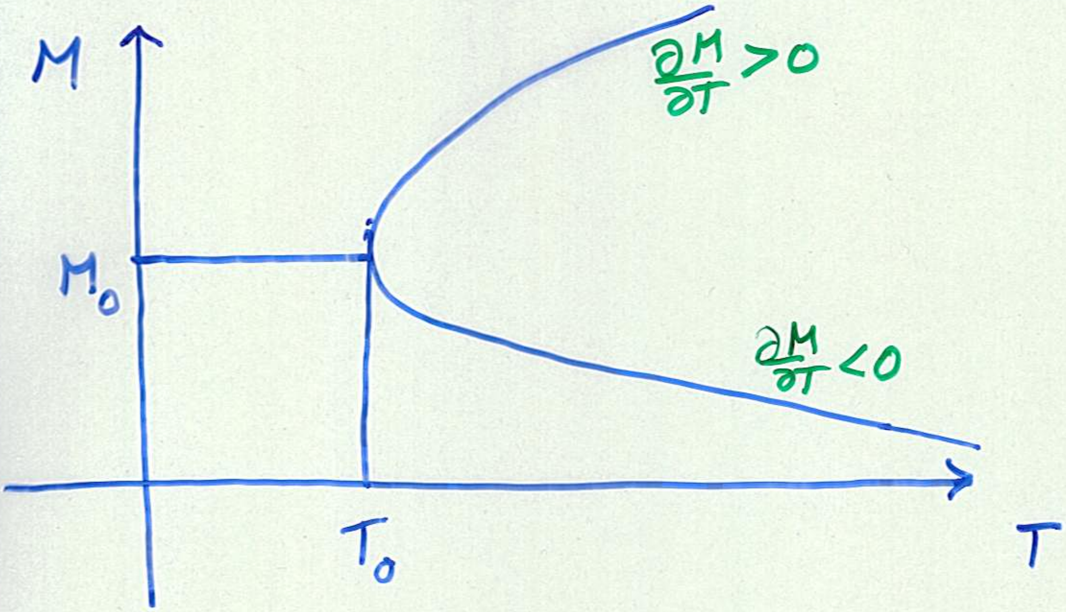
NOTE:

- The temperatures we have been discussing are 'low':

$$T \sim \frac{1}{R} \quad \leftrightarrow \quad TR \sim 1$$

- as $(r_+)_- \to l_s$, $T \sim \frac{1}{l_s}$ (high temp.).

failure of SUGRA



(digression)

$$M_0 = \frac{3}{4} \frac{R^2}{\omega_4} \qquad t \equiv \frac{T}{T_0}$$

$$\frac{\partial M}{\partial t} \sim \frac{1}{\sqrt{t-1}} \quad \text{near } t = 1^+$$

$$\Rightarrow \gamma = -\frac{1}{2}$$

Entropy: $S \sim \dfrac{(r_+)_\pm^3}{G_N}$

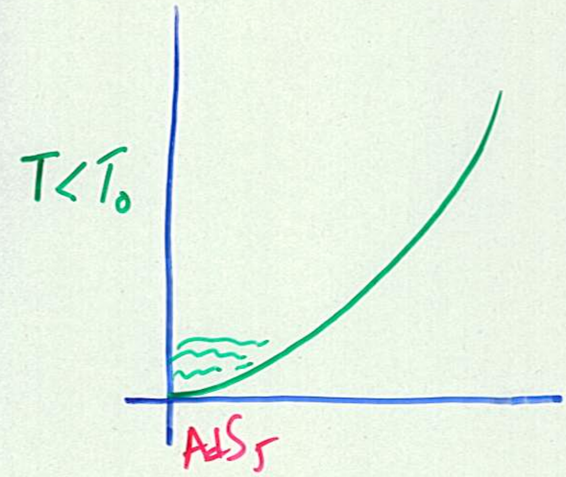
Canonical ensemble OK

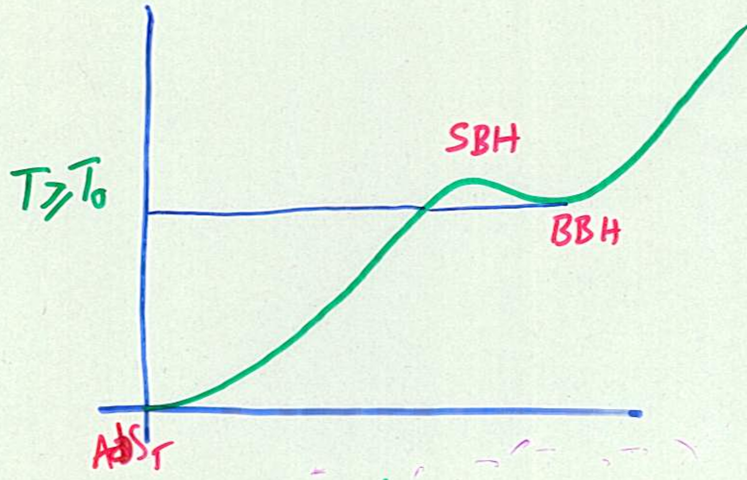
BBH $(r_+)_+ \gg R$, $S_{BBH} \sim M^{\frac{3}{4}} \dfrac{(R^2 \omega_4)^{1/4}}{\omega_4}$

SBH $(r_+)_- \ll R$, $S_{SBH} \sim M^{3/2} \left(\dfrac{\omega_4}{G_N}\right)^{1/2}$

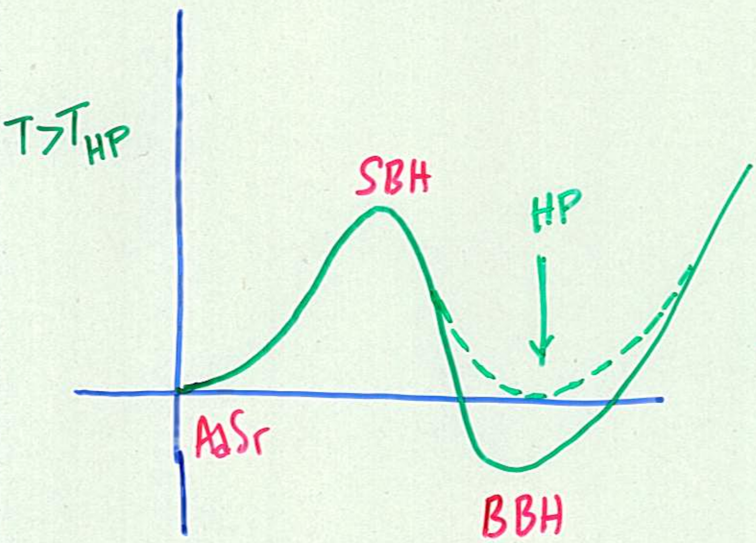
Schwarzschild bh in 4+1.

'Potential' picture in SUGRA :

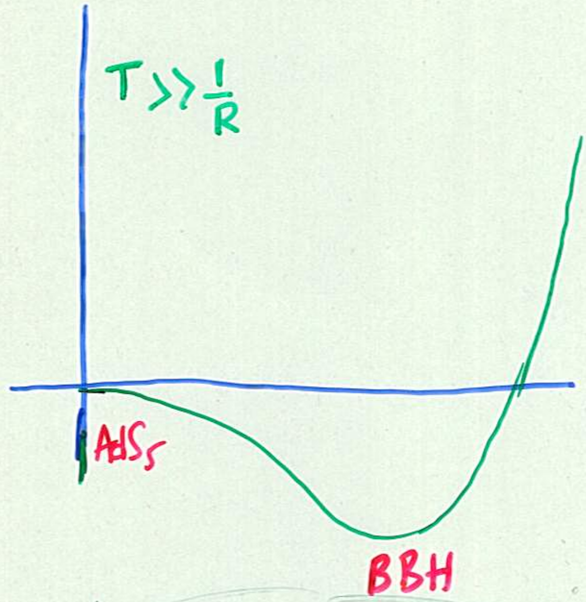




$T_0 \sim \frac{1}{R}$



$T_{HP} \sim \frac{1}{R}$



Phase Transitions in Gauge theories

Finite temperature $\Rightarrow$ $t \to it = \tau$, Euclidean

and $\qquad R^1 \to S^1 \qquad 0 \leq \tau \leq \beta = \frac{1}{T}$

Order Parameter: Polyakov line

$$U(\vec{x}) = e^{i\int_0^\beta A_0(\vec{x},\tau)\,d\tau} \in \underset{\text{fund. rep.}}{SU(N)}$$

$\frac{\mathrm{Tr}}{N} U^n(\vec{x})$ is gauge invariant, $n = \pm 1, \pm 2, \ldots$

Symmetry of order parameter:

Z_N is the center of $SU(N)$, $z_n = e^{i\frac{2\pi n}{N}}$

$g \in Z_N$, $\quad g = \mathrm{diag}\,[1, z_1, z_2, \ldots, z_{N-1}]$

Consider the global symmetry

$$U(\vec{x}) \to g\,U(\vec{x})$$

Action is invariant but $\langle \mathrm{tr}\, U \rangle$ is not.

(exactly like in ferromagnets)

Confinement - deconfinement transition

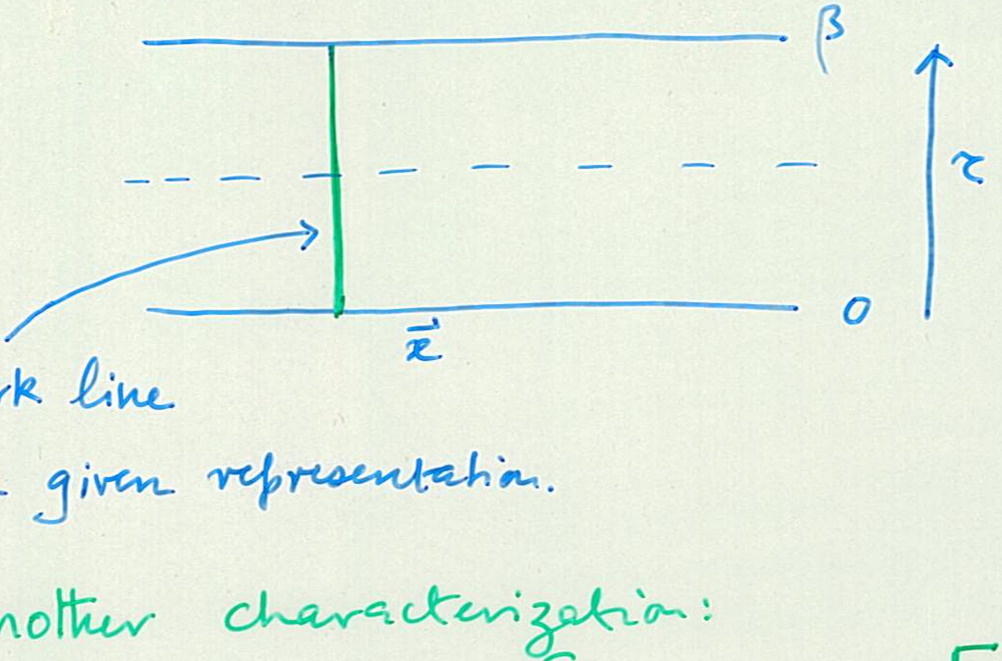
Confinement $\xrightarrow[T_c]{}$ deconfinement

$\langle \mathrm{tr}\, U \rangle = 0$ $\qquad\qquad$ $\langle \mathrm{tr}\, U \rangle \neq 0$

$\langle \mathrm{tr}\, U \rangle = e^{-\beta F_q}$ $\qquad\qquad$ $\langle \mathrm{tr}\, U \rangle \neq 0$

F_q = free energy of a single quark $\qquad$ $\Rightarrow F_q < \infty$.

$\langle \mathrm{tr}\, U \rangle = 0 \Rightarrow F_q = \infty$



quark line
in a given representation.

Another characterization:

$$Z = \int dA_\mu \, d\phi^a \, d\psi \,..\, e^{-S} \equiv e^{-\beta F(\beta)}$$

Confinement : $F \sim o(1) = N^0$

deconfinement : $F \sim o(N^2)$

Hawking-Page $\longleftrightarrow$ conf. - deconfinement.

deconfinement phase $\longleftrightarrow$ big blackhole of AdS_5

$F \propto N^2 \beta^{-4}$ $\qquad\qquad$ $\beta I_+ \propto \frac{1}{G_N} \beta^{-4}$

confinement phase $\longleftrightarrow$ $AdS_5 \times S^5$ (Thermal)

$F \sim 0$ $\qquad\qquad$ $F \sim 0$

Hawking-Page transition in which $AdS_5 \times S^5$ flops into the big blackhole of $AdS_5 \times S^5$ is the bulk description of the confinement-deconfinement transition which occurs for <u>large</u> $g_{YM}^2 N$

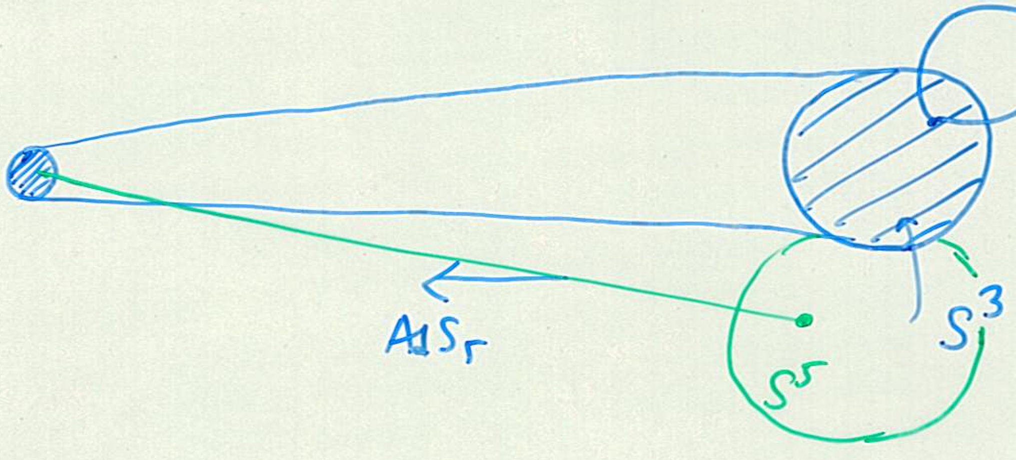
Further applications of AdS/CFT:

Type IIB string theory in $AdS_5 \times S^5$ has a very complicated landscape.

We have just studied the dominant saddle points at low and intermediate temperatures ($TR \ll 1$ + $TR \sim 1$)

Another phenomenon one can study is the small Schwarzschild bh ↔ 'strings' transition (Horowitz-Polchinski). no SUSY

- 10-dim. Small Schwarzschild bh can be embedded in $AdS_5 \times S^5$ (Horowitz-Hubeny)
- metric is uniform on asymptotic S^3

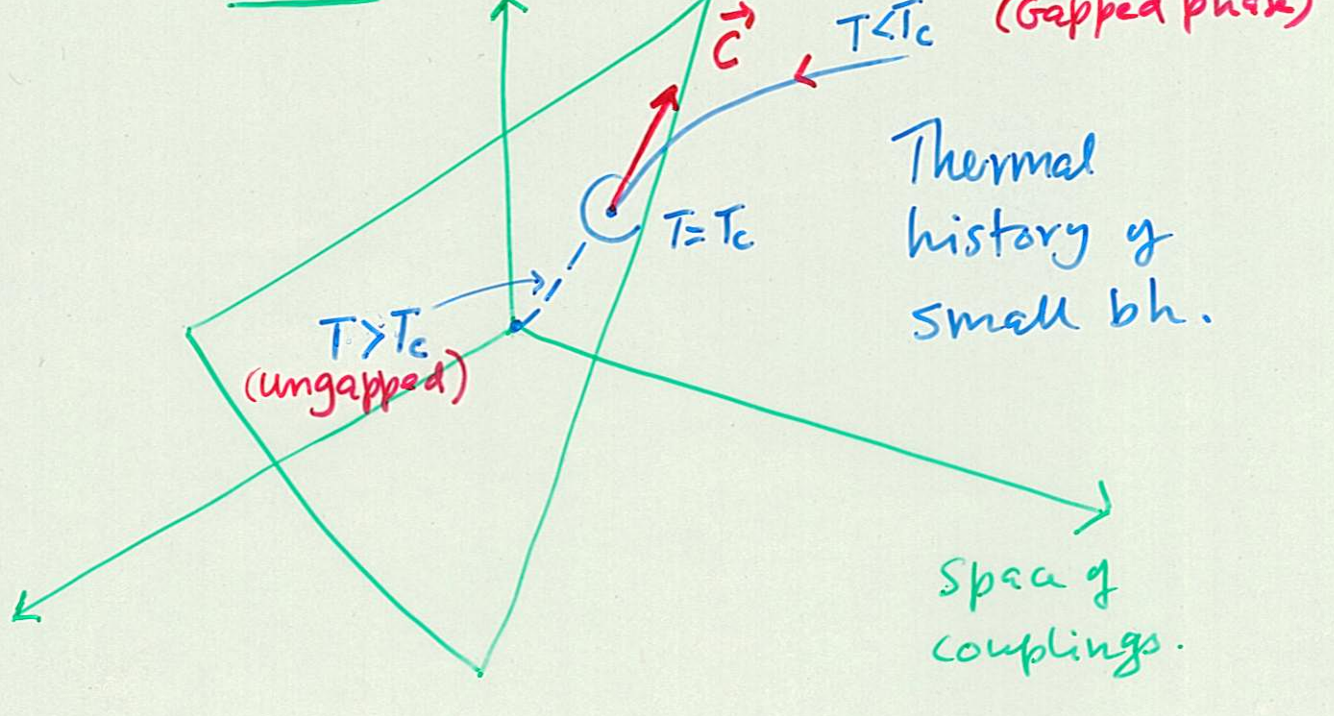


$\Rightarrow$ Constant Polyakov line on S^3.
(the non-constant part can be integrated out)
There are no Nambu-Goldstone modes associated with SO(6) symmetry breaking by the bh.

$\Rightarrow$ Effective lagrangian is a unitary matrix model

$$S_{eff}(U) = \sum_{k_1 \ldots} a_{k_1, k_2 \ldots} \, \mathrm{tr}\, U^{k_1} \mathrm{tr}\, U^{k_2} \ldots$$

This model can be exactly analysed in the large N limit in the vicinity of the critical <u>plane</u>



$$Z \sim e^{F(\vec{c}\cdot\vec{t})}$$

$\vec{c}$ is normal to the critical plane

$$\vec{t} = N^{2/3}(\vec{g} - \vec{g}_c)$$

$\downarrow$ ∞ $\quad$ $\downarrow$ 0

double Scaling limit at GWW transition

$$\vec{c}\cdot\vec{t} = t, \qquad \frac{\partial^2 F}{\partial t^2} = -f^2(t)$$

$$\frac{1}{2}\frac{\partial^2 f}{\partial t^2} = tf + f^3 \qquad \text{(Painleve II)}$$

$F(t)$ is smooth in the range $(-\infty, \infty)$.

Singularity of the Lorentzian bh seems resolved

Condensation of Polyakov lines at crossover:

$$N^{2/3}\left\langle \frac{\mathrm{tr}}{N} U^k \right\rangle_{sub} \sim C_k \frac{\partial}{\partial t} F(t).$$

$$\frac{\partial F(t)}{\partial t} \to 0 \quad \text{as } t \to -\infty$$

$$\frac{\partial F}{\partial t} \not\to 0 \quad \text{as } t \to +\infty$$

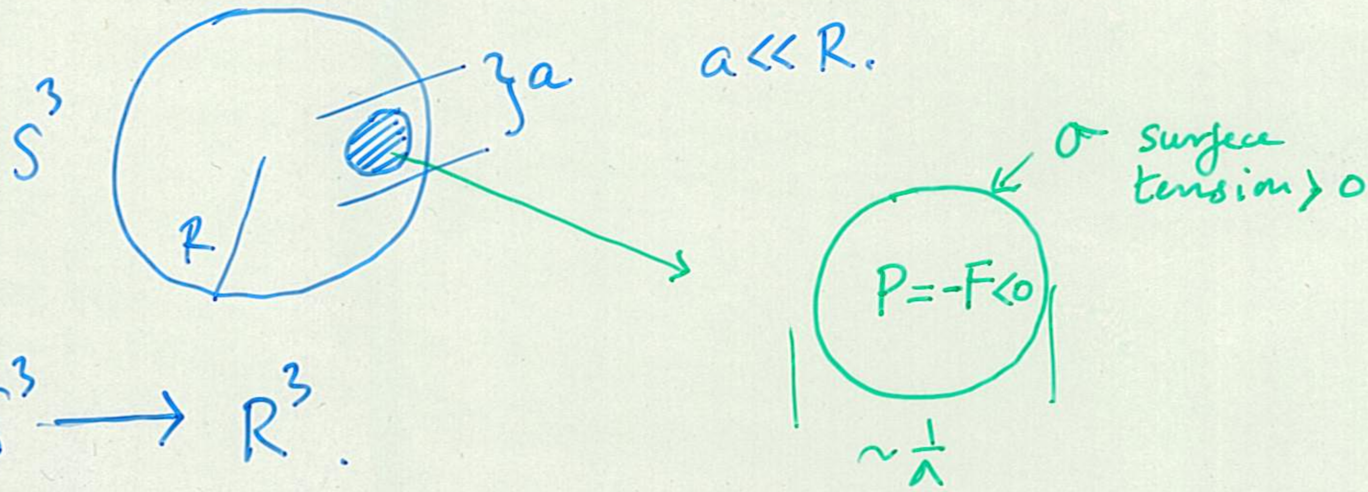
hepth: 0609052 / 0605041

Alvarez-Gaume
Basu
Marino
SRW

Plasma Balls

Till now we have discussed bhs in the context of a conformal gauge theory at finite temperature. Such bhs are uniformly spread over the S^3 in the boundary of AdS_5.

Now consider a confining gauge theory with an additional scale $\Lambda \equiv \frac{1}{a}$.



$S^3 \longrightarrow R^3$.

We can have a localized 'deconfinement' phase of size $a \sim \frac{1}{\Lambda}$, $T = T_c + \epsilon$, $M \sim O(N^2)$

(Aharony, Minwalla and Wisemann)

Explicit model:

$N=4$ gauge theory on $R^2 \times S^1_{2\pi} \times S^1_\beta$

both $S^1_{2\pi}$ + S^1_β have anti-periodic bd. conds. for the fermions (breaking SUSY)

Dual Picture:

Sherck-Schwarz compactification of AdS_5.

$$ds^2 = L^2\alpha' \left(e^{2u}\left[d\tau^2 + T_{2\pi}(u)\, d\theta^2 + dw_i^2\right] + \frac{1}{T_{2\pi}(u)}\, du^2 \right)$$

$$T_x(u) = 1 - \left(\frac{x}{\pi} e^u\right)^{-4}$$

AdS soliton

$$T_x(u_0) = 0$$

$$ds^2 = L^2\alpha' \left(e^{2u}\left[T_\beta(u)\, d\tau^2 + d\theta^2 + dw_i^2\right] + \frac{1}{T_\beta(u)}\, du^2 \right)$$

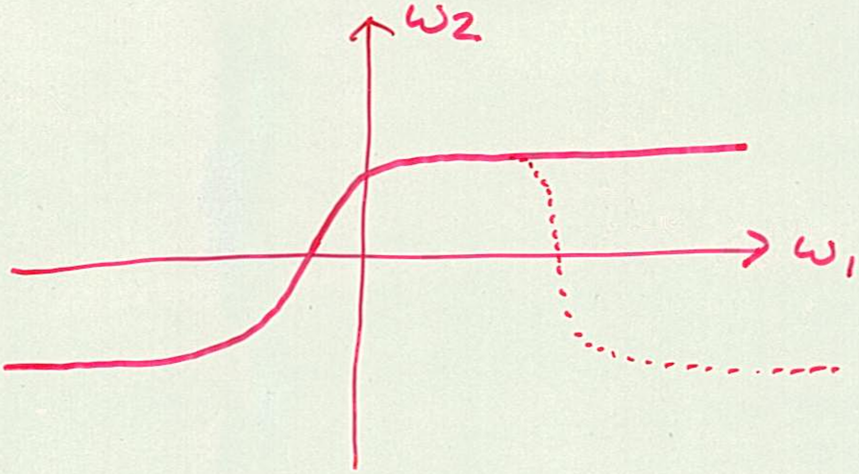
Black brane

$\beta < 2\pi$ AdS soliton dominate path integral

$\beta > 2\pi$ black brane dominates path int.

$\beta = 2\pi$ free energy =

$\exists$ interpolating solution in the non-compact directions (ω_1, ω_2) - numerical



The plasma ball whose inference comes from the bulk dual description can actually be constructed as a soliton of $M \sim O(N^2)$ directly in the gauge theory: 2 order parameters

$U_{2\pi}(\omega_1, \omega_2)$, $U_\beta(\omega_1, \omega_2)$ (Basu, Ezhuthachan, SRW)

$\rightarrow$ 1-dim many fermion problem.

"Precision test of String theory"

Small supersymmetric black holes:

Stringy corrections and finite horizon.

(Dabholkar, Kallosh, Maloney, Denef, Moore, Pioline, de Wit, Mohaupt, Cardoso, Ooguri, Strominger, Vafa)

Small (zero classical area bhs) in Heterotic string on $T^4 \times T^2$ or $\mathrm{II}A$ on $K_3 \times T^2$ — black holes of charges (P, Q)

Partition function

$$Z \sim e^{4\pi\sqrt{|PQ|}} \left(|PQ|\right)^{-\frac{27}{4}} \left(1 - \frac{675}{32\pi} \frac{1}{\sqrt{|PQ|}} + \cdots\right)$$

$$\sim \hat{I}_{13}\left(4\pi\sqrt{|PQ|}\right) \quad \text{modified Bessel fn.}$$

$$\Rightarrow S = \underbrace{4\pi\sqrt{|PQ|}}_{\downarrow} - \frac{27}{4}\ln|PQ| - \frac{675}{32\pi}\frac{1}{\sqrt{|PQ|}}$$

matched by calculating bh entropy (Wald) in string corrected Einstein gravity

$$\text{Action} \sim \frac{1}{l_P^8}\int\left(R + \text{"}l_s^2 R^2\text{"}\right)\sqrt{g} \quad \Rightarrow \quad S = 4\pi\sqrt{|PQ|}$$

Stringy bh sol. has a string scale horizon

$\odot\ l_s$

"classical" with area $\quad A = 8\pi\sqrt{|PQ|}$

$$S_{bh} = \frac{A}{4} + \frac{A}{4} = 4\pi\sqrt{|PQ|}$$

OSV conjecture* can be verified and all subleading corrections can be reproduced!

The above 'stringy' story can be repeated for other solutions like 'black rings'.

* generalizes Wald's entropy formula to quantum case.